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STATISTICAL METHODS IN QUALITY MANAGEMENT – PROCESS CAPABILITY ANALYSIS

1. INTRODUCTION

Statistical methods belong to a canon of quality instruments and a formal requirement of their application is contained in each contemporary quality management system i.e. both in ISO standards of 9000 group and in branch standards coming from ISO standards as well as in every other "quality philosophy" (e.g. in the Six Sigma organization culture) [1, 2, 3].

In a practical approach the statistical methods in quality management are first of all: statistical process control (SPC), measurement system analysis (MSA), acceptance sampling techniques and process and product reliability assessment. Statistical methods take particular place in a process improvement; in this scope the following statistical instruments are applied: hypotheses and test procedures, analysis of variance (ANOVA), regression and correlation analysis, design of experiment (DOE) etc. [2, 4, 5].

This work concerns the statistical process control (SPC). The estimation results of process capability indices for steel forgings production are presented in this article.

2. CHARACTERISTIC OF SPC METHODS

It is stressed widely that SPC methods are situated in an „avoiding loss" convention – information achieved by these methods we use „on-line" in order to assure a desirable performance of the process (instead of concentrate on process effects that not always are desirable and expected, we focus on the process that generates these results!).

The basic tasks of SPC are the following [5, 6, 7]:

– Assessment of capabilities of a process/ machine relating to expectations of a client widely understood (a client can be the subsequent process) – in this range it determines capability indices of a process/machine.

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– Current monitoring of a process in order to verify its stability in a statistical meaning. The stable process in a statistical meaning is a predictable process, and a predictability – it does not require wide explanation – is a very important attribute. Relating to the predictable process we just know what expect, how it will behave, how much it will satisfy client's expectations etc. To monitor the process the run charts are used and – first of all – the control charts (mainly Shewhart's control charts).

SPC is a verified and effective method enabling continuous process improvement. Information coming from SPC methods can be used to make decisions on different management levels (for planning, designing, analyzing of reliability).

Writing about SPC methods their useable nature should be stressed. There are two aspects of this usability. First, SPC methods in a statistical aspect should be easy to use and comprehensible. Second, when we measure and assess anything, we should administer the achieved information maximally, that means we ought to draw the most conclusions and take maximum advantages of the information from measurement or assessment, even if it is the simplest.

3. CAPABILITY PROCESS ESTIMATION

3.1. Variability

Natural need of statistics comes from a variability. The variability is an inseparable feature of each process that is comprehended as a specified system converting inputs into outputs. Where is the variability in a process from – an operator, even very experienced, does not act twice exactly the same, machines work in certain tolerances, tools are wearing, raw materials come from different suppliers, and even they are from one supplier, they can be characterized by a certain heterogeneity etc. In consequence, even if the process is set up for the nominal value, for the sake of all impossible to be predicted factors generating variability, values of the considered parameter will change in a certain range and will not correspond exactly to the nominal value. In other words, each process is a random experiment – after generating the value we know what realization took place, before generating – we are not able to state what realization will occur.

So, as each process is a random experiment, the only possible way to describe it is statistics – statistics is a method for a description of variability. Let's say also clearly – a variability reduction is betterment of process quality!

3.2. Variability versus client's expectations

We consider the process from the point of view of a specified technological parameter or a product parameter deciding about quality. According to a terminology used in Six Sigma it can be a feature critical to costs (CTC), critical to satisfaction (CTS), critical to quality (CTQ), critical to process (CTP), critical to delivery (CTD) etc. [8]. The considered parameter is characterized by a determined variability, it is mathematically correctly a random variable. Disposing of a representative sample we can characterize statistically this parameter by determining the following parameters:

$\overline{x}$ – mean value; position measure,
s – standard deviation; dispersion measure.

Taking into consideration additionally that the considered parameter is subject to a normal distribution, we can define the scope in which practically all the realizations of this parameter should be situated. This scope, according to the rule of three standard deviations, corresponds to six standard deviations (6s, s – standard deviations). In SPC, the 6s scope is often defined as a so-called natural tolerance of the process [5].

Formulating this the most fundamentally, estimation of a process capability boils to a comparison of the process variability with the expected, tolerable variability defined with the aid of specification limits. The expression of this comparison is a series of different capability indices, among others capability factors of the so-called first generation C_p, C_{pk} (Tab. 1) [5, 9].

Table1. Capability indices of the first generation – calculation formulae

Parameters	potential capability $\mathbf{C_p}$	real capability $\mathbf{C_{pk}}$
double-sided specification limits	$\frac{USL - LSL}{6 \cdot s}$ (1)	$\min\left(\frac{\overline{x} - LSL}{3 \cdot s}; \frac{USL - \overline{x}}{3 \cdot s}\right)$ (2)
lower specification limit only	N / A	$\frac{\overline{x} - LSL}{3 \cdot s}$ (3)
upper specification limit only	N / A	$\frac{USL - \overline{x}}{3 \cdot s}$ (4)
Symbols: USL – upper specification limit, LSL – lower specification limit, N / A – not available		

The calculation formulae presented in the Table 1 are right when the analyzed parameter is subject to a normal distribution or at least has the distribution close to the normal one.

In accordance with the given definitions (Tab. 1) , there is a relationship $C_{pk} <= Cp$; if the process is centered i.e. the mean value $\overline{x}$ corresponds to a midpoint of a tolerance limit, then $C_{pk} = C_p$; in each other case $C_{pk} < C_p$. The value of C_{pk} factor does not depend on a direction of a process displacement.

In order to interpret process capability factors C_p, C_{pk} let's use the following analogy. In a production process a specified product is manufactured, in a measurement process we "produce" numbers. A good measurement should be correct and precise. From the point of view of correctness, a good measurement means the measurement without a systematic error. Precision is connected with a dispersion of measurement results round an unknown value of the measured parameter. At more precise measurement the dispersion of measurement results is the smallest. Because, regarding the variability, a measurement and production process can be treated the same, we can say also that a good process should be – a good measurement similarly – correct and precise, too. The relative measure of a process correctness is the real capability factor C_{pk}; if $C_{pk} = C_p$, the process is centered that means the expected realization of the process corresponds to the target (the midpoint of a specification limit). If $C_{pk} < C_p$, the position of the process is improper, the process is displaced.

The relative measure of a process precision is the potential capability factor C_p; the higher value of this factor, the lower variability of the process relating to the variability declared by specification limits.

Of course, there is an unequivocal relationship (for given distribution) between values of capability indicecs C_p, C_{pk} and a fraction of the realization outside specification limits (limit).

From the quality point of view, it is expected the most often that factors C_p, C_{pk} take values more than 1,33 in case of on-going processes and at least 1,67 in case of new processes [5]. Detailed definition of requirements belongs to the client.

3.3. Making use of capability factors

Relating to a client, process capability indices C_p, C_{pk} are, first of all, information on technological potential of a process, so they can be a criterion respecting selection of a producer. Relating to a producer, the factors can be a proof for process improvement or an alarm signal regarding a process performance.

4. EXPERIMENTAL PROCEDURE

4.1. Aim, material, experimental results

The aim of tests is estimation of a process capability indices C_p, C_{pk} for steel forgings production (steel S550, according to PN-EN 10137-2:2002) taking into account tensile strength R_m and deviation from a nominal Δ (Tab. 2). The measurement system which has been used for assessment of analyzed parameters fulfilled all criteria of ability.

Table 2. Analyzed parameters – quality requirements

Parameters	LSL, lower specification limit	USL, upper specification limit
R_m, tensile strength, MPa	640 *	820 *
Δ, deviation from nominal, mm	N / A	+ 0.5 **
*) according to PN-EN 10137-2:2002; **) according to client expectation; N / A – not available		

Data for analysis came from the laboratory of mechanical and geometrical features and concerned a representative period (Tab. 3).

Table 3. Statistical characteristic of data

Parameters	n	$\overline{x}$	s	x_{min}, x_{max}	S_k	K_u
R_m	186	747,76	15,27	708,6; 782,2	–0,84	–1,09
Δ	152	0,177	0,067	–0,006; 0,385	0,43	0,42
(n – simple size, – mean value, s – standard deviation, x_{min}, x_{max} – min, max value respectively, S_k – skewness, K_u – kurtosis)						

In the framework of estimation of the process capability, the factors indices C_p, C_{pk} and the fraction of realization outside specification limits have been determined (Figs. 1–4, Tab. 4).

Statistical analysis has been made with the application of STATGRAPHICS 5.1

Table 4. Results – capability indices C_p, C_{pk}

Parameters	C_p acc. to (1)	C_{pk} acc. to (2)	p(ppm)*	C_p 95% Confidence Interval	C_{pk} 95% Confidence Interval
R_m	1.965	1,577	1.11	(1.765; 2.165)	(1.409; 1.745)
Δ	N / A	1.603	0.76	N / A	(1.414; 1.792)
p = fraction nonconforming; ppm – part per million					

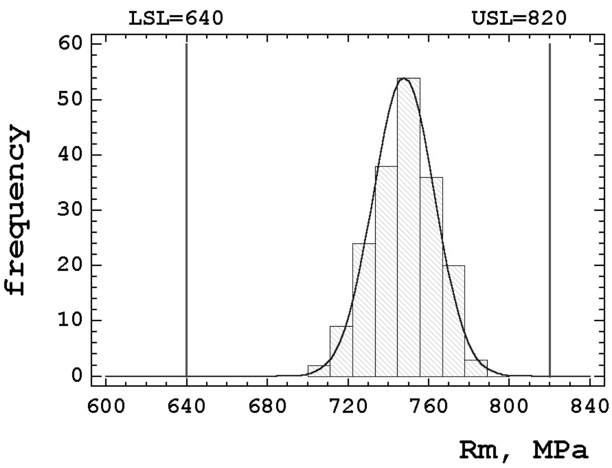

Fig. 1. Strength R_m – empirical distribution

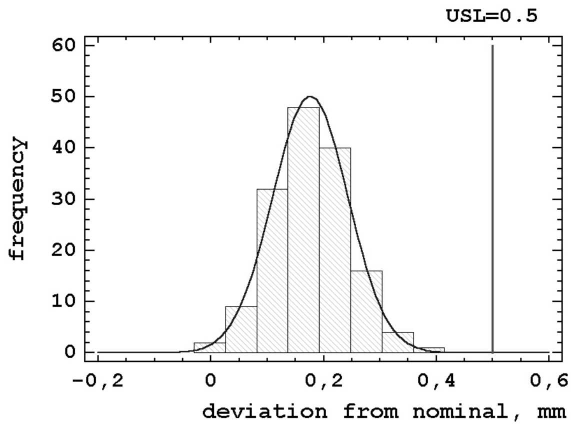

Fig. 2. *Deviation from nominal Δ – empirical distribution*

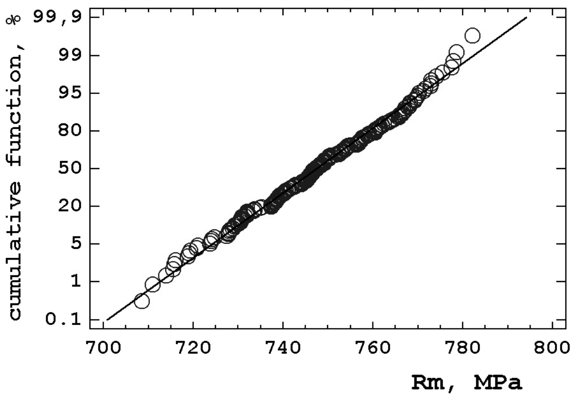

Fig. 3. *Strength R_m – normal probability plot*

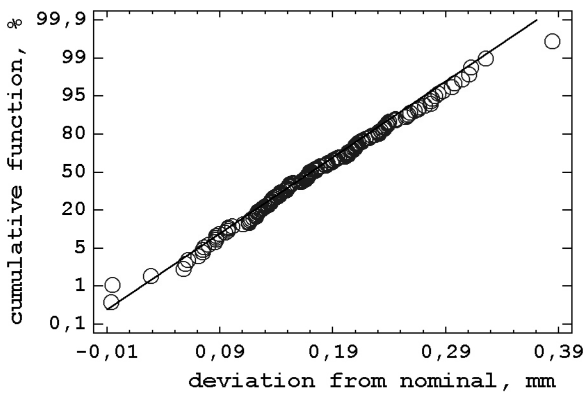

Fig. 4. *Deviation from nominal Δ – normal probability plot*

4.2. Discussion of results

The variability in case of both parameters analyzed is subject to normal distribution, that is indicated by shape parameters i.e. kurtosis and skewness (Tab. 3) and – first of all – the normal probability plot (Fig. 2, 4).

Tensile strength R_m. The process is distinctly shifted towards grater values (Fig. 1), therefore $C_{pk} < C_p$ (Tab. 4). The values of the indices C_p, C_{pk} (Tab. 4) are, from the client expectations point of view, satisfactory (the minimal expected value of C_{pk} was 1,5).

Deviation from nominal Δ. The value of the indice C_{pk} is very high (Tab. 4). From the client expectations point of view, the process is satisfactory (the minimal expected value of C_{pk} was 1,3).

5. SUMMARY

The estimation of a process capability is one of the basic tasks of the statistical process control. The process capability indices C_p, C_{pk} i.e. the capability indices of first generation are used in practice the most often. There are also the capability indices of second and third generation, besides in order to describe a process regarding variability some other capability indices are applied, such as – let's mention at least the names – performance indices P_p, P_{pk}, capability indices for unstable process T_p, T_{pk}. Whereas to assess the ability of measurement systems the capability indices C_g, C_{gk} are used.

The results presented in this article are the another proof of usability of statistical methods in engineering, and this should convince about necessity of statistical education in technical universities.

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