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AN APPLICATION OF RESPONSE SURFACE METHOD
TO DESIGN OPTIMIZATION OF A MODEL OF RAIL VEHICLE
CONSIDERING UNCERTAINTIES

SUMMARY

An application of design optimization methodology performed for a multibody model of real-live rail structure stands for the scope of present work. For carried out analyses a model of five-piece tram has been elaborated and then parameterized to allow for an effective and easy change of checked design configuration. There has been assumed the index of travelling comfort as an object of optimization process. Performed analyses have taken into account the uncertainty dealing with the number of passengers on board. The stiffness coefficients of springs of the primary and secondary suspension systems have been chosen as design parameters. Mentioned index has been calculated with virtually measured accelerations in the passengers area for assumed velocity of a run. For modeled rail there has been introduced roughness defined to represent real geometric imperfections. Response surface modeling based on polynomial regression has been applied as a surrogate for the full model to speed up the dynamic analyses. The work has used genetic algorithms for the optimization. Improved values of studied index confirm better comfort of traveling.

Keywords: *response surface method, metamodel, rail vehicle, tram, suspension system, travelling comfort, multibody model, design optimization, uncertainty propagation, genetic algorithms*

ZASTOSOWANIE METODY POWIERZCHNI ODPOWIEDZI
W OPTYMALIZACJI MODELU POJAZDU SZYNOWEGO Z UWZGLĘDNIENIEM NIEPEWNOŚCI

Artykuł przedstawia wyniki optymalizacji przeprowadzonej dla modelu pięcioczłonowego tramwaju opracowanego z zastosowaniem metody układów wieloczłonowych. Jako cel zadania optymalizacji przyjęto poprawę komfortu podróżowania ocenianego współczynnikiem sformułowanym zgodnie z wytycznymi normy UIC 513. Zbudowany model tramwaju po sparametryzowaniu pozwolił na ocenę własności dynamicznych dla wybranych konfiguracji projektowych, a w szczególności zbadanie wpływu zmian parametrów charakteryzujących układ zawieszenia, przy uwzględnieniu niepewności związanych z masą pasażerów, na zmienność przyspieszeń drgań w przestrzeni pasa-żerskiej. W modelu uwzględniono nierówności torów reprezentujące rzeczywistą zmienność ich geometrii zgodnie z normą ORE B176 RP1. W celu przyspieszenia obliczeń zastosowano metodę powierzchni odpowiedzi z wielomia-nową aproksymacją współczynnika komfortu. W optymalizacji zastosowano algorytmy genetyczne. Dla znalezio-nych wartości parametrów projektowych charakteryzujących układ zawieszenia uzyskano poprawę sformułowanego współczynnika.

Słowa kluczowe: *metoda powierzchni odpowiedzi, metamodel, pojazd szynowy, tramwaj, układ zawieszenia, komfort podróżowania, model układów wieloczłonowych, optymalizacja konstrukcji, propagacja niepewności, algoryt-my genetyczne*

1. INTRODUCTION

Virtual prototyping procedures stand for a key point while launching a new rail vehicle. Wide range of possible kinds of numerical simulations allows for comprehensive analyses determining safety, performance as well as comfort of travelling (Beretta *et al*. 2005; Martowicz *et al*. 2009a; Uhl and Chudzikiewicz 2002). As far as save run is of concern there have to be satisfied respective requirements mainly dealing with strength and run stability. On one hand designed construction should prevent from fatigue failure that may occur during scheduled period of operation and applied loads. On the other hand properties of contact area established between wheel and rail, i.e. including their curvatures etc., and characteristics of suspension systems have to guarantee the stability of running vehicle for assumed speed, radius of curves and rail roughness. The second issue is related to the vehicle performance. The lighter vehicle the less power is required to control its speed. Moreover the shape of body may be a key point especially in case of high-speed trains. The application of materials characterizing less mass density and still acceptable strength may also help to reduce power consumption. Eventually, the comfort of travelling can be improved in terms of the reduction of acoustic emission and vibration. Mentioned above three aspects of design process should be taken into account for save, fast and comfort travel.

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In the work there are presented the results of optimization task performed to increase the travelling comfort for a multibody model of five-piece tram. Determined index of travelling comfort, called as comfort index, has been taken as the object of optimization (standard UIC 513 1994). Basically it has been defined with the values of virtually measured accelerations in the passengers area in all sections and therefore is assumed to stand for the assessment that could be formulated by passengers. In case of vibrations felt by passengers the issue of comfort is a matter of subjective assessment rather than objective one, however indexes based on measured acceleration of vibrations seem to be quite conclusive quantities (standard UIC 513 1994). The elaborated parameterized multibody model has allowed for the change of stiffness coefficients of springs mounted in suspension systems. Transient analyses have been performed to simulate the run of tram with constant velocity only. There has been assumed for present work that neither accelerating nor braking is taken into account. The model of rail has included roughness. Time-history plots of vibration acceleration have been registered for each checked design configuration and then applied to calculate the value of comfort index. Apart from two design parameters characterizing properties of suspension systems there has been considered the uncertainty on the number of passengers on board. Each design configuration has been checked in terms of the influence of variation of mass of passenger on the comfort index. The worst case scenario has always been taken into account with the highest value of index which, on the contrary, means the lowest comfort.

The task of design optimization usually requires many configurations of design to be checked. This appears most frequently in cases when relationships between design parameters and optimized quantities are not monotonic. Therefore effective search through the input design domain cannot be performed on the basis of fast gradient-based method merely. Eventually it results in considerably great number of dynamic analyses which have to be carried out depending on applied optimization method. The application of response surface method (RSM) may stand for a solution and enables for significant reduction of the number of necessary time-consuming analyses (Box and Draper 1986; Gallina, Martowicz and Uhl, 2006; Myers and Montgomery 1995). Metamodels created during fitting procedures approximate relationships between input, i.e. design, and output parameters, i.e. being optimized. Moreover RSM allows for the calculation of sensitivity of optimized quantities on the variation of design parameters. Mentioned feature additionally enables for the search of the most influential parameters which at first should be taken into account while optimization. The application of RSM in present work has allowed to find metamodels with a limited number of the simulations used to determine the properties of the multibody model of tram. The important issue that should be faced while applying RSM is however the type of approximation functions used to build the metamodel. Incorrectly selected structure of the approximation formula may result in artifacts. Crude form of metamodel, in turn, artificially smoothes the shape of response and may prevent from finding of global optimum. The study also covers this topic. A number of metamodels including different regressors have been applied to approximate the variation of optimized parameter. Genetic algorithms (GA) have been used to select best metamodel depending on assumed number of included regressors. Chosen metamodels have been applied for the optimization task. Best configurations characterizing improved value of the comfort index have been found again with the applications of GA. Determined metamodels have also been used to assess the propagation of uncertainty with Monte Carlo simulation (MCS) (Martowicz *et al.* 2007b; Schueller 1997). Obtained results have been verified with the outcomes from multibody simulations.

Performed optimization has eventually helped to find such configuration of two design parameters which characterizes possible smallest value of the comfort index taking into consideration allowed variation of mass of passengers. Note that the definition of comfort index assumes decrease of its value while increasing the comfort of travelling. Used approach is known as the robust optimization (Martowicz *et al.* 2009a; Poloni, Geremia, and Clarich 2006) with Taguchi-based techniques of the search in input domain. Found robust configuration defined with controlled parameters stands for the design characterizing smallest sensitivity to introduced uncertainties, i.e. uncontrollable parameters (Gallina, Martowicz and Uhl 2007).

The following sections of the paper subsequently present: the description of numerical model and performed dynamic analyses, definition of comfort index, description of introduced design and uncertain parameters, design of experiment (DOE) and metamodel fitting procedure, design optimization and exemplary results of uncertainty analysis, summary and concluding remarks.

2. MULTIBODY MODEL AND NUMERICAL ANALYSES OF ITS PROPERTIES

Multibody model of an articulated five-piece tram has been chosen for the task of design optimization. The model including both rail and vehicle has been elaborated in MSC/Adams software and is presented in Figure 1. All structural elements have been modeled as discrete components. The tram is equipped with three bogies consisting of bumpstops, dampers, springs for primary and secondary systems. The following elements have been modeled as rigid ones: frames of bogies, wheels, axles, grease-boxes and bogies of wagons. Applied model of rails includes their roughness. It has been assumed to represent the superposition of sin wave-type geometrical irregularities defined according to the standard ORE B176 RP1 (1989). There has
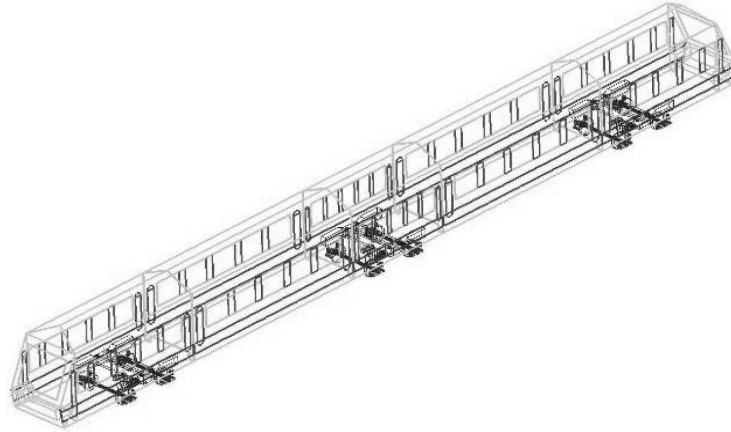
Fig. 1. Multibody model of tram

been assumed that vertical roughness S_{ZZ}, horizontal roughness S_{YY} and inclination $S_{\phi\phi}$ are defined as follows:

$$S_{ZZ} = \frac{A_V \Omega_c^2}{\left(\Omega^2 + \Omega_r^2\right)\left(\Omega^2 + \Omega_c^2\right)} \left[\frac{m^2}{rad/m}\right] \qquad (1)$$

$$S_{YY} = \frac{A_A \Omega_c^2}{\left(\Omega^2 + \Omega_r^2\right)\left(\Omega^2 + \Omega_c^2\right)} \left[\frac{m^2}{rad/m}\right] \qquad (2)$$

$$S_{\phi\phi} = \frac{A_V \Omega_c^2 \Omega^2}{\left(\Omega^2 + \Omega_r^2\right)\left(\Omega^2 + \Omega_c^2\right)\left(\Omega^2 + \Omega_s^2\right)} \frac{1}{b^2} \left[\frac{rad^2}{rad/m}\right] \qquad (3)$$

and are arbitrary determined with the following values of characteristic parameters: $\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m, $\Omega_s = 0{,}4380$ rad/m, $b = 0.75$ m, $A_V = 1.08 \cdot 10^{-6}$ m·rad, $A_A = 6.125 \cdot 10^{-7}$ m·rad. Assumed model of roughness results in wave lengths within the range from 2 m to 100 m. The mass of empty vehicle equals 29.3 t.

The model is parameterized so that the uncertainty on the mass of passengers could also be taken into account. The presence of passengers is considered with lumped elements, i.e. discrete masses which are spread through the tram pieces. Hence, it makes the total mass of the tram vary up to 48,8 t accordingly to the design specification on maximal density of passengers per given floor area. There is assumed the proportional growth of the number of passengers in all wagons. Given percentage deals with uniform distribution of the passengers in all wagons. Performed parameterization also allows for the change of stiffness coefficients of springs mounted in both primary and secondary system.

Elaborated multibody model has been used to find acceleration of vibrations measured in each wagon. The measurement process stands for the check of amplitude of vibration with five virtual three-axis accelerometers localized in the passengers area of each wagon. During numerical analyses a vehicle run is performed with constant, exemplary assumed, speed 10 m/s. As mentioned before there have not been considered neither accelerating nor braking to ob-

tain preliminary results. One full evaluation of a run takes approximately 5 minutes in terms of CPU time using standard desktop PC configuration. The plots of all accelerations calculated in MSC/ADAMS software are registered for assumed period of time. The time domain is however shortened at the beginning of simulation. Starting 5-second period of the plot is removed to prevent from the analysis of declining vibrations resulting only from the static vertical displacement observed in presence of gravity when vehicle is released and starts running.

Introduced rail roughness acts as a kinematic excitation in the model. The work deals however with the resultant accelerations which are measured in the passengers area on a floor level. This virtual measurement is feasible since used software allows for the modeling of vibration propagation through the suspension systems. Generally the higher amplitude of the acceleration of vibrations the lower subjectively assessed comfort of travelling felt by passengers. This phenomenon is especially seen for the range of low frequencies when a travel sickness is more often observed.

3. DEFINITION OF COMFORT INDEX

The procedure which has been applied for the calculation of comfort index consists of the steps presented in Figure 2 (standard UIC 513 1994). First accordingly to applied DOE the current configuration of input parameters is established. Subsequent input configurations can be set according to procedures of sensitivity, uncertainty analysis or RSM. Prepared realization of the multibody model is then used for the simulation of its dynamic properties and to find plots of accelerations in all directions and in each tram piece in the passenger areas. The next step stands for the calculation of root mean square (RMS) values of measured accelerations for given time windows. The representative resultant acceleration is considered to be a percentile 95 calculated for RMS values denoted a_{XP95}, a_{YP95}, a_{ZP95} accordingly to applied standard UIC 513. Exemplary plots of measured acceleration on the floor level in central wagon as well as its RMS value are presented in Figures 3 and 4.
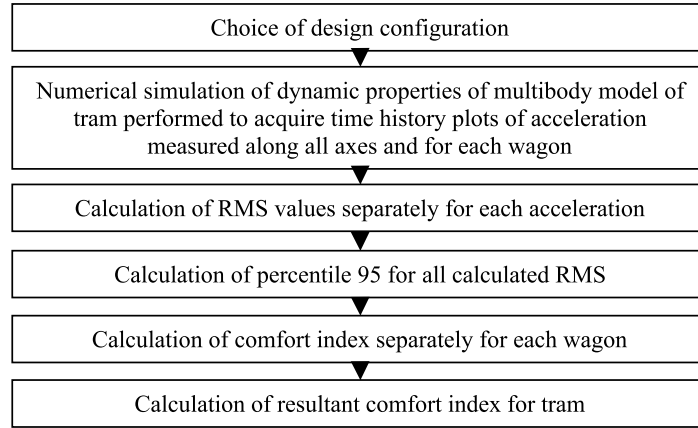
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│            Choice of design configuration                 │
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│ Numerical simulation of dynamic properties of multibody   │
│ model of tram performed to acquire time history plots of  │
│ acceleration measured along all axes and for each wagon   │
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┌─────────────────────────────────────────────────────────┐
│  Calculation of RMS values separately for each acceleration │
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│      Calculation of percentile 95 for all calculated RMS  │
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│   Calculation of comfort index separately for each wagon  │
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│     Calculation of resultant comfort index for tram       │
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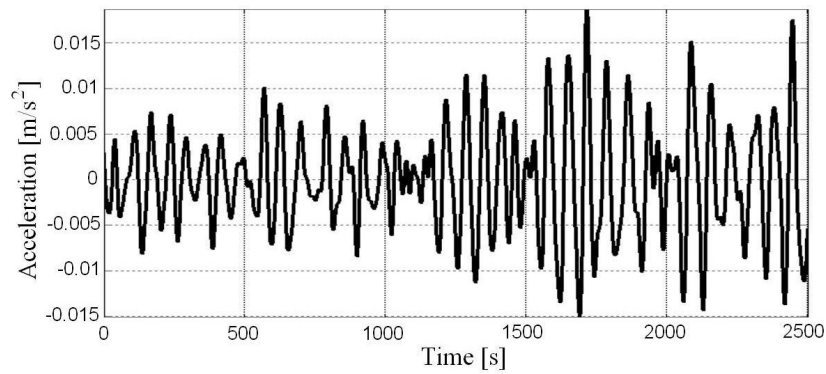**Fig. 2.** Procedure applied to calculate comfort index

Fig. 3. Exemplary plot of acceleration measured in central wagon along axis x
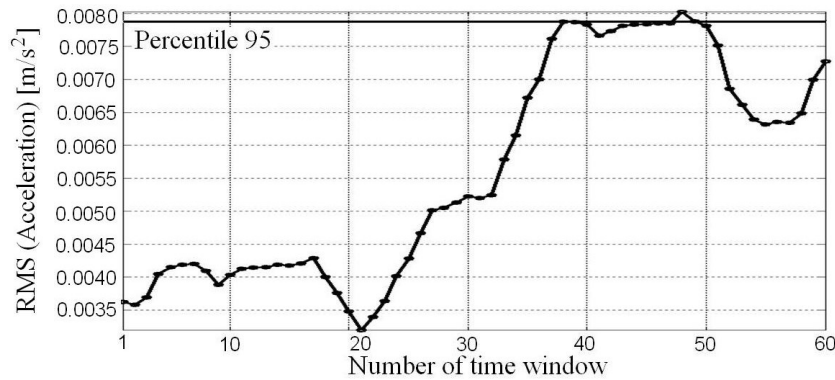

Fig. 4. RMS values and percentile 95 calculated for exemplary plot of acceleration

The comfort index for chosen wagon is separately calculated according to the following formula (standard UIC 513 1994):

$$N_{MV} = 6\sqrt{\left(a_{XP\,95}\right)^2 + \left(a_{YP\,95}\right)^2 + \left(a_{ZP\,95}\right)^2} \qquad (4)$$

Finally, the resultant comfort index is found as follows:

$$N = \sqrt{\frac{1}{5}\left(\sum_{i=1}^{5} N_{MVi}^2\right)} \qquad (5)$$

with the indexes N_{MVi} calculated for all wagons, i.e. for $i = 1, ..., 5$. The comfort index N is then taken as the objective of the optimization. Since N is basically calculated with the values of measured acceleration it means that the smaller value of N the better comfort of travelling. There is considered a good comfort when N is smaller than 2. Poor comfort of travelling occurs when N is greater than 5.

4. DESIGN AND UNCERTAIN PARAMETERS

As already mentioned there have been considered both design and uncertain parameters in the study. Since the travelling comfort is of concern the coefficients of stiffness of springs mounted in the suspension systems have been

Table 1

Input design and uncertain parameters

No.	Description	Type of parameter	Nominal value	Range of variation
1	Multiplier for stiffness coefficients of primary suspension system – p	Design	1	$\langle 1, 1.5 \rangle$
2	Multiplier for stiffness coefficients of secondary suspension system – s	Design	1	$\langle 0.5, 1.5 \rangle$
3	Mass of passengers – multiplier m	Uncertain	1	$\langle 0, 1 \rangle$

chosen as two design parameters. The nominal values of stiffness coefficients are:

- 5 300 000 N/m for the primary suspension system along horizontal axes,
- 626 000 N/m for the primary suspension system along vertical axis,
- 78 000 N/m for the secondary suspension system along horizontal axes,
- 1 074 600 N/m for the secondary suspension system along vertical axis.

The bound for which the coefficients are specified is +/–0.2 m.

The mass of passengers is considered to be uncertain parameter. It is assumed that the total mass of all passengers who may be on board, i.e. nominal passengers mass, equals 19.5 t. Description of input design and uncertain parameters and their allowed ranges of variation are presented in Table 1.

In case of parameters p and s the changes in stiffness coefficients are made simultaneously for all springs in each bogie within allowed ranges of variation.

5. DOE AND METAMODEL FITTING PROCEDURE

RSM has been applied for the description of relationships between input three design and uncertainty parameters p, s, m and output parameter N (Box and Draper 1986; Gallina, Martowicz and Uhl, 2006; Myers and Montgomery 1995). The whole input domain defined in Table 1 has been covered uniformly with the samples of configuration. The uniform mesh of 405 points has been arbitrary assumed to consider changes of 12.5% of nominal values for all input parameters which respectively means 9 values for both s and m as well as 5 values for p. Applied DOE has allowed for the creation of metamodels consisting of regressors of up to 4-th power. Detailed information on used types of regressors is presented in Table 2.

For specified DOE 405 multibody analyses have been performed accordingly to the procedure for the calculation of comfort index presented in Figure 2. Yielded values of N and determined list of 125 regressors have been finally applied for the metamodel fitting procedure. Coefficients of regressors have been found with the least square method (LSM) (Box and Draper 1986). The quality of metamodel has been assessed with the coefficient of multiple determination R^2 and adjusted coefficient of multiple determination R^2_{adj} (Montgomery and Runger 2007), which equal 0.847 and 0.846 respectively. Especially the later parameter, which considers the number of input parameters, stands for the objective assessment of the quality how the variation of modeled output parameter is projected in the variation represented by elaborated metamodel. In Figure 5 there are presented coefficients of the first 20 most influential regressors. Since the input domain has been defined with unitless parameters and additionally normalized there is possibility to compare directly the regressor coefficients. The procedure of data normalization has been performed with the original ranges presented in Table 1 and made all input parameters vary within the same input domain defined as $\langle -1, 1 \rangle$ before metamodel fitting is accomplished.

Table 2

Regressors of metamodel

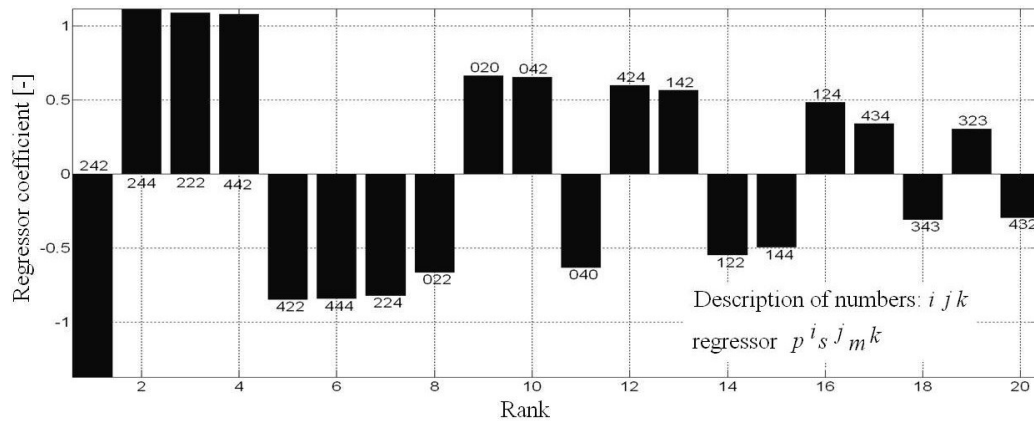| Type of regressor	Constant	Linear	Non-linear interactions with parameters of up to r-th order each	Non-linear of up to r-th power
No. of regressors of chosen type	1	n (n-no. of input parameters)	$(r+1)^n - 1 - n - nr$ (for $r = 4$)	nr (for $r = 4$)
	1	3	109	12
Total no. of regressors used in metamodel	125			

Fig. 5. Most influential regressors found in metamodel

From the analysis of the list of the most influential regressors it can be concluded strong nonlinearity of the functions that define relationships between input and output parameters of the analysis. Almost all presented regressors are of high orders. For example the first one being coded as 242 stands for the regressor $p^2 s^4 m^2$. The sixth position is occupied by the regressor of possible highest order, i.e. $p^4 s^4 m^4$. In shown order there is no linear terms and the only regressor of low order is the squared s at ninth position. The comparison between comfort indexes calculated in multibody simulations (in the following denoted by MB) and obtained with metamodel (denoted as RSM) is presented in Figure 6.

The comparison between results has been performed for the case with maximal mass of passengers on board. Apart from the region of the highest value of comfort index obtained results may be treated as similar. The surfaces are overlapping each other almost within the whole domain of design parameters. The shape of the response is of good quality as it allows for the search of global optimum in optimization task. Even though considerable differences in results appear the regions where extreme values of N can be still correctly identified.

Additionally the analysis of influence of the number of regressors included in metamodels on the quality of approximation has been carried out. For assumed numbers of regressors included in metamodel (taken from the list of regressors defined in Tab. 2) the procedure of metamodel fitting has been performed with LSM. For all degenerated metamodels (i.e. with reduced number of included regressors) there have been used the same results of multibody simulations which have been applied for a case with all 125 regressors, i.e. with 405 samples. Obtained collections of regressors are presented in Figure 7. 24 cases have been studied with the number of included regressors from 5 up to 120 with the step of 5. The procedure of regressors selection utilizes GA. Its steps are presented in Figure 8.

The number of genes in each individual equals the number of all regressors which could be included in the metamodel, i.e. 125. The value of each gene, in turn, defines whether related regressor is included in the metamodel. The greater gene value the higher probability it is taken for the metamodel. Therefore the population of individuals actually stands for the collection of chosen metamodels with selected regressors accordingly to their maximal number. All coded metamodels are then checked in terms of the
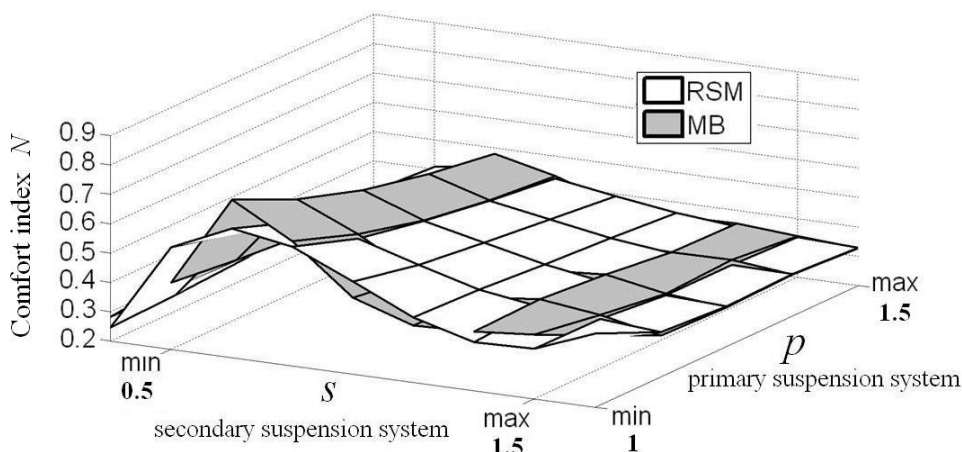

Fig. 6. Comparison between results obtained with multibody simulations and RSM
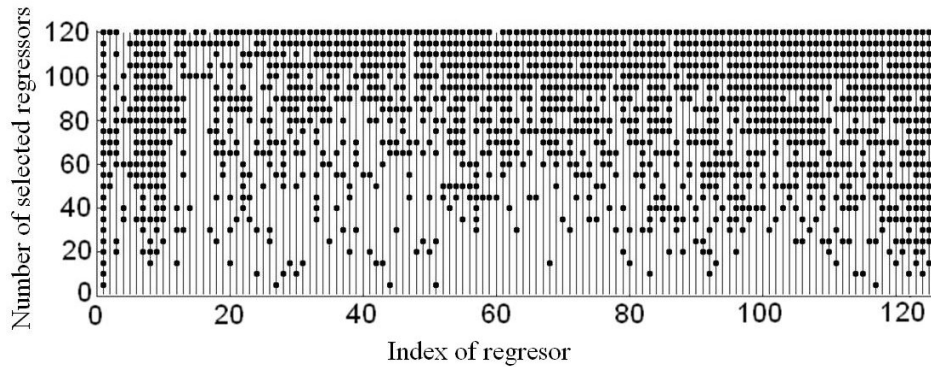
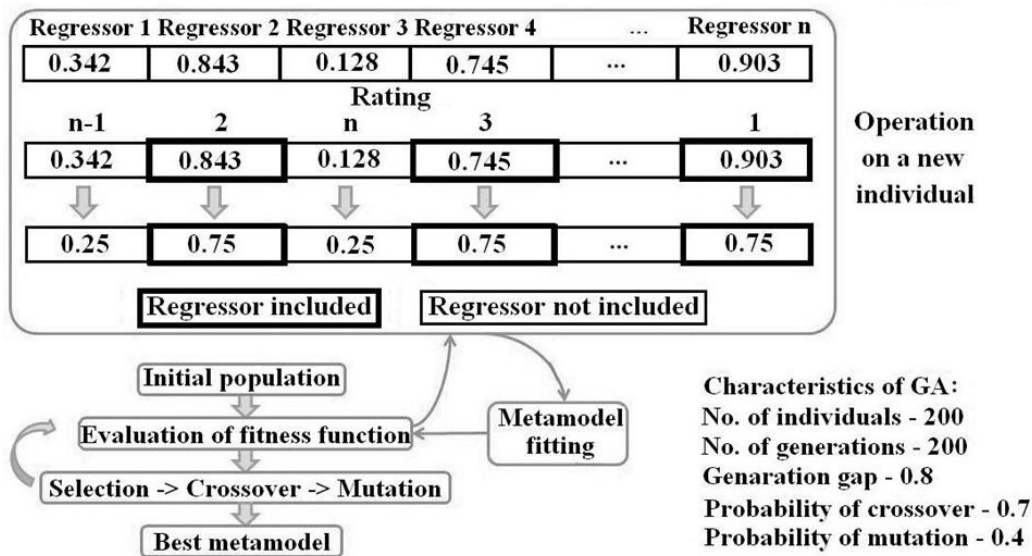**Fig. 7.** Chosen regressors for degenerated metamodels

Fig. 8. Procedure of regressor selection aided with GA

quality of representation of results obtained with multibody simulations. The fitness function has been calculated as the maximal difference between values of comfort indexes evaluated for both multibody simulations and currently coded metamodel. The difference on results is calculated considering all 405 samples of previously described DOE. Figure 7 can be considered as a kind of stabilization diagram in which it is clearly seen how the contribution of particular regressor evolves and therefore is important for the metamodel with increasing number of introduced approximating terms. Some of regressors seem to be quite important in terms of metamodel quality since they appear almost in all cases starting with cases where rather low number of regressor is considered. On the other hand there have been reported regressors characterizing low importance for the quality of the approximation as they seem to appear for random cases, preferably for these ones that characterize great number of introduced terms. Figure 9 presents the curve describing the minimal value of fitness function calculated for the best individuals and obtained with GA performed for each case, i.e. assumed number of regressors included in the metamodel, separately.

Found metamodels, which differ in the number of included regressors, have been applied for the task of optimization of comfort index. Optimal designs are described in the following section. The curve presented in Figure 9 does not converge strictly. The minimum value of the fitness function is found for the case with 100 regressors included in the metamodel which means that not for the most complicated one. This, in turn, could lead to the assumption that increasing the number of regressors does not necessarily improve the quality of the metamodel. Unnecessary regressors of higher orders may introduce additional error to the approximation rather than result in more accurate representation of modeled output parameter. It is seen that analyzed curve stabilizes in the interval $\langle 0.25, 0.28 \rangle$. It occurs for the metamodels with at least 50 introduced regressors. Maximal difference in results can be observed in Figure 6 for the region where $p = 1$ and $0.5 < s < 1$. Although the value of difference between results is quite considerable, i.e. 0.28 with respect to approximated value of N (around 0.7 for mentioned region in input domain) there has been possible to find design with lower value of the comfort index.
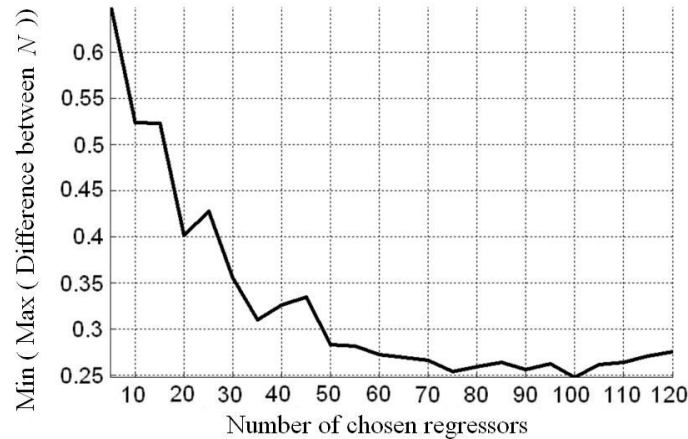
Fig. 9. Minimal value of fitness function for metamodels with different numbers of regressors

6. DESIGN OPTIMIZATION

The overall objective of performed optimization task has stood for the minimization of the value of N. The input design domain has been defined with 2 multiplier of stiffness coefficients p and s with allowed ranges of variation. The mass of passengers, defined with multiplier m has also been introduced to the optimization as an uncertain parameter. For each checked combination of design parameters the worst case scenario has been taken into account in which the greatest value of N has appeared while changing m within its whole domain $0 \leq m \leq 1$. Hence the optimization task can be formulated as follows:

$$\min_{\substack{design \\ parameters}} \quad \max_{\substack{uncertain \\ parameter}} \quad N \qquad (6)$$

The expression $\max_{\substack{uncertain \\ parameter}} N$ can be considered as the actual objective function and means N calculated for the worst case scenario for assumed variation of the mass of passengers. Again GA have been used to find optimal designs because of its widely known applications for computational mechanics (Deb and Gulati 2001; Dias and Correa 2006; Martowicz, Pieczara and Uhl 2007a; Martowicz, Stanciu and Uhl 2009b). Applied GA characterize the following parameters: number of individuals – 200, number of generations – 50, generation gap – 0.8, probabilities of crossover and mutation – 0.7 and 0.4, respectively. Each two-gene individual has coded values of p and s. The value of fitness function has been found for coded pair p and s as the maximal value of the parameter N appearing for $0 \leq m \leq 1$. The optimization of N has been performed for all previously found best metamodels for 13 cases differing in the number of included regressors (i.e. from 10 up to 125). The results of the optimization task found for exemplary metamodels are presented in Table 3. The results include also the approximation built with all considered 125 regressors. Figure 10 presents the convergences of N for the optimization procedure of the suspension system with the metamodel using all 125 regressors for 10 exemplary runs.

Table 3

Best designs of suspension system found with metamodels differing in the number of regressors

| No. of regressors	Multiplier p	Multiplier s	Comfort index			Adjusted coefficients of multiple determination for best metamodel
			N_{RSM}	$N_{Multibody}$ (reference)	error [%]	
10	1.43	0.50	0.321	0.304	5.59	0.075
20	1.50	1.07	0.265	0.350	24.3	0.420
30	1.50	0.50	0.269	0.303	11.2	0.526
40	1.48	0.50	0.288	0.303	4.95	0.607
50	1.50	0.50	0.297	0.303	1.98	0.788
60	1.50	0.50	0.292	0.303	3.63	0.794
70	1.50	0.50	0.287	0.303	5.28	0.825
80	1.50	0.50	0.287	0.303	5.28	0.823
90	1.50	0.50	0.284	0.303	6.27	0.832
100	1.50	0.50	0.285	0.303	5.94	0.828
110	1.50	0.50	0.286	0.303	5.61	0.832
120	1.50	0.50	0.287	0.303	5.28	0.845
125	1.50	0.50	0.287	0.303	5.28	0.846
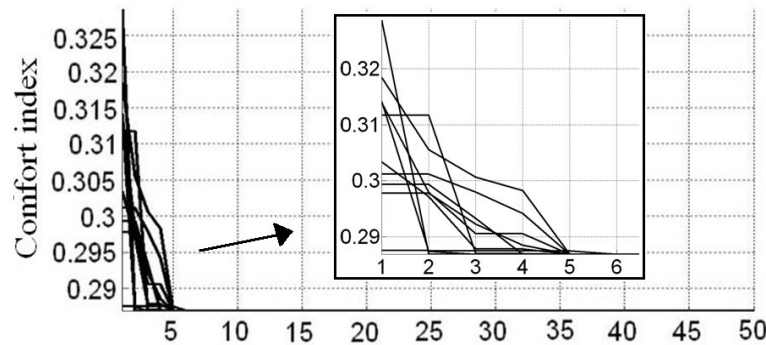
Fig. 10. Convergences of approximated comfort index for the optimization task with metamodel
which considers all 125 regressors

There has been found the configuration that characteri-zes value of the comfort index lower than calculated for the nominal case. The best design which has been found using the optimization procedure is defined with the following values of multipliers: $p = 1.5$, $s = 0.5$. This final configura-tion appears for majority of applied metamodels. It strictly corresponds to the expected global optimum which can be found in Figure 6 when analyzing the referential results of multibody simulations denoted by MB. The minimal value of N equals 0.303. This value has been obtained with the multibody simulation for found best design. For this confi-guration the error introduced by the metamodel is within the bound from 3.63% (observed for the metamodel consisting of 60 regressors) up to 6.27% (the case with 90 regressors). Calculated decrease of the value of N equals 28.4% with respect to the nominal case when $p = 1$, $s = 1$. Obtained relative decrease of the value of N seems to be significant but the authors admit that the value of N is anyway small and its absolute decrease equals 0.12. This difference, in turn, does not seem to represent considerable improvement in the comfort of travelling. The paper is however indented rather to present the workflow and results of exemplary application elaborated for the optimization of the comfort of travelling supported with GA and RSM.

It can be seen that the final configuration of multipliers p and s can be found for metamodels with at least 30 regres-sors included though the error resulting from metamo-delling is quite considerable and equals 11.2%. Mentioned metamodel characterizes small value of adjusted coefficients of multiple determination, i.e. 0.526. It means than this metamodel states for the least complicated approximation that could be effectively applied for the optimization prob-lem. Further increase of number of introduced regressors (actually starting with the case of 50 regressors) has not changed the best configuration. Additional regressors con-sidered in growing metamodels seem to play negligible role in the process of search of final design. On the other hand in case of crude approximation there may be a risk of poor quality of applied metamodel. It can be observed for the case with only 20 regressors where incorrect optimal solu-tion has been found, i.e. $p = 1.5$, $s = 1.07$ (seen with the referential results in Fig. 6). This conclusion has the con-firmation in small value of adjusted coefficients of multiple determination which equals only 0.42 and significant error of 24.3%. Respective value of the comfort index found with multibody simulation is greater of about 16% than calculat-ed for optimal solutions found with better metamodels.

The study of the convergence of comfort index presented in Figure 10 results in the conclusion that for assumed num-ber of individuals it is not necessary to exceed the number of generation over 10. Its further increase has not changed found optimal design. This behavior must have resulted from great ratio between the number of individuals in GA, i.e. 200, and the number of considered genes, i.e. 2.

7. UNCERTAINTY ANALYSIS

Found best configuration ($p = 1.5$, $s = 0.5$) has been applied to assess the propagation of defined uncertainty. There has been studied the influence of variation of multiplier m on comfort index N. MCS with 500 samples has been applied for the uncertainty analysis. The uniform probability densi-ty function for the variation of mass of passengers on board has been arbitrary assumed for uniform covering of the input parameter domain. To assess the uncertainty propaga-tion the following three cases have been considered:

– analysis denoted by MB – the values of comfort index N are found with multibody simulations; obtained results are considered as referential ones;

– analysis denoted by RSM3 – the values of comfort index N are found with the metamodel which considers all input design and uncertain parameters: p, s and m and with optimal configuration $p = 1.5$ and $s = 0.5$; the metamodel considers 125 regressors and is one of already used for the task of design optimization;

– analysis denoted by RSM1 – the values of comfort index N are found with more accurate metamodel which considers only one input parameter – uncertain parameter m describing the variation of passenger mass; applied metamodel has been determined for the optimal design $p = 1.5$, $s = 0.5$ on the basis of results yielded in additional multibody simulations; the appro-ximation introduces regressors composed only with the parameter m and of power up to 4.
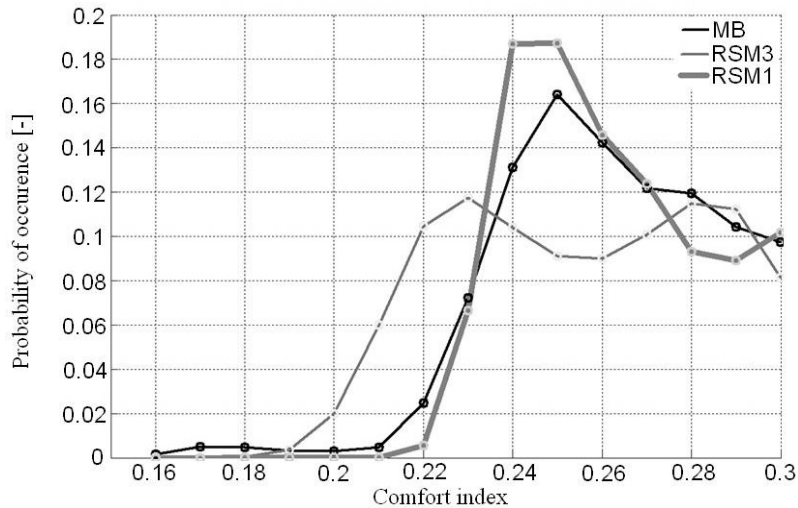
Fig. 11. Kernel densities of comfort index obtained with performed uncertainty analyses

Table 4

Comparison between results of uncertainty analyses

Analysis	Comfort index N		
	Mean (error [%])	Standard deviation (error [%])	Coefficient of variation (COV) (error [%])
MB (reference)	0.2642	0.0251	0.0948
RSM3	0.2576 (2.50)	0.0282 (12.4)	0.1096 (15.6)
RSM1	0.2642 (0.00)	0.0218 (13.1)	0.0826 (12.9)

The results comparison of performed assessment of uncertainty propagation are presented in Figure 11 and Table 4. Figure 11 presents the comparison between kernel densities of comfort index N obtained with both multibody simulations and by the application of RSM (Bowman and Azzalini 1997). Table 4, in turn, presents numerical comparison of other statistic parameters.

The application of more accurate metamodel in the analysis RSM1 has resulted in better agreement with the analysis MB as far as determined kernel densities are of concern. In case of analyses MB and RSM1 there has not been also found any difference between mean values. 2.5% error has appeared for the comparison between mean values obtained in analyses MB and RSM3. Comparable values of errors concerning standard deviations however have been found for both analyses RSM1 and RSM3. Slightly lower value of COV has been calculated for analysis RSM1. It confirms again better quality of the model prepared for the analysis RSM1.

From the previous section it can be seen that metamodel used for the task of optimization (i.e. used in the analysis RSM3) has allowed to find the correct optimal configuration (defined with the referential multibody simulations) operating with 3 input parameters. However it cannot be used for the assessment of uncertainty propagation where obviously more accurate results are necessary for the correct concluding about variation of the comfort index. Apparently included interaction terms cause decrease of quality.

Above stated observations are confirmed with the plots presented in Figure 11. Kernel density function calculated for the analysis RSM1 has managed to effectively follow the changes of respective curve found for the analysis MB. Significant difference however is seen between curves obtained with analyses MB and RSM3. Hence according to the present study one can assume that it is recommended to built additional metamodel as the one used in the analysis RSM1 which is valid only for the optimal configuration of design parameters and allows only for the variation of uncertain parameter. This approach should results in more accurate outcomes of the uncertainty analysis.

8. SUMMARY AND CONCLUDING REMARKS

In the paper there have been presented the results of optimization performed for a five-wagon tram in order to improve the quality of travelling. The optimization has considered both design and uncertain parameters respectively connected to the properties of suspension systems and the mass of passengers on board. Parameterized multibody model has allowed for the simulation of dynamic properties of tram for different combinations of input parameters. To speed up performed analyses metamodelling procedure has been applied. Finally the uncertainty analysis has been carried out for exemplary metamodel.

While the metamodelling technique is applied it is important to consider proper approximating terms so that the variation of modeled quantity could be represented correctly. It is obvious that this is not an easy issue especially when there is no additional information, guidelines which could be used for the formulation of the structure of the approximation. When relationships between input and output parameters of the analyses seem to be strongly nonlinear it is a good practice to gradually include regressors of higher orders and check the quality of current metamodel with either deterministic or statistic method. This approach has been effectively applied in presented work and the procedure of search for the best configuration of used regressors has been supported by GA. GA have been successfully applied several times for different numbers of introduced regressors. Simple form of fitting function has allowed to find actual best approximation. Calculated values of adjusted coefficients of multiple determination have been used for the assessment of metamodel quality. It has been observed that there exists minimal number of regressors that should be taken into consideration to achieve convergence of value of determined quality index. On the other hand the best approximation does not necessarily have to appear for the most complicated metamodel which leads to the assumption that unnecessary regressors of higher orders may introduce additional error to the approximation rather than lead to more accurate results. The artifacts may appear.

For the optimization task there have been also used metamodels differing in the number of included regressors. Even though the quality of metamodels is not very high, i.e. the approximation error is over 5% for majority of applied metamodels and the value of adjusted coefficients of multiple determination less than 0.85, there has been possible to find optimal design configuration which is the same as determined with multibody simulations. It turns out that mentioned above difference in results (mainly assessed with the approximation error) does not necessarily lead to wrong results of optimization task. Even though applied optimization tool operates with not perfectly accurate data, the metamodels are good enough to capture the shape of modeled response and then allow to localize the area of global optimum correctly. Actually when the optimal design is known it is not an issue to find the exact value optimized quantity by performing only one additional metamodel simulation. The risk of avoidance of global optimum has appeared for crude metamodels only.

The issue of correct approximation of modeled parameter has arose in case of uncertainty analysis however. For an analysis for which the quality of its results strictly depends on outcomes yielded from subsequent realizations of designed structures, as it happens for statistics, one should assume that the quality of determined metamodels should be possibly highest. Therefore in a case similar to the analyzed one in presented work an assumption also should be stated that it is recommended to built additional metamodel which is valid only for the optimal configuration of design parameters and allows only for the variation of uncertain parameter. This approach should results in more accurate outcomes of the uncertainty analysis.

Acknowledgements

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