MECHANICS AND CONTROL
Vol. 34 No. 2 2015 http://dx.doi.org/10.7494/mech.2015.34.2.29

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# TIME-VARIANT FREQUENCY RESPONSE FUNCTION FOR ANALYSIS OF TIME-VARYING MECHANICAL SYSTEMS

*ABSTRACT*

*System Identification is an important and often complex process in many areas of engineering. This process is not easy when the parameters of the analyzed system vary with time. In such cases, classical methods fail to properly identify the parameters. The work demonstrated in this paper deals with the identification of natural frequencies in time-variant systems. The paper presents the application of the Time-Variant Frequency Response Function for this analysis. The calculation procedure requires division of the output spectrum by the input spectrum, which often leads to division by close-to-zero values; this leads to infinite (or undefined) values of the resulting transfer function. Additional processing is required for interpretation. The major focus and challenge relate to the ridge extraction of the above time-frequency characteristics. The methods presented in the paper are illustrated using an experimental multi-degree-of-freedom system. The results show that the proposed method correctly captures the dynamics of the analyzed time-variant systems.*

**Keywords:** *system identification, time-variant systems, natural frequency, wavelet analysis, Crazy Climbers algorithm*

*CZASOWO-ZALEŻNA WIDMOWA FUNKCJA PRZEJŚCIA DLA CZASOWO-ZALEŻNYCH SYSTEMÓW MECHANICZNYCH*

*Identyfikacja układów mechanicznych jest bardzo ważnym oraz często skomplikowanym zadaniem w wielu dziedzinach inżynierii. Zadanie to jest szczególnie trudne w przypadku, gdy dotyczy ono układów mechanicznych czasowo-zależnych. W takich przypadkach klasyczne metody oparte na analizach uniezależnionych od czasu zawodzą. Artykuł przedstawia identyfikację częstotliwości własnych układu mechanicznego czasowo-zależnego oraz zastosowanie czasowo-zależnej widmowej funkcji przejścia dla analizy tego typu. Wyznaczenie tej funkcji wymaga podzielenia czasowo-zależnego widma odpowiedzi przez widmo wymuszenia, które często prowadzi do dzielenia przez wartości bliskie zeru, a to prowadzi do bardzo dużych wartości wynikowej funkcji. Dodatkowe przetwarzanie sygnałów jest wymagane do dalszej interpretacji. Głównym celem tego artykułu jest estymacja grzbietów czasowo-zależnej widmowej funkcji przejścia. Metoda ta jest przedstawiona w artykule przy użyciu danych eksperymentalnych z systemu o wielu stopniach swobody. Otrzymane wyniki pokazują, że dzięki tej metodzie można poprawnie przedstawić dynamikę czasowo-zależnego układu mechanicznego.*

**Słowa kluczowe:** *identyfikacja układów mechanicznych, układy czasowo-zależne, częstotliwości własne, analiza falkowa, algorytm „Crazy Climbers”*

## 1. INTRODUCTION

The identification of natural frequencies and their in-operational changes is important for many engineering structures. It is well known that natural frequencies depend on structural mass and stiffness. Both of the physical parameters are often constant over the entire life of the structures. However, changes in mass and stiffness are quite common during the construction or operation of structures. A stiffness gain due to the concrete curing process over the time of construction or mass gain due to additional non-structural systems and components (e.g. precast cladding) can lead to changes in the natural frequencies that are often observed during the construction process (Memari *et al.* 1999, Ni *et al.* 2011). Sudden mass additions (due to structural redesign or placement of heavy interior equipment) and stiffness reduction (that builds up over a long period of time due to structural deterioration) are also common during the operation of many structures, and these result in changes in the natural frequencies (Butt, Omenzetter 2011). The dynamics of such phenomena can be easily captured using experimental modal analysis and operation modal analysis.

Various input-output and output-only approaches can be used for the identification of time-variant systems. Altogether, these approaches can be divided into two major groups: parametric and non-parametric methods. The former methods are based on models that are often of the form of polynomials with unknown coefficients. A good review of these methods is given in (Poulimenos, Fassois 2006; Spiridonakos, Fassois 2009). The latter group of methods for the analysis of time-variant systems

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– i.e., non-parametric – assumes no a priori information about such systems. These methods usually involve the analysis of both the time and frequency contents of excitation and/or response. Therefore, Cohen's class of time-frequency representations (Cohen 1995) provides a natural non-parametric framework for the non-parametric time-variant approaches. The continuous wavelet transform (Carmona *et al.* 1998) is a time-scale method that is qualitatively different from all other members. Wavelet-based methods employ different types of shifted and dilated wavelet functions to analyze and/or decompose signals. Various wavelet-based approaches have been developed for time-variant system identification, as reviewed in (Nagarajaiah, Basu 2009; Robertson, Basu 2009).

The wavelet-based FRF proposed in (Staszewski, Giacomin 1997; Staszewski, Robertson 2007) has been recently extended to give a theoretical background along with a physical interpretation (Staszewski *et al.* 2014) and to provide a full framework for modal analysis and system identification (Dziedziech *et al.* 2014; Staszewski, Wallace 2014; Dziedziech *et al.* 2015).

The structure of the paper is as follows. Classical time-invariant methods are given in Section 2. The wavelet spectra and wavelet-based FRF are briefly described in Section 3. Numerical implementation of the method used is presented in Section 4. The proposed methodology for detecting changes in natural frequencies of time-variant systems is illustrated in experimental examples in Sections 5. The experimental work utilizes the model of a three-floor-building structure. The results presented in Section 5 demonstrate the good performance and practicality of the approach used. Finally, the paper is concluded in Section 6.

## 2. ANALYSIS OF TIME-INVARIANT SYSTEMS

Different methods can be used for signal analysis. The two most-common approaches utilize time responses and power spectra. Analysis of a time response leads to information on signal amplitude and the localization of events in time. Simplicity is the major advantage of this approach. Relatively little signal post-processing is required to obtain basic information. In contrast, analysis in the frequency domain (based, for example, on the power spectra) provides information about the frequency content of the analyzed signal.

The well-known Fourier transform defined as:

$$X(\omega) = F\left[x(t)\right] = \frac{1}{2\pi}\int x(t)e^{-j\omega t}dt \tag{1}$$

can be used to obtain the power spectra. In modal analysis, the FRF defined as the frequency-domain ratio between output (or response) $Y(\omega)$ and input (or excitation) $X(\omega)$ is as follows:

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{F\left[y(t)\right]}{F\left[x(t)\right]} \tag{2}$$

This allows modal parameters (natural frequency, damping and mode shapes) to be estimated.

The major disadvantage of the classical method described in this section is the fact that the Fourier Transform is capable of properly analyzing only time-invariant signals. The application of this approach to time-variant systems may lead to incorrect FRFs and identified physical/modal parameters. This is the reason why other approaches are required.

## 3. ANALYSIS OF TIME-VARIANT SYSTEMS

Analysis of time-variant systems is a very challenging task due to the high complexity of the phenomena itself. Mechanical properties of systems can be observed in the frequency domain, while its time-variant nature requires observation of a system in the time domain. In order to observe the time-variant nature of mechanical systems, one needs to involve the advanced signal processing method, which enables us to analyze the system in both the time and frequency domains. There are a number of time-frequency methods that can be utilized for this purpose; e.g., Short Time Fourier Transform (STFT), Continuous Wavelet Transform (CWT). The work presented in this paper will utilize STFT, which can be defined as:

$$W(\tau,\omega) = \frac{1}{2\pi}\int x(t)w(t-\tau)e^{-j\omega t}dt \tag{3}$$

where $W(\tau, \omega)$ is the time-frequency spectrum of input signal $x$ and $w(t - \tau)$ is the sliding window that enables the localization of time.

A simple approach to the analysis of time-variant mechanical systems will utilize an analysis of the output signal's amplitude in the time-frequency domain. It is straightforward that a system's natural frequency will be located in the neighborhood of the highest values on a time-frequency distribution. For this purpose, the auto-power spectrum should be calculated as:

$$G_{YY}(\tau,\omega) = Y(\tau,\omega)Y^*(\tau,\omega) \tag{4}$$

where $G_{YY}$ is the auto-power spectrum of output signal $y$. Such an obtained time-frequency distribution does not require additional processing and can be interpreted directly; however, this can lead to the wrong conclusions, even if the excitation signal was white noise (Dziedziech *et al.* 2014). It is possible to overcome this problem (at a cost of additional post-processing) by defining the Time-Variant Frequency Response Function (TVFRF). The classical FRF can be intuitively extended for time-variant systems to provide time-frequency localization capability. When the analysis is limited to small periods of time that exhibit time-invariant behavior, time-variations are negligibly small. The TVFRF can then be defined as:

$$H(\tau,\omega)=\frac{Y(\tau,\omega)}{X(\tau,\omega)} \tag{5}$$

where $H(\tau, \omega)$ is TVFRF. Equation (5) presents a definition of TVFRF. The calculation procedure is a little bit different. More information about this ratio and its calculation procedure can be found in (Dziedziech *et al.* 2014; Staszewski, Wallace 2014; Dziedziech *et al.* 2015).

Equation (5) is relatively simple, yet it is often not easy to use in practice. For values of $X(\tau, \omega)$ that are close to zero, $H(\tau, \omega)$ tends to reach infinity. This makes it very difficult to interpret. That is why additional post-processing is required to avoid the problem above.

## 4. "CRAZY CLIMBERS" ALGORITHM

The TVFRF defined by Equation (5) does not involve any data averaging in the time domain. Additionally, when the data analyzed is noisy and close vibration modes are involved, the process of ridge extraction and ridge chaining can lead to significant numerical errors and a difficult interpretation. Various post-processing algorithms can be applied to avoid such difficulties. The so-called "Crazy Climbers" algorithm is one of the possible methods that can be used in practice. This method is based on Monte Carlo Markov Chain (MCMC) simulations. The main idea of the method is to use the TVFRF to generate a random walk on the time-frequency plane in such a way that the random walker is attracted by the ridges of the hills. In addition, the random walk is done at a given "temperature," which changes over time. The temperature is gradually decreased in the process, as in the simulated annealing algorithm. However, contrary to the simulated annealing procedure, the motion of the walker is unrestricted in one direction, and the walker is never stuck on a ridge. Thanks to the temperature schedule, each climber is expected to spend most of his time walking along one ridge involved or another. Therefore, there are a number of walkers instead of just one, and the entire procedure is suitable for multi-degree-of-freedom (MDOF) systems. Thanks to these random walks, one can create the so-called occupation measures. These occupation measures are created for each point on the time-frequency plane. They represent numbers of time when certain points were visited by walkers. The occupation measures are expected to have higher values near the ridges, as they are more attractive for the climbers. The "Crazy Climbers" algorithm has a great advantage in terms of possible mode separation when closed vibration modes are involved. A more detailed description of the entire algorithm can be found in (Carmona *et al.* 1998). When occupation measures are created, the results are chained to obtain skeletons. The chaining procedure consists of two major steps. First, a thresholding of the occupation measures is performed. Values below the pre-defined fixed value $\lambda$ are forced to zero; i.e.,

$$\rho(\tau,\omega_n)=\begin{cases}\rho(\tau,\omega_n, \textit{if } \rho(\tau,\omega_n)\geq\lambda\\ 0, \text{ otherwise}\end{cases} \tag{6}$$

where $\rho$ is the occupation measure obtained by means of the "Crazy Climbers" algorithm. The second step considers chaining the relevant ridges into a number of skeletons in such a way that the maximal points are connected together when moving along the time direction. The entire procedure allows for ridge extraction, which is essential for system identification.

## 5. EXPERIMENTAL VALIDATION

A simple experiment was conducted to obtain vibration data from a time-variant system. The system analyzed was the time-variant three-floor-building model shown in Figure 1. The building model consisted of three 10 × 10 × 0.3 cm plates connected with four 1 mm diameter rods. On top plate empty tank was installed into which 200 ml of water was poured in during the experiment, this forced change of structures mass. Three-floor building model was excited using an electrodynamic shaker with band-limited white noise excitation signal up to 25 Hz. PCB 288D01 impedance head was connecting electrodynamic shaker with the base of the three-floor building structure and was measuring the force and acceleration signals. PCB 333B30 accelerometer was attached to the second floor plate and was measuring acceleration signal. Sampling frequency was set to 200 Hz. Although a number of tests were conducted to check the repeatability, data from only one test was used for further analysis. In other words, averaging was not applied (as explained in Section 3).

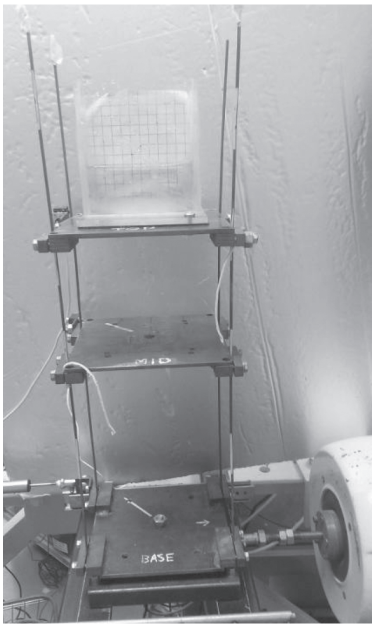


**Fig. 1.** Time-variant three-floor-building model

Before and after the time-variant experiment, a classical time-invariant experiment was conducted. Results of these experiments revealed, among others, one frequency mode of particular interest. This vibration mode was observed in the empty-tank state at 20.36 Hz and in the full-tank state at 20.17 Hz.

The data acquired during the time-variant experiment was subjected to a classical analysis. Figure 2 presents the classical FRF from time-variant experiment. A vibration mode of interest was found at 20.12 Hz. Clearly, a classical analysis failed to identify the time-variant nature of the structure.

After applying the procedures described in Section 3, the TVFRF was obtained (presented in Figure 3). Although a high level of uncertainty is present due to numerical problems with the calculation procedure, these characteristics provide a rough idea of the time-variant nature of the three-floor-building model.

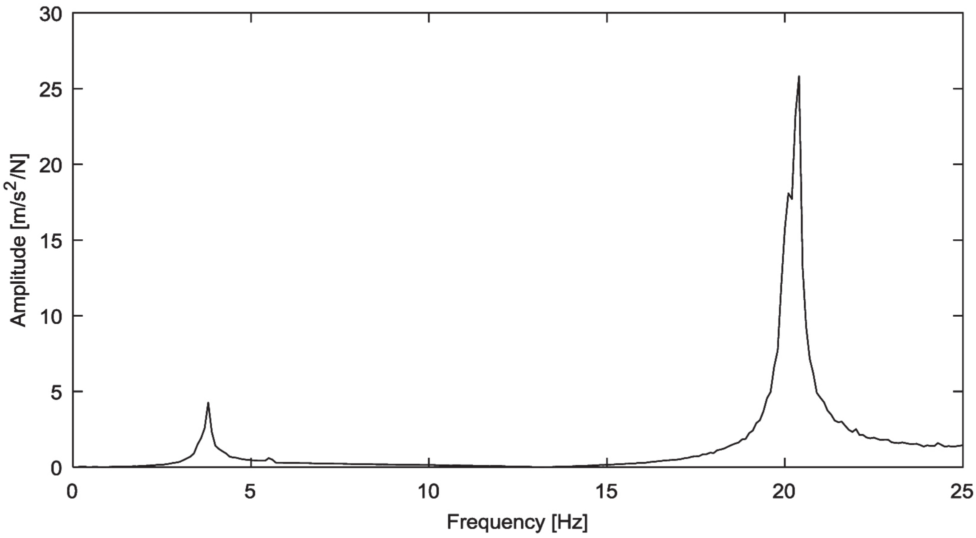


**Fig. 2.** Classical FRF limited up to 25 Hz from time-variant experiment

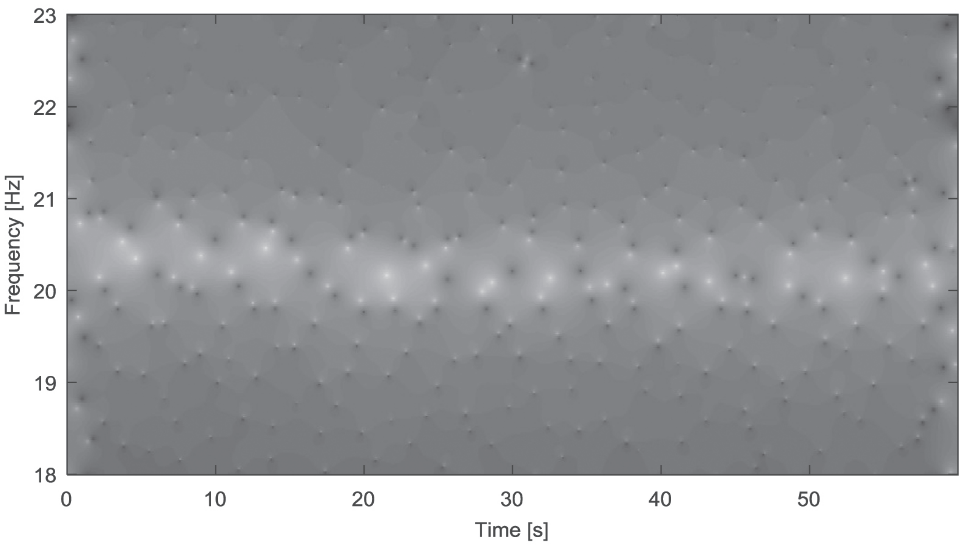


**Fig. 3.** TVFRF for considered system

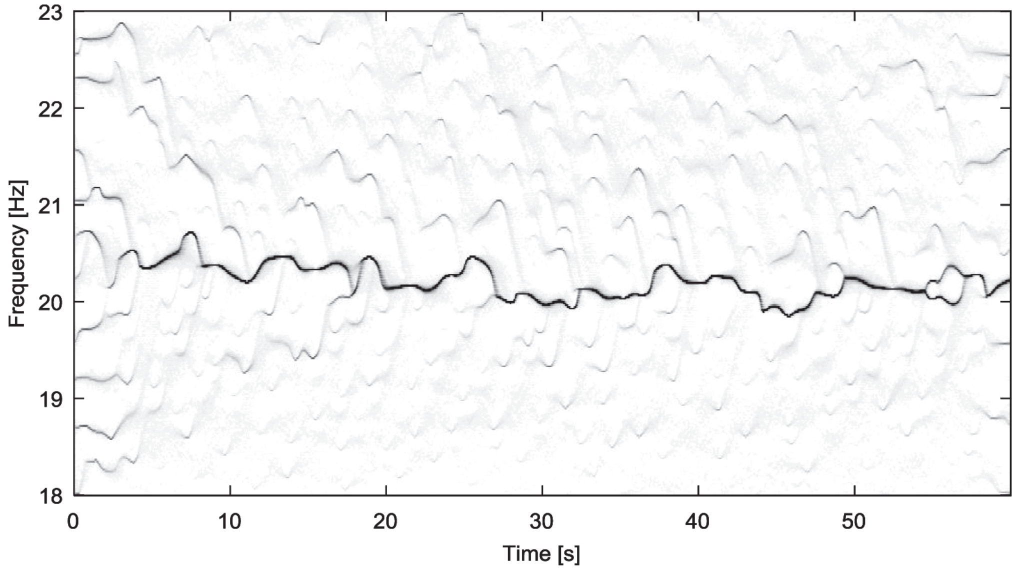


**Fig. 4.** Occupation measure for TVFRF shown in Figure 3

Calculation of the occupation measure with the use of the "Crazy Climbers" algorithm improved the localization of the natural frequency during the time-variant experiment as shown in Figure 4. At the beginning of the time-variant experiment, the natural frequency was found to be 20.4 Hz and decreased to 20.1 Hz over time. After comparing the results from the time-invariant experiments and time-variant experiment, one can conclude that the combination of the input-output analysis with the "Crazy Climbers" algorithm made it possible to properly capture the time-variant nature of the three-floor-building model.

## 6. CONCLUSIONS

The TVFRF based on the STFT was used for the identification of time-variant systems. The "Crazy Climbers" algorithm was applied to reveal the varying natural frequencies of a three-floor-building model. The results show that the method can be used to reveal the time-variant behavior of the system and to extract its varying natural frequencies. It is important to note that the identification performance of the method has been presented on one identified modal parameter.

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