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DAMPING OF THE WAVES MOTION IN A SPAN BY THE ACTIVE DISTRIBUTED FORCE

SUMMARY

In the paper the problem of optimal damping of the waves – traveling along the cable in a span and reflected – by the active distributed damping force segment located near the support is analyzed. The string equation is chosen as a model of the vibrating cable. As an objective function of the optimization problem the energy dissipated by the damper force is assumed. The damping algorithm is based upon the expression for the optimal concentrated damping force. The distributed force is assumed to be proportional to the component of the cable velocity resulting from the superposed motion of the original running and the original reflected waves. The original running waves are assumed in the form of packet waves with amplitude modulation. Illustrative numerical results demonstrating the optimal damping coefficients are included.

***Keywords**: cable vibration, running waves, active methods*

TŁUMIENIE RUCHU FAL W PRZĘŚLE ZA POMOCĄ AKTYWNEGO OBCIĄŻENIA CIĄGŁEGO

W pracy przedstawiono rozważania dotyczące optymalnego tłumienia fal - biegnących wzdłuż liny w przęśle i odbijanych – za pomocą aktywnego ciągłego pasma tłumiącego umiejscowionego w pobliżu podpory. Ruch liny opisano liniowym równaniem struny. Za funkcję celu przyjęto energię rozproszoną przez tłumik. Wykorzystując wyrażenie na optymalną skupioną siłę tłumiącą przyjęto ciągłe obciążenie tłumiące w paśmie proporcjonalne do składowej prędkości przekroju liny pochodzącej od sumy pierwotnej fali padającej oraz pierwotnej fali odbitej. Fale biegnące w linach przyjęto w postaci paczek falowych z modulowaną amplitudą. Przedstawiono wyniki obliczeń numerycznych optymalnych współczynników tłumienia w paśmie.

***Słowa kluczowe**: drgania przewodów, fale biegnące, metody aktywne*

1. INTRODUCTION

Long steel suspended cables, mainly used in suspension bridges and electric transmission lines (Snamina 2003), are structures with low damping prone to vibration induced by wind with moderate speeds combined with the presence of rain or ice on the cables. Large vibrations may cause cumulative fatigue damages to the cable assemblies. Increasing of the cable internal damping and using of the special dampers, damping loops and spacers are the main wave energy dissipation methods (Johnson *et al*., Pachero *et al*. 1993).

The forced motion is usually analyzed as a superposition of modes for short cables (Burgess and Triantafyllou 1988), whereas for very long ones, where the wavelength of waves is small relative to the cable span – superposition of the traveling waves is used (Perkins and Behbahani-Nejad 1995a, 1995b, 1996; Whitham 1999).

In the present paper the latter method is applied and the problem of optimal active distributed damping force necessary to suppress the waves traveling along the span and reflected from the support is considered.

2. CABLE MODEL AND THE EQUATION OF MOTION

As a model of the cable the string equation is taken

$$\mu \frac{\partial^2 u}{\partial t^2} - T \frac{\partial^2 u}{\partial x^2} = \tilde{q}(x, t) \tag{1}$$

with the general solution (without the components dependent on the initial conditions) for the case with the free ends in the form (Babicz and Kapilewicz 1970)

$$u(x, t) = \frac{1}{2c\mu} \int_0^t \int_{x-c(t-\tau)}^{x+c(t-\tau)} \tilde{q}(\xi, \tau)\, d\xi\, d\tau \tag{2}$$

In the expressions above the symbols used denote: μ – linear mass density, T – tension force, c – velocity of the traveling waves equal to $\sqrt{T/\mu}$, $\tilde{q}$ – the excitation load applied to the segment given by the expression (Fig. 1)

$$\tilde{q}(x, t) = \begin{cases} q(x, t) & 0 \le x \le L \\ 0 & x < 0,\ x > L \end{cases} \tag{3}$$

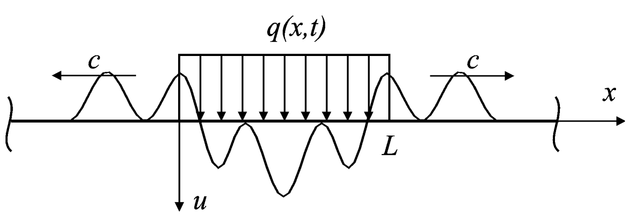

Fig. 1. Distributed force as a source of waves

For the purpose of numerical calculations of energies of the traveling waves it is convenient to utilize the expression for the cable velocity derived from the formula (2)

$$\frac{\partial u(x, t)}{\partial t} = \frac{1}{2\mu} \int_0^t \left[\tilde{q}(x - c(t-\tau), \tau) + \tilde{q}(x + c(t-\tau), \tau) \right] d\tau \tag{4}$$

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3. OPTIMIZATION PROBLEM

The general problem is to find the optimal way to suppress the waves traveling along the cable. In the paper (Łatas and Snamina 2007) it was shown that in the case of the concentrated damping force the optimal one carries out an action in the opposite direction to the component of velocity vector associated with the original wave and the force magnitude is proportional to the magnitude of the velocity components mentioned. The maximum dissipated energy acquired is the half of the original wave energy. This result was obtained with assumption that the damping force was far enough from the supports so the waves reflected from borders were neglected.

The similar strategy was proposed in the papers (Łatas and Snamina 2007; Łatas 2008a; Łatas 2008b) for suppressing the traveling waves by the active damping segment – the distributed force was assumed to be proportional to the component of the cable velocity resulting from motion of the original wave u_0

$$q(x,t) = -\alpha \frac{\partial u_0(x,t)}{\partial t} \tag{5}$$

where α denotes the damping coefficient which can be a constant value or, more generally, a function of x and t.

In the papers (Łatas and Snamina 2007; Łatas 2008a) the aim of calculations was to find the optimal value of the constant relative damping coefficient α/μ which maximized the energy dissipated in the active segment. Analogously in (Łatas 2008b) was carried out parametric optimization of the damping coefficient α assumed as a linear or segmentary linear function of x. Numerical results obtained proved that the distributed force damper prevailed over the concentrated force damper.

Basing upon the presented results the distributed active damper (shown schematically in Fig. 2) located near the support is considered. It is assumed that at $x = 0$ the cable is clamped.

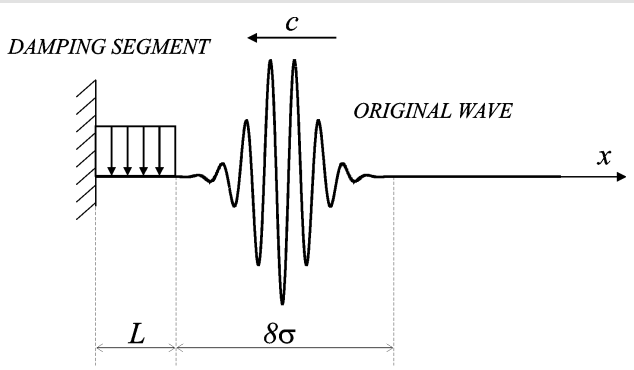

Fig. 2. Damping segment and the original wave for $t = 0$

Upon an absence of the damping segment the original, running toward the left, wave u_0 incident on the support would give rise to the original reflected, running toward the right, wave u_0^*.

The distributed damping force is assumed to be proportional to the component of the cable velocity resulting from the superposed motion of the waves u_0 and u_0^*

$$q(x,t) = -\alpha\left(\frac{\partial u_0(x,t)}{\partial t} + \frac{\partial u_0^*(x,t)}{\partial t}\right) \tag{6}$$

Because in the considered case $u(0, t) = 0$, in order to utilize the formula (2) and the expression (4) the function $\tilde{q}(x,t)$ has to be negatively extended for $x < 0$.

The loading applied to the cable by the active segment have the effect of injecting the secondary waves into the structure, running toward the left and right, which are transmitted (directly or after reflecting at the support) outside the segment. These secondary and secondary reflected waves superposing with the original reflected wave u_0^* finally form the total reflected wave.

The original wave used in numerical simulations was assumed in the form of packet wave with amplitude modulation, convenient in modeling of disturbances observed in cables

$$u_0(x,t) = \Psi_0 \sin\left(\frac{2\pi}{\lambda}(x + ct - L - 4\sigma)\right) \cdot \exp\left(-\frac{(x + ct - L - 4\sigma)^2}{4\sigma^2}\right) \tag{7}$$

It is postulated (regarding a shape of the packet wave function) that a width of the packet wave with non-zero amplitude of motion necessary to consider is 8σ, so the above expression displays the wave (presented in Fig. 2) running to the left and reaching the damping segment at the moment $t = 0$.

Employing the reflection law to the original wave (7) the original reflected wave is given by

$$u_0^*(x,t) = -\Psi_0 \sin\left(\frac{2\pi}{\lambda}(ct - x - L - 4\sigma)\right) \cdot \exp\left(-\frac{(ct - x - L - 4\sigma)^2}{4\sigma^2}\right) \tag{8}$$

To determine the energy E_D dissipated in the damping segment the energy of the total reflected wave E_T has to be calculated

$$E_T = \int_0^{\frac{8\sigma+4L}{c}} \sqrt{T\mu}\left(\frac{\partial u(L,t)}{\partial t} + \frac{\partial u_0^*(L,t)}{\partial t}\right)^2 dt \tag{9}$$

The energy balance furnishes with

$$E_D = E_O - E_T \tag{10}$$

where E_O denotes the energy of the original wave

$$E_O = \int_0^{\frac{8\sigma}{c}} \sqrt{T\mu}\left(\frac{\partial u_0(L,t)}{\partial t}\right)^2 dt \tag{11}$$

The dissipation efficiency η is defined as a ratio of the energy dissipated by the damping segment to the energy of the original wave

$$\eta = \frac{E_D}{E_O} = 1 - \frac{E_T}{E_O} \tag{12}$$

For a given length of the active segment, limited due to the technological restrictions, the damper should be efficient in a wide range of the packet wavelength λ and the packet width parameter σ, so the optimization problem may be described using the mean dissipation efficiency defined as

$$\eta^*(\tilde{\alpha}) = \frac{\iint \eta(\tilde{\alpha}, \bar{\sigma}, \bar{\lambda}) W(\bar{\sigma}, \bar{\lambda}) d\bar{\sigma} d\bar{\lambda}}{\iint d\bar{\sigma} d\bar{\lambda}} \tag{13}$$

where: $\bar{\sigma} = \frac{\sigma}{L}, \bar{\lambda} = \frac{\lambda}{L}, \tilde{\alpha} = \frac{\alpha}{\mu}$.

The weighing function W may be related, for example, to the probability distribution of the packet wave parameters. The aim is to find the optimal value of the relative damping coefficient, denoted by $\tilde{\alpha}^*_{OPT}$ [1/s], maximizing the mean dissipation efficiency given by formula (13).

4. NUMERICAL RESULTS

Figure 3 illustrates the dissipation efficiency vs. the relative damping coefficient obtained for different values of $\bar{\sigma}$ and $\bar{\lambda}$.

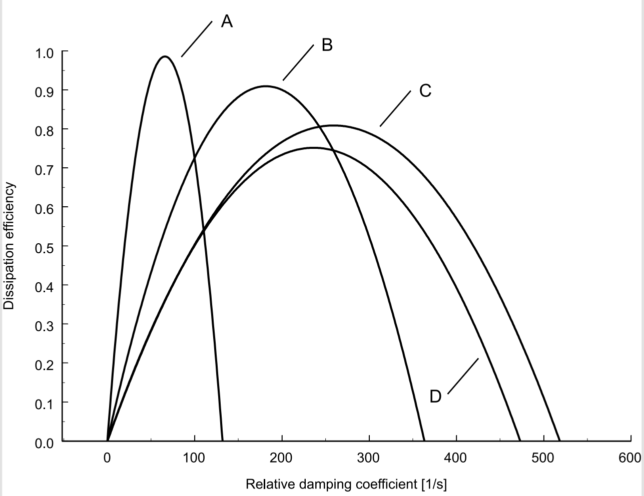

Fig. 3. Dissipation efficiency η vs. relative damping coefficient $\tilde{\alpha}$: A) $\bar{\sigma} = 6, \bar{\lambda} = 6$; B) $\bar{\sigma} = 5, \bar{\lambda} = 12$; C) $\bar{\sigma} = 4, \bar{\lambda} = 18$; D) $\bar{\sigma} = 3, \bar{\lambda} = 24$

It occurs that the dissipation efficiency as a function of the damping coefficient has a maximum.

Table 1 shows the illustrative examples of the optimal values of the relative damping coefficient, denoted by $\tilde{\alpha}_{OPT}$ [1/s], altogether with the maximal dissipation efficiency η_{MAX} obtained for different values of $\bar{\sigma}$ and $\bar{\lambda}$.

Figures 4 and 5 demonstrate in detail the optimal values of the relative damping coefficient and the maximal dissipation efficiency vs. the dimensionless wavelength $\bar{\lambda}$ for different values of the packet wave width parameter $\bar{\sigma}$.

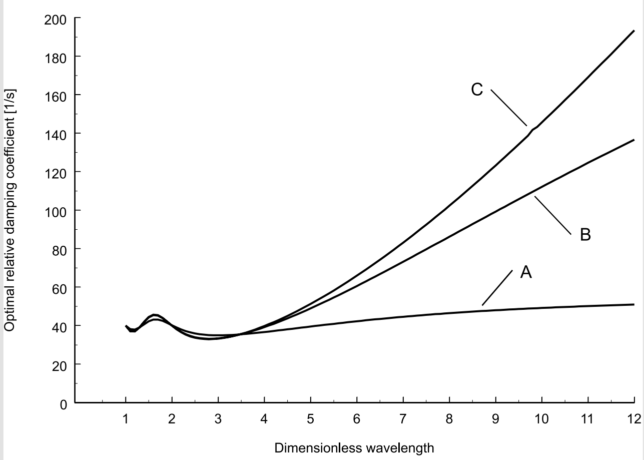

Fig. 4. Optimal relative damping coefficient $\tilde{\alpha}_{OPT}$ vs. dimensionless wavelength $\bar{\lambda}$: A) $\bar{\sigma} = 1$; B) $\bar{\sigma} = 3$; C) $\bar{\sigma} = 6$

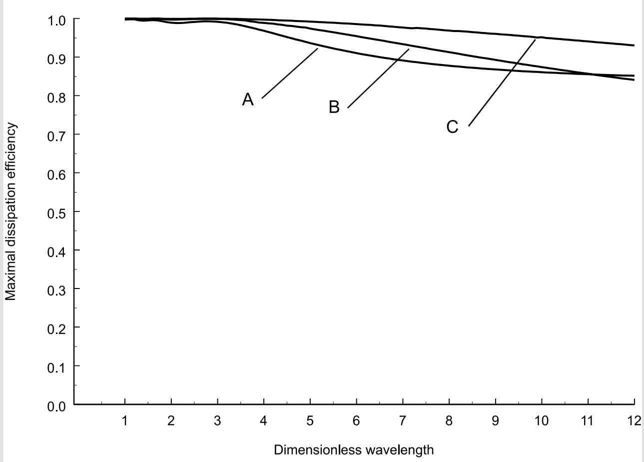

Fig. 5. Maximal dissipation efficiency η_{MAX} vs. dimensionless wavelength $\bar{\lambda}$: A) $\bar{\sigma} = 1$; B) $\bar{\sigma} = 3$; C) $\bar{\sigma} = 6$

As an additional example optimization of the function (13) with the weighing function W equal to 1.0 and in the area $[\bar{\sigma} \in \langle 1, 6 \rangle] \times [\bar{\lambda} \in \langle 1, 6 \rangle]$ was carried out

$$\eta^*(\tilde{\alpha}) = \frac{\int_1^6 \int_1^6 \eta(\tilde{\alpha}, \bar{\sigma}, \bar{\lambda}) d\bar{\sigma} d\bar{\lambda}}{25} \tag{14}$$

Table 1
Optimal relative damping coefficient and maximal dissipation efficiency

	$\bar{\lambda} = 1$		$\bar{\lambda} = 3$		$\bar{\lambda} = 6$	
$\bar{\sigma} = 1$	$\tilde{\alpha}_{OPT} = 40$	$\eta_{MAX} = 0.997$	$\tilde{\alpha}_{OPT} = 35$	$\eta_{MAX} = 0.991$	$\tilde{\alpha}_{OPT} = 42$	$\eta_{MAX} = 0.910$
$\bar{\sigma} = 3$	$\tilde{\alpha}_{OPT} = 40$	$\eta_{MAX} = 0.999$	$\tilde{\alpha}_{OPT} = 33$	$\eta_{MAX} = 0.999$	$\tilde{\alpha}_{OPT} = 61$	$\eta_{MAX} = 0.954$
$\bar{\sigma} = 6$	$\tilde{\alpha}_{OPT} = 40$	$\eta_{MAX} = 0.999$	$\tilde{\alpha}_{OPT} = 33$	$\eta_{MAX} = 0.999$	$\tilde{\alpha}_{OPT} = 66$	$\eta_{MAX} = 0.985$

giving the following results:

$$\tilde{\alpha}^*_{OPT} = 39.0 \text{ 1/s}$$

$$\eta^*_{MAX} = 0.984$$

5. CONCLUSIONS

Numerical calculations performed confirm the evident fact that the maximal dissipation efficiency tends to 1.0 with $\bar{\lambda} \to 0$, it means with the length of the damping segment tending to infinity. Even a relatively short damping segment (compared to the packet wavelength λ and to the packet width parameter σ) occurs to be very effective – the maximal dissipation efficiency ranges from 0.841 to 0.999 in the area $[\bar{\sigma} \in \langle 1,6 \rangle] \times [\bar{\lambda} \in \langle 1,12 \rangle]$. For a limited length of the damping segment it is possible to adjust the damping coefficient so the damper is effective in a given ranges of the packet wave parameters $\bar{\lambda}$ and $\bar{\sigma}$.

The results obtained for continuous damping segment may help to find the control algorithm for optimal suppressing the traveling and reflecting waves by the discrete system of concentrated forces.

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