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Jacek SNAMINA*, Paweł ORKISZ*

ASSESSMENT OF PARAMETERS OF LABORATORY MODEL OF TELECOMMUNICATION MAST

SUMMARY

This work presents an algorithm for determining values of parameters of a laboratory model of a telecommunication mast. The algorithm is constituted by several steps finally leading to a physically executable model, being a good representation of the dynamics of the mast construction. The calculations carried out were used for design a laboratory model of a telecommunications mast.

Keywords: *model reduction, physically executable model, telecommunication mast*

WYZNACZANIE PARAMETRÓW KONSTRUKCYJNYCH MODELU LABORATORYJNEGO MASZTU TELEKOMUNIKACYJNEGO

W pracy przedstawiono algorytm wyznaczania wartości parametrów laboratoryjnego modelu masztu telekomunikacyjnego. Algorytm składa się z kilku kroków, w wyniku których uzyskano fizycznie realizowalny model. Otrzymany model dobrze odwzorowuje dynamikę konstrukcji masztu. Przeprowadzone obliczenia wykorzystano do zaprojektowania modelu laboratoryjnego masztu telekomunikacyjnego.

Słowa kluczowe: *redukcja modelu, model fizycznie realizowalny, maszt telekomunikacyjny*

1. INTRODUCTION

Dynamic analysis of complex constructions is related to development of mathematical models with many degrees of freedom. The programs most often used for the analysis of such constructions base on a finite element method. One of the methods for increasing the accuracy of calculations is increasing the amount of finite elements, which results in a significant increase the number of degrees of freedom. From the point of view of control algorithms the formation of models with a relatively small number of state variables, and simultaneously with the sufficiently accuracy, should be aimed for. Similarly in order to design laboratory stands, the complex structure should be reduced to a relatively simple system. For both problems, reduction of the complex model to a simple model under strictly defined conditions should be carried out. The base of modeling of dynamic systems has been presented in (Meirovitch 1990). Methods for creation of models and exemplary results are described in (Preumont *et al.* 2008). A significant part of these algorithms are methods of model reduction. Examples of discrete models creation can be found in (Balas 1982) or (Seto *et al.* 1992).

This article presents an algorithm for determining values of parameters of a model of a telecommunication mast. The assessment of parameters was used for design a laboratory stand.

2. ALGORITHM CONSTRUCTION

Calculations were divided into several stages schematically presented in Figure 1. In the first stage, the finite elements method was used. A detailed model and a simplified beam model of the mast were done. Using these models, the modal analysis of the mast was carried out.

In the next step, the matrix equation of vibrations of the construction can be written by introducing a vector of principal coordinate $\mathbf{q}$ and modal matrices of mass $\mathbf{M}$ and stiffness $\mathbf{K}$. The modal description is very simple and fully reflects the basic properties of the construction. Thus the reduction of the complex model can be carried out in the simplest way in a modal space. Reduction is based on limitation of the number of modes and the number of principal coordinates.

Most often, the set of modes is limited by rejection of vectors related with the highest frequencies. Selection of modes and principal coordinates is equivalent to the introduction a new structurally simpler model called the reduced model. The kinetic and potential energies of the reduced model are smaller than the corresponding energies of the construction. However, the frequencies of reduced model are the same as the corresponding frequencies of the construction. The equations of vibrations of the reduced model can be written by using reduced matrices of mass $\tilde{\mathbf{M}}$ and stiffness $\tilde{\mathbf{K}}$ and reduced vector $\tilde{\mathbf{q}}$, including principal coordinates. Principal coordinates are generalized coordinates and cannot be directly identified with the physical coordinates describing the motion of selected points belonging to the constructions. The final description of the reduced model is related to define physical coordinates. Physical coordinates can be selected in many ways. Often, the minimization of interactions of omitted modes with modes of reduced model is considered ("spillover" effect (Braun *et al.* 2002).

* AGH University of Science and Technology, Faculty of Mechanical Engineering and Robotics, Department of Process Control, al. A. Mickiewicza 30, 30-059 Krakow, Poland; Snamina@agh.edu.pl; Orkisz@agh.edu.pl

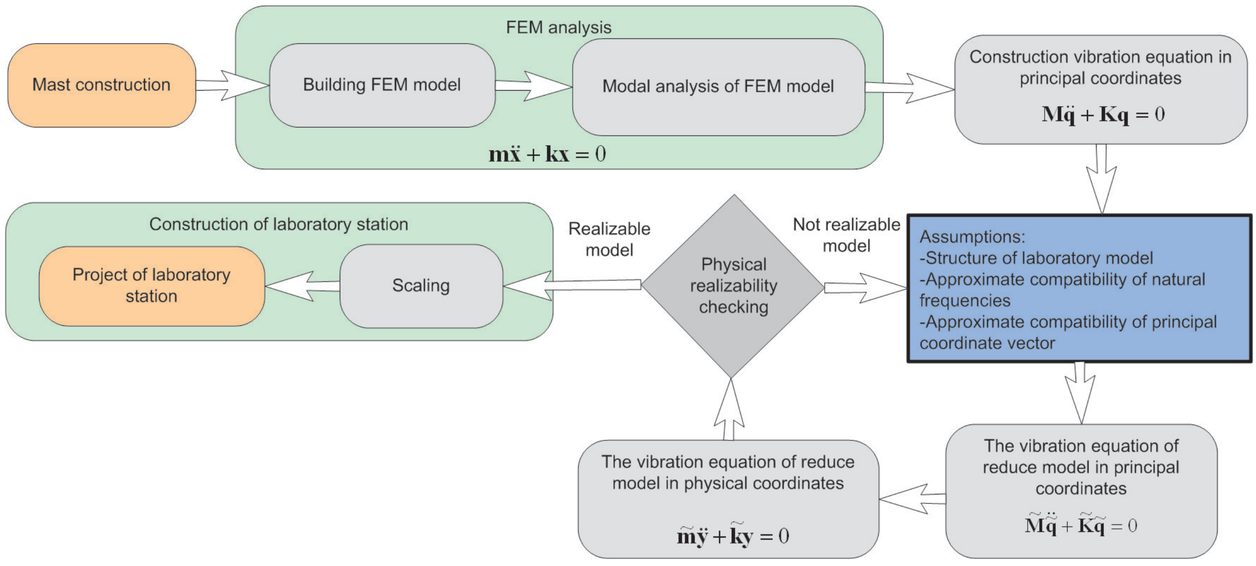

Fig. 1. Scheme of the discrete model construction

This stage of the creation of the reduced model is related to the derivation of equations in the system of physical coordinates. The obtained matrices of mass $\tilde{\mathbf{m}}$ and stiffness $\tilde{\mathbf{k}}$ describe the object, which may be realized as a passive system, or such realization of the system may not be possible. In order to solve this problem in algorithm, a decision block, is introduced (Fig. 1). The final stage of calculations is the scaling. The determination of a scaling coefficient is associated with possible size of the model in the laboratory.

3. FEM MODEL OF THE TELECOMMUNICATION MAST

The considered mast is a steel lattice structure of a height of 45 m consisting of two sections comprised of forty-one segments as shown in Figure 2. The lower section is comprised of twenty-three segments made from angle bars of various dimensions. The base of the section is a square with a side length of 2.04 m. The section has the shape of a truncated pyramid with a height of 30 m. The upper cross-section has the shape of a square with a side length equal to 0.726 m. The upper section is made up of 18 segments having the shape of rectangular prisms of various heights.

Eighteen antennas with a total mass of 800 kg are installed on the mast from a height of about 25 m. The construction of the mast is asymmetrically reinforced with four fixed stay ropes. The total mass of the mast with stay lines and the upper platform (mass of 300 kg) is equal to about 5400 kg.

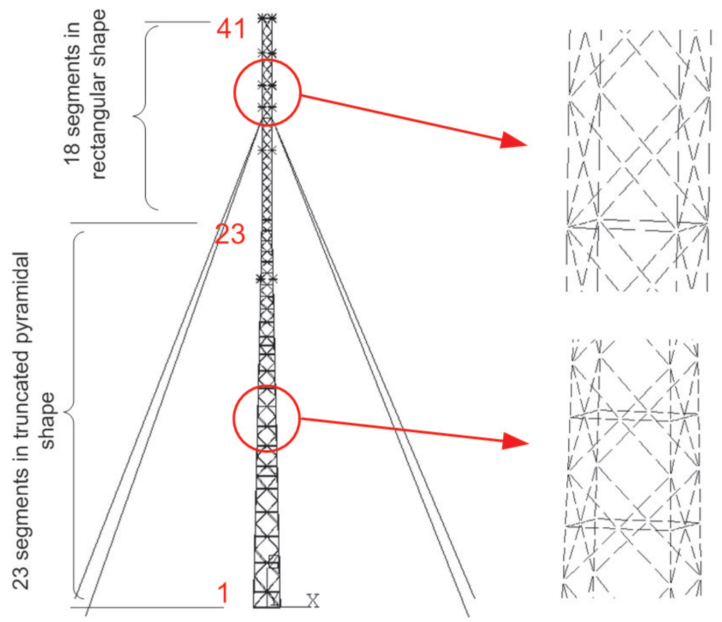

Fig. 2. Telecommunication mast construction

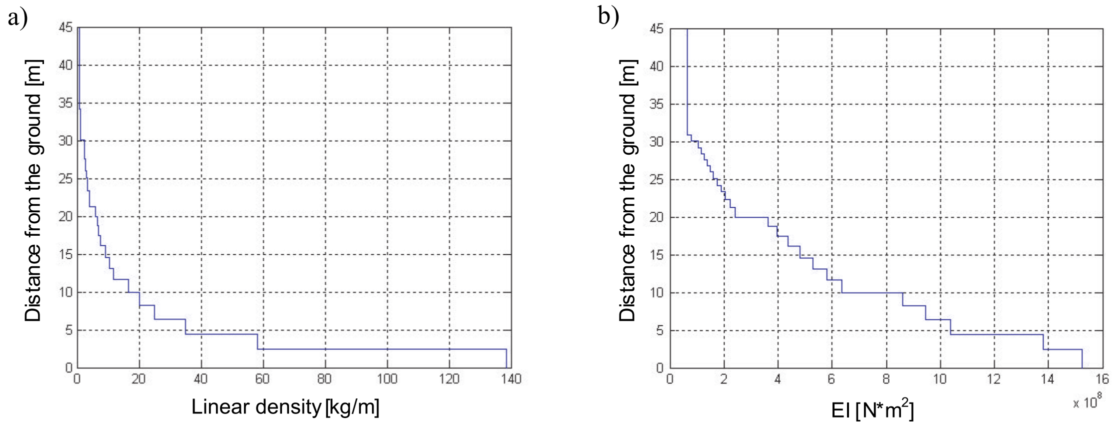

Fig. 3. Linear density of mast segments (a), reduced bending stiffness of the mast (b)

On the basis of technical documentation of the telecommunications mast, a detailed FEM model of the construction containing about 3000 nodes is prepared. The construction model shown in Figure 2 was described in further detail in (Kowal 2009). If equations of vibration of the detailed construction model are used, determination of model parameters is very time consuming. Thus a simplified model is constructed, in which individual mast segments are modeled beam elements with specifically defined parameters of mass and stiffness. This model is called the beam model of the mast.

Linear density and stiffness of elements in the beam model are determined by calculating the mass of the segments and their average bending stiffness. The determined values of linear density and stiffness of cross-sections in various distances from the ground are presented in Figures 3a and 3b.

The mast model created in this way, in the form of a beam with variable stiffness and linear density, is compared to the detailed model by means of determination of the first four modes and corresponding frequencies. The results are presented in Figure 4.

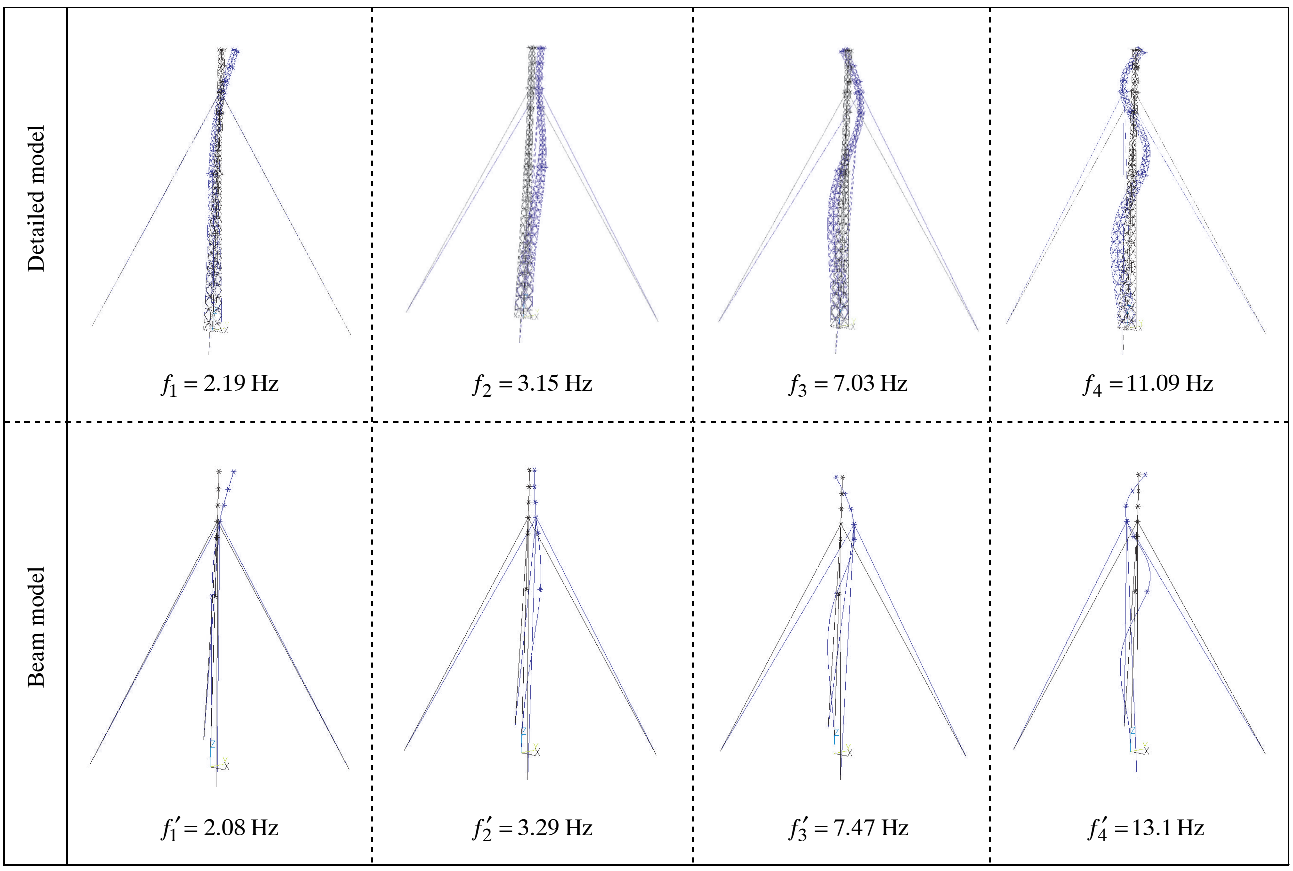

Fig. 4. Mode shapes and corresponding frequencies of the telecommunication mast

4. DISCRETE MODEL OF THE MAST

Modal analysis carried out in the ANSYS environment for the beam model allows for the calculation of modal matrices of mass **M** and stiffness **K**. The matrix equation of vibrations in principal coordinates takes the following form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \tag{1}$$

The suggested method of creation of the reduced model is based on the limiting of the number of eigenvectors with simultaneous reduction of the principal coordinates number. Taking into account the expected range of frequencies of excitations associated with earthquakes or wind (Levy 1976, Conte *et al.* 1997), the set of modes is limited to the first three vectors. In accordance with (Hori *et al.* 2000), the points that the motion are described by the reduced model should be placed near the nodes of the first omitted mode of the mast. This allows for minimization of the "spillover" effect related to the influence of modes omitted in calculations on the motion of the construction. According to this principle, the mass m_1 is situated at point 132, the mass m_2 in point 42 and the mass m_3 in point 89. The numbering of points as shown in Figure 5a is accepted according to the numbering of nodes introduced by the ANSYS program. For better imaging the points choosing process, the stay lines of mast were not shown in Figure 5.

Additionally, a schematic structure of the reduced model with three mass moving in the direction of the horizontal axis is shown in Figure 5b. The matrix equation of reduced model vibrations has the following form:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix} + \begin{bmatrix} k_1+k_4+k_5 & -k_4 & -k_1 \\ -k_4 & k_2+k_3+k_4 & -k_2 \\ -k_1 & -k_2 & k_1+k_2+k_6 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 0 \tag{2}$$

The results of calculations done in procedure for finding parameters of the reduced model are shown in Table 1. The first column contains matrices of mass, and the second, matrices of stiffness of the reduced model. The last column of the table contains modal vectors matrices. The table also contains physical parameters of the reduced model, with a structure as shown in Figure 5b. Matrices of masses and stiffness determined for the reduced model in principal coordinates are shown in the first row of the table. Matrices of mass and stiffness in physical coordinates can be appointed by comparing the kinetic and potential energy of model in physical and principal coordinates:

$$\begin{matrix} \frac{1}{2}\dot{\mathbf{y}}^T\tilde{\mathbf{m}}\dot{\mathbf{y}} = \frac{1}{2}\dot{\tilde{\mathbf{q}}}^T\tilde{\mathbf{M}}\dot{\tilde{\mathbf{q}}} \\ \frac{1}{2}\mathbf{y}^T\tilde{\mathbf{k}}\mathbf{y} = \frac{1}{2}\tilde{\mathbf{q}}^T\tilde{\mathbf{K}}\tilde{\mathbf{q}} \end{matrix} \rightarrow \begin{matrix} \tilde{\mathbf{m}} = \left(\tilde{\mathbf{X}}^T\right)^{-1}\cdot\tilde{\mathbf{M}}\cdot\left(\tilde{\mathbf{X}}\right)^{-1} \\ \tilde{\mathbf{k}} = \left(\tilde{\mathbf{X}}^T\right)^{-1}\cdot\tilde{\mathbf{K}}\cdot\left(\tilde{\mathbf{X}}\right)^{-1} \end{matrix} \tag{3}$$

The second row of the table contains matrices of mass and stiffness as well as modal vectors matrix of the reduced model described in physical coordinates.

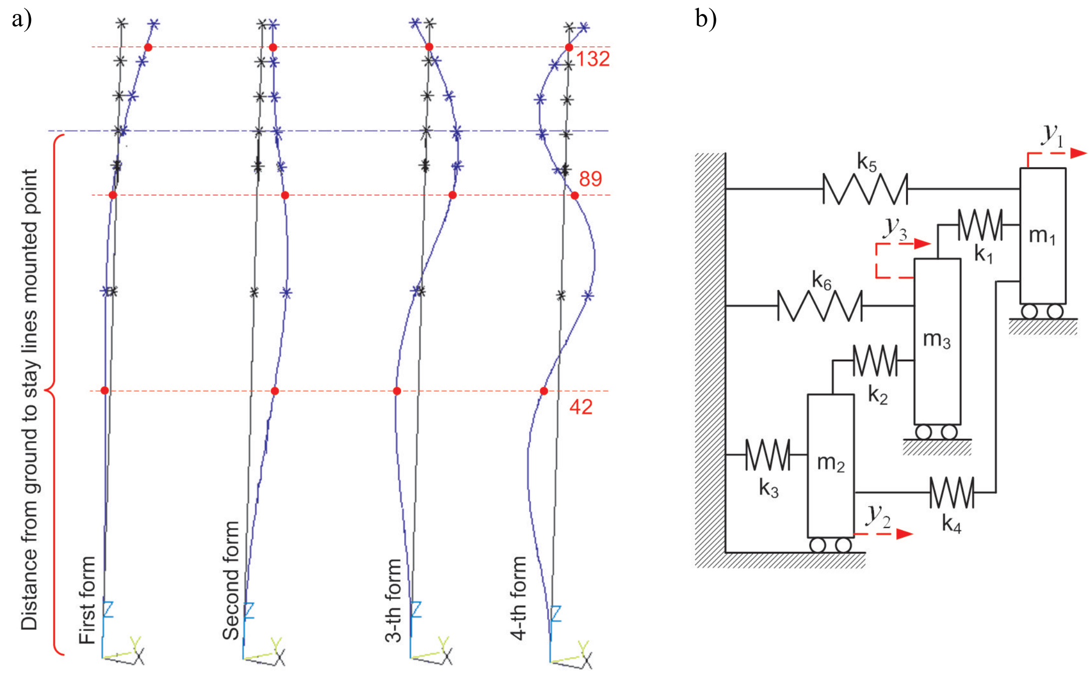

Fig. 5. Mode shapes of the beam model (a), scheme of the reduced model with three masses (b)

Table 1
List of selected matrices

No	Mass matrix	Stiffness matrix	Modal vectors matrix
1	$\tilde{\mathbf{M}} = \begin{bmatrix} 1111 & 0 & 0 \\ 0 & 3581 & 0 \\ 0 & 0 & 2235 \end{bmatrix}$	$\tilde{\mathbf{K}} = \begin{bmatrix} 0.19 & 0 & 0 \\ 0 & 1.531 & 0 \\ 0 & 0 & 4.924 \end{bmatrix} \cdot 10^6$	
2	$\tilde{\mathbf{m}} = \begin{bmatrix} 1025 & 2.4 & 12.9 \\ 2.4 & 1596 & -0.9 \\ 12.9 & -0.9 & 1107 \end{bmatrix}$	$\tilde{\mathbf{k}} = \begin{bmatrix} 0.200 & 0.137 & -0.005 \\ 0.137 & 2.088 & -1.191 \\ -0.005 & -1.191 & 1.439 \end{bmatrix} \cdot 10^6$	$\tilde{\mathbf{X}} = \begin{bmatrix} 1 & 0.513 & -0.07 \\ -0.19 & 1 & -0.839 \\ -0.175 & 1.239 & 1 \end{bmatrix}$
3	$\tilde{\mathbf{m}}' = \begin{bmatrix} 1025 & 0 & 0 \\ 0 & 1597 & 0 \\ 0 & 0 & 1106 \end{bmatrix}$	$\tilde{\mathbf{k}}' = \begin{bmatrix} 0.2 & 0.142 & -0.014 \\ 0.142 & 2.088 & -1.191 \\ -0.014 & -1.191 & 1.438 \end{bmatrix} \cdot 10^6$	$\tilde{\mathbf{X}}' = \begin{bmatrix} 0.9987 & 0.5212 & -0.0648 \\ -0.1890 & 0.9998 & -0.8393 \\ -0.1689 & 1.2420 & 1 \end{bmatrix}$
	parameters of the first reduced model	$m_1 = 1.025\times10^3$ kg $m_2 = 1.597\times10^3$ kg $m_3 = 1.106\times10^3$ kg	$k_1 = 0.014\times10^6$ N/m, $k_2 = 1.191\times10^6$ N/m $k_3 = 1.039\times10^6$ N/m, $k_4 = -0.142\times10^6$ N/m $k_5 = 0.328\times10^6$ N/m, $k_6 = 0.233\times10^6$ N/m
4	$\tilde{\mathbf{m}}'' = \begin{bmatrix} 752.2 & 0 & 0 \\ 0 & 1596.4 & 0 \\ 0 & 0 & 2219.1 \end{bmatrix}$	$\tilde{\mathbf{k}}'' = \begin{bmatrix} 0.157 & -0.001 & -0.157 \\ -0.001 & 2.090 & -1.699 \\ -0.157 & -1.699 & 2.850 \end{bmatrix} \cdot 10^6$	$\tilde{\mathbf{X}}'' = \begin{bmatrix} -1.1320 & -0.7891 & -0.0755 \\ -0.1904 & 0.9979 & -0.8401 \\ -0.1997 & 0.8288 & 0.7060 \end{bmatrix}$
	parameters of the second reduced model	$m_1 = 0.752\times10^3$ kg $m_2 = 1.596\times10^3$ kg $m_3 = 2.219\times10^3$ kg	$k_1 = 0.157\times10^6$ N/m, $k_2 = 1.699\times10^6$ N/m $k_3 = 1.152\times10^6$ N/m, $k_4 = 0$ N/m $k_5 = 0$ N/m, $k_6 = 0.234\times10^6$ N/m
5	scaling parameters of the laboratory stand	$m_1 = 27$ kg $m_2 = 57$ kg $m_3 = 79$ kg	$k_1 = 0.56\times10^4$ N/m, $k_2 = 6.07\times10^4$ N/m $k_3 = 4.11\times10^4$ N/m, $k_4 = 0$ N/m $k_5 = 0$ N/m, $k_6 = 0.836\times10^4$ N/m

Elements of modal vectors can be approximately read from the graphs showing mode shape (Fig. 5a). The mass matrix, according to equation (2), should be a diagonal matrix, and the elements of the stiffness matrix should be possible to determine after the correct selection of stiffness coefficients of the springs. In successive rows of the table, attempts at adaptation of the reduced model to the accepted structure are presented. The data in row third corresponds to a model which cannot be realized due to negative spring stiffness coefficient. Adaptation is carried out through the changing of the coordinates of eigenvectors, which is associated with tantamount to partial abandonment of the selected points motion description. Finally the parameters determined for the reduced model have been presented in the fourth row of the table. Parameters of the laboratory stand obtained as a result of scaling are presented in the last row of Table 1.

5. LABORATORY STAND OF THE MAST

The values of parameters determined for the reduced model made it possible to design a laboratory stand for testing active vibration systems of slender constructions at the Department of Process Automation of the AGH Technical University. The structure of the laboratory mast model is comprised of three segments, constituted by steel plates connected by four aluminum angle bars. Additionally, the mast is equipped with four stay lines. The equivalent stiffness coefficient of lines is equal to k_6.

The presented algorithm does not account for problems related to the stability of a slender construction subjected to the influence of the gravity forces. For this reason, before laboratory model was built, the stability condition was checked. The structure performs the stability condition with a slight surplus. Taking into account the safe exploitation of

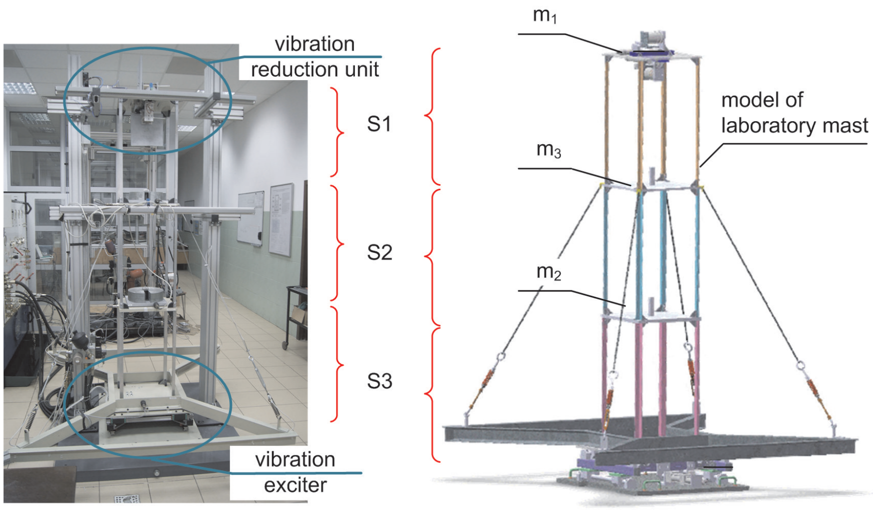

Fig. 6. Laboratory stand for testing the active systems of vibrations reduction

the stand, especially for large amplitudes, possible replacement of the angle bars in the top section (Fig. 6) is foreseen. Changing angle bars in the top section caused the change of masses of remaining construction elements. The mechanical part of the stand is equipped with a bi-axial exciter of mechanical vibrations. The exciter, powered by a hydraulic aggregate, makes it possible to generate displacements in two axes, which considerably increases the testing capabilities of the stand (Pluta *et al. 2010*). An electrodynamic vibration reduction unit, used for testing control algorithms is installed on the upper platform. The view of the stand has been shown in Figure 6.

6. CONCLUSIONS

The calculations indicated significant problems related to the physical realization of models of complex constructions. Because of using of available materials the calculated parameters should be usually verified. The presented algorithm of calculations allows for the assessment of displacements and forces occurring in real construction on the basis of the test results obtained using the laboratory model.

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