Andrzej Czarski* , Piotr Matusiewicz*

# INFLUENCE OF MEASUREMENT SYSTEM QUALITY ON THE EVALUATION OF PROCESS CAPABILITY INDICES

## 1. INTRODUCTION

Statistical methods – statistical process control (SPC), statistical acceptance plans, reliability analysis, statistics in improvement of processes (ANOVA, DOE) etc. – belong to basic quality instruments. Statistical analysis can be performed in relation to measurable parameters or, on the grounds of alternative evaluation (yes/ no, conforming/ nonconforming). Apart from a kind, data being a basis of statistical analysis, must be the result of – let's determine this generally – a good measurement. It can be formulated, that without a good measurement there is no good statistics, performance of a statistical analysis requires valuable data achieved from measurement systems of adequate quality. Measurement system analysis (MSA) is concerned with the evaluation of quality of measurement systems. Here, it should be emphasized that, first of all, the statistical properties decide about the quality of a measurement system and its ability for processes' monitoring with sufficient accuracy.

In this work there are mutual relations between Statistical Process Control (SPC) and Measurement System Analysis (MSA) presented in the example of the influence of measurement system variability on the assessment of process capability indices.

## 2. STATISTICAL PROCESS CONTROL (SPC)

SPC is a proven and efficient method for: (1) current process monitoring relating to variability, (2) evaluation of process capability relating to variability with respect to expectations of a wide-understood client, and (3) continuous process improvement i.e. reduction of its variability. Information coming from SPC methods can be used for decisions at different levels of management (for planning, for designing, in reliability analysis etc.).

---

* Ph.D.: AGH University of Science and Technology, Faculty of Metals Engineering and Industrial Computer Science, Krakow, Poland; e-mail: czarski@agh.edu.pl

It is stressed widely that SPC methods are situated in an „avoiding loss" convention – information achieved by these methods we use „on-line" in order to assure a desirable performance of the process (instead of concentration on process effects that not always are desirable and expected, we focus on the process that generates those results!).

In a more detailed and practical approach the tasks of SPC are the following [1, 2]:

– Assessment of capability of a process/ machine relating to expectations of a client widely understood (a client can be the subsequent process) – in this range it determines capability indices of a process/ machine such as *Cp*, *Cpk*, *Pp*, *Ppk*, *Cm*, *Cmk*, *Tp*, *Tpk* etc.
– Current monitoring of a process in order to verify its stability in a statistical meaning, or meeting the specified requirements. To monitor the process the run charts are used and – first of all – the control charts (mainly Shewhart's control charts).

It should be emphasized a useful character of the SPC methods. There are two aspects of this usefulness. First, the SPC methods in a statistical aspect should be easy to use and comprehensible. Second, information coming from the SPC methods should be utilized maximally, that means we ought to draw the most conclusions and take maximum advantages.

## 3.  MEASUREMENT SYSTEM ANALYSIS (MSA)

As it has been already mentioned, first of all, the statistical properties decide about the quality and the utility of the measurement systems, because a measurement process is a process of producing numbers in variable conditions.

A good measurement should be correct and precise. The correctness of measurement means a lack of a systematic error (bias), while the precision is connected with dispersion of measurement results; the more precise the measurement, the less dispersion of measurement results.

Qualification of a measurement system relating to correctness should comprise the evaluation of a bias, linearity (linearity – the change in bias over the normal operating system) and stability (stability – the change in bias over time).

 Qualification of a measurement system relating to the precision comprises the repeatability evaluation (repeatability – variation in measurements obtained with one measuring instrument when used several times by an appraiser while measuring the identical characteristic on the same part) and reproducibility (reproducibility – variation in the average of the measurements made by different appraisers using the same ganse when measuring a characteristic on one part).

The simplest method of evaluation of a measurement system relating to the precision is a range method (so called the R method), but it does not allow for separating reproducibility and repeatability from total variability. It is possible thanks to the average and range method (it is so called R&R method) and the analysis of variance method (ANOVA); in practice the most popular is the R&R method [1, 5, 6].

The following criteria of assessment of measurement system capability are obligatory [5]:

- If the variability of the measurement system is under 10% of a process variability (or variability declared by specification limits) the measurement system is suitable without any restrictions.
- If the variability of the measurement system is from 10% to 30% of a process variability (or variability declared by specification limits) the measurement system is suitable conditionally (e.g. relating to costs and may be acceptable for some applications).
- If the variability of the measurement system is over 30% of a process variability (or variability declared by specification limits) the measurement system is not suitable to control the process (every effort should be made to improve the measurement system).

Let's give attention that in the presented criteria a variability of the measurement system relates to a process variability or a variability declared by specification limits. This is a very rational approach; the point is that a measurement system cannot be too good (because then it surely costs too much!) not bad (because then it does not "see" a process variability).

## 4. THE INFLUENCE OF MEASUREMENT SYSTEM VARIABILITY ON THE ASSESSMENT OF PROCESS CAPABILITY

On the basis of data resulting from the process achieved by using the specified measurement system, process capability indices $Cp$, $Cpk$ are determined as follows [1–4]:

- potential capability index $Cp$

$$Cp = \frac{USL - LSL}{6s_{total}} \qquad (1)$$

- actual capability index $Cpk$

$$Cpk = \min\left( \frac{\bar{x} - LSL}{3s_{total}}; \frac{USL - \bar{x}}{3s_{total}} \right) \qquad (2)$$

where:

$\bar{x}$ – mean value,
$s_{total}$ – standard deviation,
$LSL$, $USL$ – specification limits.

Standard deviation $s_{total}$ is a measure of a total variability, i.e. of a variability coming from the process and a variability coming from a measurement system:

$$s_{total}^2 = s_{process}^2 + s_{MSA}^2 \qquad (3)$$

where:

$s_{process}$ – standard deviation of a process; a measure of a process variability
$s_{MSA}$ – standard deviation of a measurement system; a measure of a measurement system variability.

In industrial conditions, a measurement system variability we assess using of the average and range R&R method [4, 5].

The index $Cp$ (equation 1) is determined on the basis of a total variability. Let $Cp_{process}$ mean an index of potential capability determined only on the basis of a process variability i.e. on the basis of $s_{process}$ (equation (1) – in place of $s_{total}$ we insert $s_{process}$). Dependence between the mentioned indices – in a context of variability measurement system and the R&R method presents as follows [5]:

$$Cp_{process} = \frac{Cp}{\sqrt{1-(GRR_{process}/100)^2}} \tag{4}$$

$$Cp_{process} = \frac{Cp}{\sqrt{1-Cp^2(GRR_{tolerance}/100)^2}} \tag{5}$$

where:

$GRR_{process}$ – percentage fraction of a measurement system variability ($s_{MSA}$) relating to a process variability ($s_{process}$), i.e. ($s_{MSA}/s_{process}$)·100%,

$GRR_{tolerance}$ – percentage fraction of a measurement system variability ($s_{MSA}$) relating to a variability declared by the width of specification limits $T$ ($T = USL - LSL$), i.e. ($6s_{MSA}/T$)·100%.

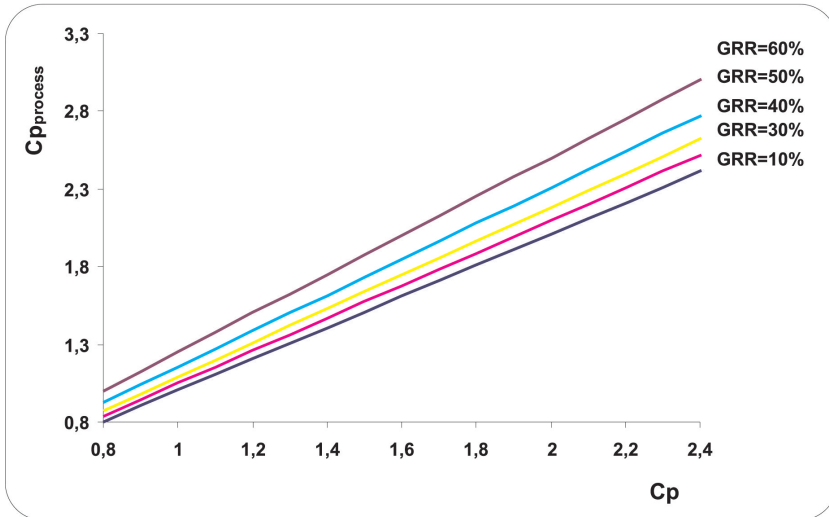Relationships (4), (5) are graphically presented respectively in Figure 1 and Figure 2.



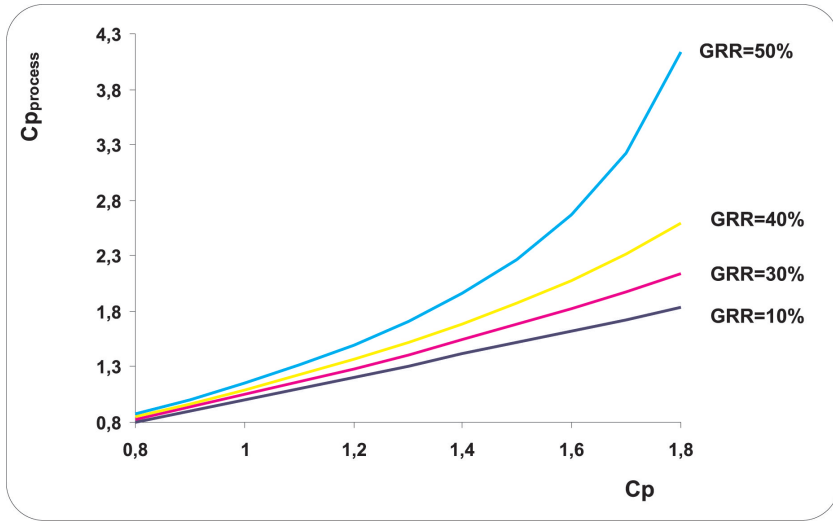*Fig. 1. Process capability (Cp_process) depending on a measurement system variability (GRR_process ), equation (4)*

**Fig. 2.** *Process capability (Cp$_{process}$) depending on a measurement system variability (GRR$_{tolerance}$), equation (5)*

## 5. EXPERIMENTAL PROCEDURE

### 5.1. Aim, scope, material for the tests

The aim of the tests was an assessment of the capability of the process of 41Cr4 steel drop forgings' heat treatment related to a hardness. Data for analysis came from a laboratory of mechanical and geometrical properties.

Measurements were made by means of a KP15002P Rockwell-Brinell's hardness tester. In relation to the measurement system used (hardness tester, operators etc.) the evaluation of repeatability and reproducibility was executed with the R&R average and range method (2 operators, 10 samples covering practically all ranges of hardness variability, 3 measuring trials). The results of the evaluation are demonstrated in Table 1.

**Table 1.** Results of the evaluation of repeatability and reproducibility, R&R method

| Parameter | Description | Value |
|---|---|---|
| *EV* | Equipment Variation | 0.230 |
| *AV* | Appraiser Variation | 0.320 |
| *GRR* | Repeatability&Reproducibility (= $s_{MSA}$) | 0.394 |
| *PV* | Part Variation | 1.260 |
| *GRR$_{process}$* | *(GRR/PV)*100%* | 31.3% |
| *GRR$_{tolerance}$* | *(GRR/T)*100% (T = USL − LSL); T = 10* | 23.6% |

### 5.2. Results of tests

In the meaning of results of a graphical test of normality (Fig. 3) there is no basis to reject the hypothesis that the analyzed parameter is subject to a normal distribution.

In relation to the parameter analyzed the following specification limits have been assumed $LSL = 30$ HRC, $USL = 40$ HRC. The results of the evaluation of process capability indices $Cp$, $Cpk$ (equation 1, 2) and – additionally – of the expected fraction of realization outside the p specification limits (assuming that the parameter is subject to a normal distribution) are presented in Figure 4.
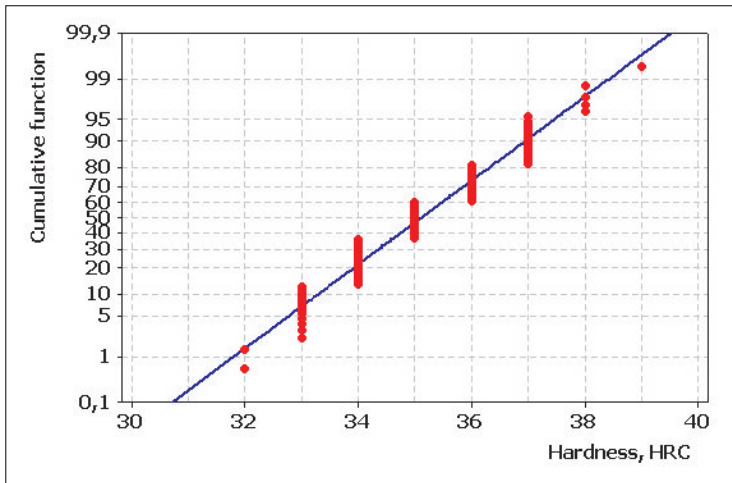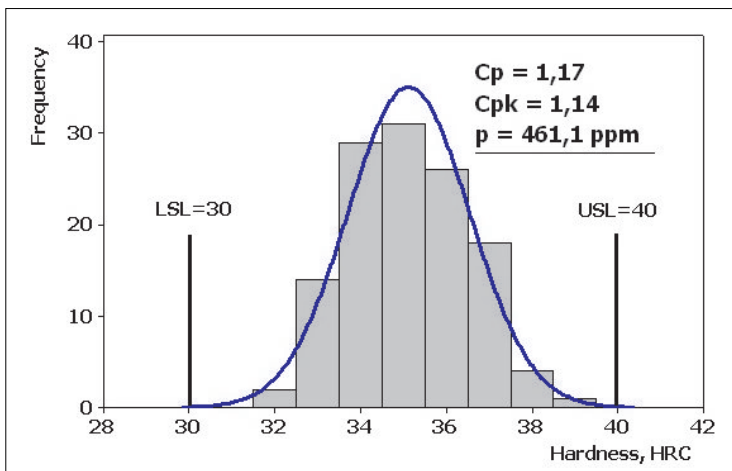


**Fig. 3.** *Normal probability plot*



**Fig. 4.** *Results of assessment of process capability; indices Cp, Cpk have been determined on the basis of equations (1), (2) respectively*

Considering the results of the evaluation of repeatability and reproducibility of the measurement system (Tab. 1) and expression (3) the $Cp_{process}$, $Cpk_{process}$ capability indices have been determined with taking into account only process variability (Tab. 2).

**Table 2.** Analysis of process capability with consideration of a measurement system variability

| Parameter | Description | Value |
|---|---|---|
| $s_{total}$ | Total variation | 1.423 |
| $s_{MSA}$ | Measurement system variation | 0.394 |
| $s_{process}$ | Process variation | 1.367 |
| $Cp$ | Potential capability, eq. (1), $s_{total}$ | 1.17 |
| $Cpk$ | Actual capability, eq. (2), $s_{total}$ | 1.14 |
| $Cp_{process}$ | Potential capability, eq. (1), $s_{MSA}$ instead of $s_{total}$ | 1.22 |
| $Cpk_{process}$ | Potential capability, eq. (2), $s_{MSA}$ instead of $s_{total}$ | 1.19 |

### 5.3. Discussion of results

The measurement system variability (the R&R method, Tab. 1) relating to process variability is quite large ($GRR_{process}$ = 31,3%). In the meaning of obligatory capability criteria (see: Chapter 3), if $GRR_{process}$ > 30% the measurement system is not capable of controlling the process; we are practically on a permission limit. Relating a variability of the measurement system to a variability declared by specification limits it brings $GRR_{tolerance}$ = 23,6%, so as for the control of a product the measurement system is conditionally capable (20% < $GRR_{tolerance}$ < 30%).

The measurement system variability comprises a variability from the equipment (repeatability) and a variability from the operators (reproducibility). In the analyzed case a component from reproducibility is a little larger comparing to a component from repeatability (Tab. 1, $EV$ = 0,31, $AV$ = 0,42); possible improvement actions should go to a reduction of variability from operators.

Process capability is not high, the process is practically centered (Fig. 4, Tab. 2, $Cp$ = 1,17, $Cpk$ = 1,14) which proves that a process setting is correct, however, its variability relating to expectations defined by specification limits could be lower; such a reduction of variability certainly requires undertaking some technical and technological actions.

Considering the rather high variability of the measurement system in comparison to the process variability and the variability declared by the specification limits, the potential of the process, only the process, goes off a little better relating to the variability (Tab. 3, $Cp_{process}$ = 1,22, $Cpk_{process}$ = 1,19), but differences between $Cp$ and $Cp_{process}$ as well as $Cpk$ and $Cpk_{process}$ are not significant, approximately 5%.

Summarizing, we can state that at observed process variability and given specification limits, the measurement system variability does not influence significantly the results the evaluation of process capability, and deducing and decision making on their basis.

## 6.  SUMMARY

The measurement system fulfills a service function relating to methods of statistical process control – provides with results, data for analysis. Each statistical analysis of a process within the framework of SPC should be accompanied by a conviction, even evidence, that the measurement system is capable of controlling this process. In consequence, it requires performing many assessments in the scope of an analysis of measurement systems i.e. MSA. In this way there is a particular relationship between MSA and SPC; without documented MSA analysis, in practice, it is not possible to take advantage of information delivered by SPC about the process to the full.

The evaluation of the process capability is one of the basic tasks of statistical process control. The analysis of the influence of the measurement system variability on the capability assessment performed in this work presents not only the mutual relationships between MSA and SPC, but also demonstrates a computational way of proceeding that can be used practically in such a type of analyses.

### REFERENCES

[1] *Montgomery D.C.*: Introduction to Statistical Quality Control, Third Edition, John Wiley & Sons Inc, New York-Singapore (1997)

[2] *Ryan T.P.*: Statistical Methods for Quality Improvement, A Wiley-Interscience Publication, Second Edition (2000)

[3] *Pyzdek T.*: Quality Engineering Handbook, Marcel Dekker, Inc. Second Edition (2003)

[4] Statistical Process Control (SPC), Reference Manual, Second Edition (2005) DaimlerChrysler Corporation, Ford Motor Company, and General Motors Corporation

[5] Measurement Systems Analysis (MSA), Reference Manual, Fourth Edition (2010) DaimlerChrysler Corporation, Ford Motor Company, and General Motors Corporation

[6] *Breyfogle F.W.*: Implementing Six Sigma, Smarter Solutions Using Statistical Methods, John Wiley&Sons, Inc. Second Edition (2003)