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Jacek SNAMINA*

MODEL BASED PREDICTIVE CONTROL OF BEAM WITH MAGNETORHEOLOGICAL FLUID

SUMMARY

In the work a model predictive control method was applied to control the beam vibrations. Model predictive control (MPC) is widely used as advanced control methodology. The considered beam consists of two outer layers made of aluminium and MR fluid layer in between. Activation of the MR fluid is realized by magnetic field.
The analysis of strain and stress in three-layered beam were done. Then the equation of forced vibration of beam in the vicinity of the first resonances was derived. Based on this equation the simulation of MPC application was performed for the sinusoidal and random excitation.

***Keywords**: beam vibrations, magnetorheological fluid, model predictive control*

STEROWANIE PREDYKCYJNE DRGANIAMI BELKI Z CIECZĄ MAGNETOREOLOGICZNĄ

W pracy zastosowano algorytm sterowania predykcyjnego przy wykorzystaniu modelu do redukcji drgań belki z cieczą magnetoreologiczną (MR). Rozważana belka składa się z dwóch zewnętrznych warstw wykonanych z aluminium oraz umieszczonej pomiędzy nimi warstwy cieczy MR. Zmiana własności cieczy MR następuje w wyniku wytworzenia przez elektromagnes pola magnetycznego obejmującego część belki.
Przedstawiono analizę stanu odkształcenia i stanu naprężenia w poszczególnych warstwach belki. Wyprowadzono równania opisujące drgania w otoczeniu pierwszego rezonansu. Na podstawie zbudowanego modelu zaproponowano algorytm sterowania predykcyjnego, którego celem była redukcja drgań belki. Wykonano symulację drgań belki dla wymuszenia sinusoidalnego oraz stochastycznego.

***Słowa kluczowe**: drgania belek, ciecze magnetoreologiczne, sterowanie predykcyjne*

1. INTRODUCTION

MR fluids are non-colloidal suspensions consisting of high concentration magnetically polarizable particles suspended in a non-magnetic liquid carrier. The particles of MR fluids, made of iron or iron oxides, have a size of a few microns. MR fluid changes itself from linear viscous liquids to semi-solids when it is exposed to a magnetic field (Yalcinitas and Dai 2004). The rheological properties of MR fluids depend on the concentration of particles, particle size and properties of the carrier fluid. The response time of MR fluids under fluctuating magnetic fields is of the order of milliseconds. Thereby MR fluids are excellent for applications where strong dynamic features are required. Many control strategies are described for such semi-active damping systems (Holnicki-Szulc and Marzec 2000, Mróz *et al.* 2010).

Magnetorheological fluids are used, among others, in various types of dampers. MR dampers are applied in many structures such as bridges, automobiles, and space platforms. In recent years a new method of the exploitation of MR fluids is developed. Fluids are embedded in multi-layer beams and plates to control its vibration. Due to MR fluids properties, the damping and stiffness of the whole structure can be changed by using the magnetic field.

This paper presents the modeling and the simple control of a three-layered beam filled with MR fluid (Snamina *et al.* 2010a, 2010b). The beam is composed of two aluminium outer layers and an MR fluid layer placed between them. A model predictive control method was applied to control the beam vibrations. Model predictive control (MPC) is widely used as advanced control technique (Arnold and Schiehlen 2008, Camacho and Bordons 2004).

2. ANALYSIS OF STRAIN AND STRESS IN THREE-LAYERED BEAM

A schematic drawing of the cantilever beam is presented in Figure 1. The main dimensions of the beam, shown in the drawing, are: l, b – length and width, h and h_0 – thickness of the outer layers and the MR fluid layer.

We consider small vibrations of the beam. The description of the motion is based on the following assumptions:

- all points placed on the normal to the beam in its equilibrium state move with the same transverse displacement,
- aluminium layers are pure elastic, the energy is not dissipated in the outer layers,
- there is no slip at the interface of the MR fluid and the outer layers,
- longitudinal stresses in the MR fluid layer are negligible even though the longitudinal strain is present, the MR fluid layer undergoes shear, but no direct stress.

* AGH University of Science and Technology, Faculty of Mechanical Engineering and Robotics, Department of Process Control, al. A. Mickiewicza 30, 30-059 Krakow, Poland; snamina@agh.edu.pl

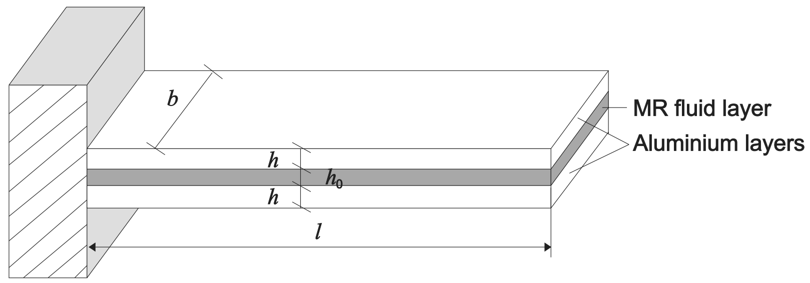

Fig. 1. Schematic drawing of the beam

The transversal section of all layers in the non-deformed state and in the deformed state of the beam is shown in Figure 2. Additionally, the coordinate systems used are depicted in this figure.

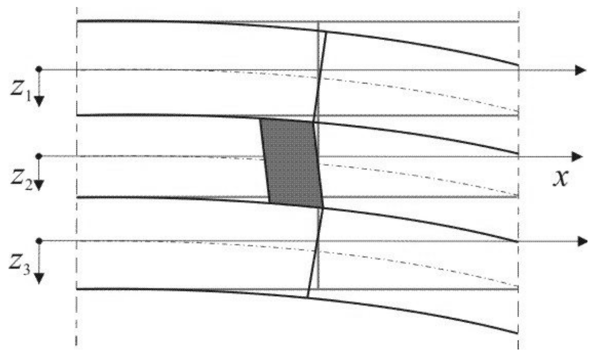

Fig. 2. Longitudinal section of the beam

Due to the displacements of the points placed on the aluminium surfaces adhering to MR layer, segments of MR fluid layer deform in shear. The deformed segment of the MR fluid layer is shown by the shaded area in Figure 2.

The tangent forces between MR fluid layer and aluminium layers cause the stretching (or compressing) of the aluminium beams. The displacement of the middle plane of the upper layer is described by function $u(x, t)$. Due to the symmetry of the beam, the displacement of the middle plane of the lower layer is in the opposite direction in relation to the upper layer. Thus the motion of the beam can be described in terms of transverse displacement $w(x, t)$ and axial displacement $u(x, t)$ of the middle plane of the upper aluminium layer.

Points in the upper aluminium layer move according to the displacement vector $\vec{u}_1$. The components of $\vec{u}_1$ are:

$$\begin{cases} (u_1)_x = u - z_1 \dfrac{\partial w}{\partial x} \\ (u_1)_z = w(x,t) \end{cases} \tag{1}$$

Similarly, the components of vector $\vec{u}_3$ describing the displacement of the points given by co-ordinate z_3 in the lower aluminium layer are:

$$\begin{cases} (u_3)_x = -u - z_3 \dfrac{\partial w}{\partial x} \\ (u_3)_z = w(x,\ t) \end{cases} \tag{2}$$

Taking into account that there are no slips at the interfaces of the MR fluid layer and outer layers, we assume the following displacement $\vec{u}_2$ in the MR fluid layer:

$$\begin{cases} (u_2)_x = z_2 \left(\dfrac{h}{h_0} \dfrac{\partial w}{\partial x} - \dfrac{2u}{h_0} \right) \\ (u_2)_z = w(x,\ t) \end{cases} \tag{3}$$

The components of strain in each layer can be calculated using the known strain-displacement relations.

The non-zero components of strain are:

– in the upper aluminium layer:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z_1 \frac{\partial^2 w}{\partial x^2} \tag{4}$$

– in the lower aluminium layer:

$$\varepsilon_x = -\frac{\partial u}{\partial x} - z_3 \frac{\partial^2 w}{\partial x^2} \tag{5}$$

– in the MR fluid layer:

$$\begin{cases} \varepsilon_x = z_2 \left(\dfrac{h}{h_0} \dfrac{\partial^2 w}{\partial x^2} - \dfrac{2}{h_0} \dfrac{\partial u_x}{\partial x} \right) \\ \gamma_{xz} = \left(\dfrac{h}{h_0} + 1 \right) \dfrac{\partial w}{\partial x} - \dfrac{2}{h_0} u \end{cases} \tag{6}$$

As it was assumed, the aluminium layers are pure elastic. Thus the stress components can be calculated using Hook's law. Taking into account the properties of the MR fluid we assumed that the non-zero longitudinal strains ε_x in MR fluid layer do not develop longitudinal stresses. In the pre-yield regime, the MR fluid demonstrates a visco-elastic behaviour. The stress and strain can be related using the complex shear modulus. The complex modulus G represents simultaneously the stiffness and damping properties of the MR fluid during harmonic motion.

3. FORCED VIBRATIONS OF THE BEAM IN THE VICINITY OF RESONANCES

Beams are often used as elements of constructions executing vibrations. In this case the points, where the beam is clamped, are sources of beam vibrations. In calculations the vertical displacement of the beam handle were assumed. In this case, it is very convenient, to describe the motion of the beam in relation to its handle position because the relationships between the relative displacements and the strain components have the same form as those introduced in previous section.

For frequencies of excitation in vicinity of arbitrary natural frequency, the vibration of the beam can be described taking into account only one mode shape, corresponding to the chosen natural frequency.

$$\begin{bmatrix} u(x,t) \\ w(x,t) \end{bmatrix} = \begin{bmatrix} U(x) \\ W(x) \end{bmatrix} \cdot q(t) \tag{7}$$

where $[U(x),\ W(x)]^{\mathrm{T}}$ is the mode shape vector. The first component $U(x)$ is associated with the axial displacement of the middle plane of the upper layer and the second component $W(x)$ with the transversal displacement of the whole beam section. The term $q(t)$ is the time dependent function describing the motion of the beam with assuming mode.

When the tangent forces between the middle layer and the outer layers of the beam are equal to zero, the outer layers are not stretched and therefore $U(x) = 0$. It can be said that the layers are completely uncoupled in axial direction. When all layers are made of aluminium and there are no slides between them, $U(x)$ is related with $W(x)$ by the following equation

$$U(x) = \frac{h_0 + h}{2} \frac{dW}{dx} \tag{8}$$

In this case the layers are fully coupled. It is evident that MR fluid, placed in the internal layer, links the outer layers in lesser degree than aluminium. Thus the following relationship between $U(x)$ and $W(x)$ can be proposed in this case

$$U(x) = \kappa \frac{h_0 + h}{2} \frac{dW}{dx} \tag{9}$$

where κ is the coefficient describing the degree of coupling between the layers. When outer layers are uncoupled the coefficient κ is equal to zero. Taking into account the above relationship, the mode shape can be described by only one function $W(x)$.

During vibrations the horizontal displacements of the middle planes of aluminium layers depend on elasticity forces and inertial forces in axial direction. It is apparent that for low natural frequencies of beam the horizontal displacements are, first of all, a result of the equilibrium of elasticity forces, because the axial components of inertial forces are small and they can be neglected. Using this property, the coupling coefficient κ can be derived by equating to zero the first derivative of the sum of the potential energy of outer layers stretching and the potential energy of the MR layer shearing.

Finally the beam displacement can be described by the following matrix equation

$$\begin{bmatrix} u(x,t) \\ w(x,t) \end{bmatrix} = \begin{bmatrix} \kappa \frac{h + h_0}{2} \frac{dW}{dx} \\ W(x) \end{bmatrix} \cdot q(t) \tag{10}$$

In exact calculations, the function $W(x)$ is determined by solving the eigenvalue problem. In approximate calculations, the function $W(x)$ can be assumed in the simplified form satisfying the kinematic boundary conditions.

Taking into account the physical interpretation of complex modulus and using Ritz-Galerkin method of discretization of continuous systems, after a number of transformations, the equation of motion can be written in the simple form

$$m\ddot{q} + b\dot{q} + kq = -m_w \ddot{z}(t) \tag{11}$$

where m is the modal mass, k is the modal stiffness coefficient and b is the modal damping. The modal mass is a sum of the substitute mass m_{MR} associated with MR fluid and the substitute mass m_{Al} associated with aluminium layers. By analogy the modal stiffness coefficient is a sum of the substitute stiffness k_{MR} associated with MR fluid and the substitute stiffness k_{Al} associated with aluminium layers. The symbol m_{w} stands for the mass in the formula for inertial force calculation. Symbols m, k, b and m_{w} can also be interpreted as operators that allow calculating the substitute mass, stiffness and damping using modal shape functions.

4. MODEL-BASED PREDICTIVE CONTROL OF BEAM VIBRATION

In control engineering the feedback control with fixed controller parameters is in common use. The physical properties of the object and the operating conditions are the base of a controller selection.

Recently, due to the fast development of computers and microcontrollers, the methods of control based on the repeated solution of state equations are more often used. One of these methods is the Model Predictive Control (MPC) method. The formulation of MPC method is very simple.

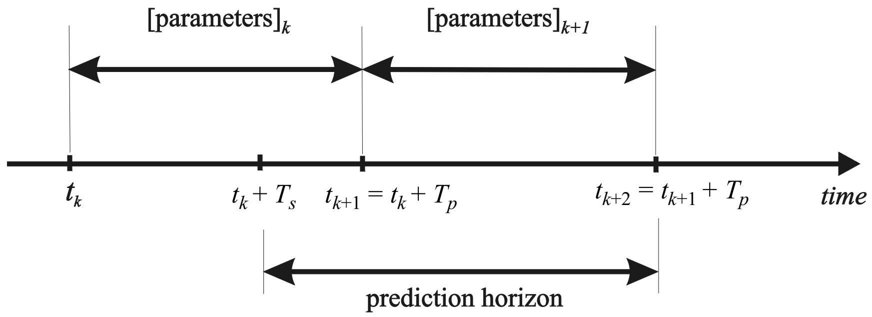

Fig. 3. Time intervals used in MPC method

As in general formulations of control problems, the system is described by the state equations of the following form

$$\dot{x} = f(x,u) \tag{12}$$

where x is a state vector and u is a control vector. The optimal control vector is determined by minimization of the cost function

$$\min_{u} J(u,x) \tag{13}$$

with the appropriate constrains.

In MPC method, the problem of determining of optimal control vector is solved on-line. The optimization is done over the time interval (t_k+T_s, t_{k+2}) as shown in Figure 3. The length of this interval is called the prediction horizon. Optimal parameters are used in the interval (t_{k+1}, t_{k+2}). The time T_s is a bit less than T_p. The difference T_p–T_s is a time designed for calculating the new optimal parameters. The ranges in which optimal parameters are calculated and used against the time axis are shown in Figure 3.

The MPC method can be adopted to control the damping and stiffness coefficients of the beam. In the formulation of the problem the permissible values of stiffness and permissible values of damping are assembled in two separate sets. The cost function was assumed as an average value of the absolute value of displacement of the end point of the beam. The optimal parameters are the result of the minimization of the cost function over the interval (t_k+T_s, t_{k+2}). Like in the basis version of MPC method, the optimal parameters are used in the interval (t_{k+1}, t_{k+2}). The procedure of parameters determination is repeated on-line.

5. NUMERICAL CALCULATIONS

The calculations were performed for the following geometric parameters of the beam: l = 0.4 m, b = 0.03 m, h = 0.001 m, h_0 = 0.002 m. The mechanical properties of aluminium layers are described by Young modulus $E = 0.7 \times 10^{11}$ N/m^2 and density $\rho = 2.7 \times 10^3$ kg/m^3. The density of MR fluid $\rho_{MR} = 3.5 \times 10^3$ kg/m^3.

In order to identify the parameters of the model, introduced in previous section, the coefficients m, b, k, m_w should be determined. The mass coefficients m, m_w can be calculated with satisfactory precision using geometric dimensions and density of aluminium and MR fluid. The stiffness coefficient and damping coefficient have to be calculated on the basis of experimental results. The experiments were conducted on the laboratory stand described in (Snamina *et al.* 2010a) and schematically shown in Figure 4.

The recorded displacements of the end point of the beam are used in identification process. The parameters of the model are as follows: m_{Al} = 0.015 kg, m_{MR} = 0.019 kg, m_w = 0.054 kg. Three values of stiffness coefficient {95, 121, 122} N/m and corresponding values of damping coefficients {0.22, 0.55, 0.61} Ns/m were chosen to the next calculations. These values were obtained for the electromagnet placement y_m = 0.115 m and the following values of the current {5, 7.5, 10} A.

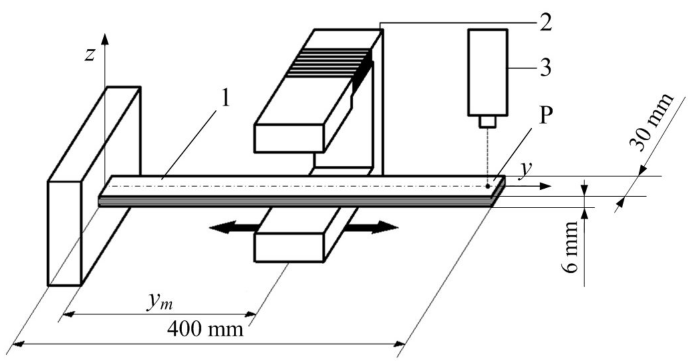

Fig. 4. Experimental set-up: 1 – beam, 2 – electromagnet, 3 – laser vibrometer

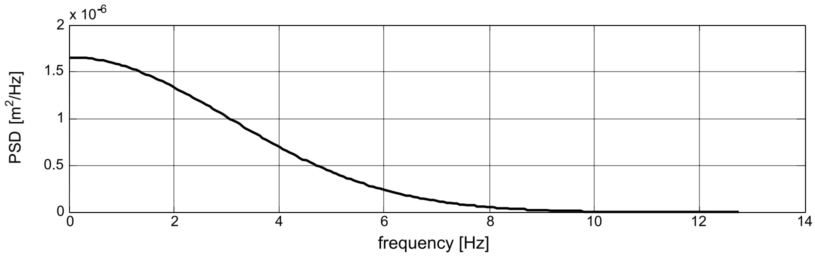

Fig. 5. Power spectral density of the kinematic excitation

The calculations were done in the vicinity of the first natural frequency (8.4 Hz). The prediction horizon was assumed to be five times larger than period corresponding to natural frequency. For prediction horizons less than this value, the performance of control was dissatisfied.

Two types of excitation were applied – the random excitation and the sinusoidal excitation. The power spectral density of the kinematic random excitation is shown in Figure 5.

The results of simulation of the beam motion with control and, for comparison, without control are presented in Figures 6–8. The displacement of the end point of the beam is calculated. Figure 6 presents the results in the case of random excitation, Figure 7 presents the results in the case of sinusoidal excitation with frequency 8 Hz (below the first natural frequency) and Figure 8 presents the results in the case of sinusoidal excitation with frequency 9 Hz (above the first natural frequency).

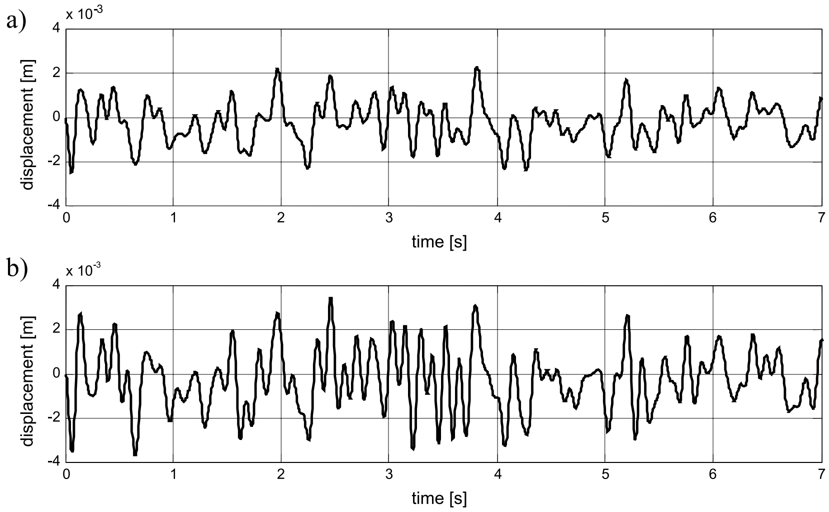

Fig. 6. Simulation of the relative displacement of the end of the beam: a) with control; b) without control (stochastic excitation)

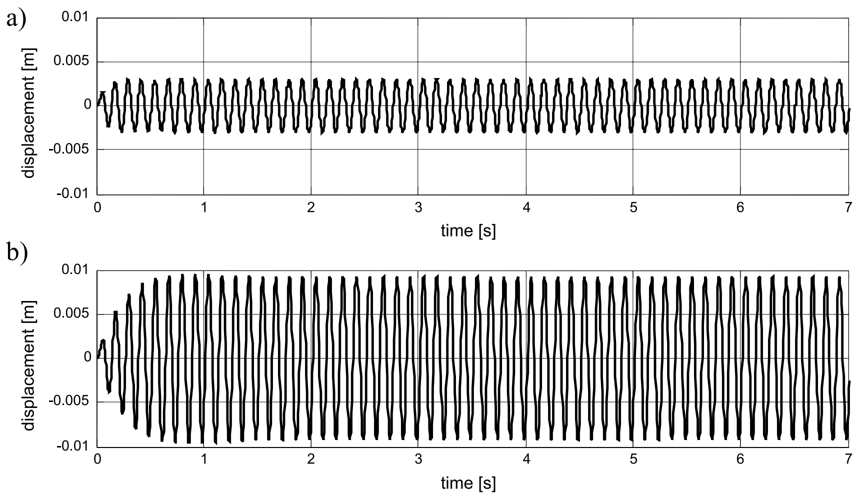

Fig. 7. Simulation of the relative displacement of the end of the beam: a) with control; b) without control (sinusoidal excitation with frequency bellow resonance)

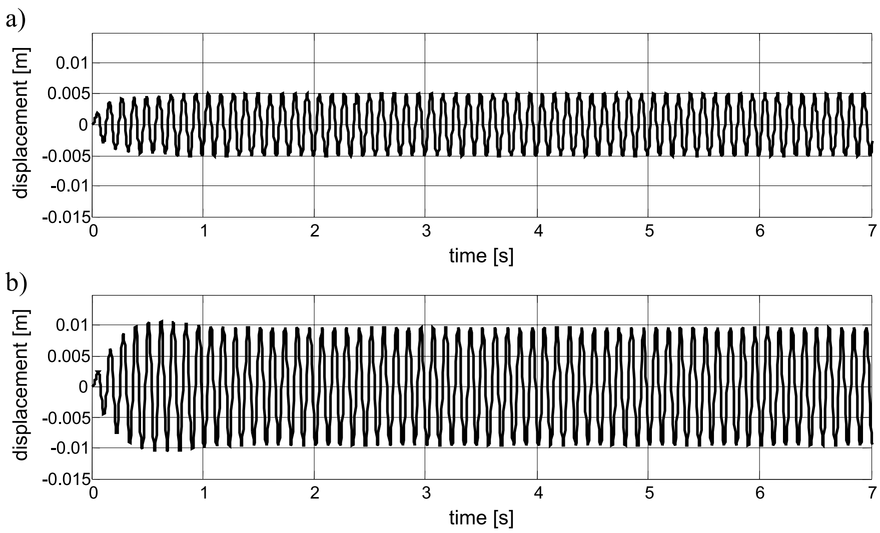

Fig. 8. Simulation of the relative displacement of the end of the beam: a) with control; b) without control (sinusoidal excitation with frequency above resonance)

6. CONCLUSIONS

In this study the forced vibration of the cantilever beam with MR fluid layer was analyzed. In order to reduce the beam vibrations the method of model predictive control was adopted. The results of calculations prove that this method of control the stiffness and damping coefficients can be useful in semi-active vibroisolation systems.

The performance of control method depends on choosing the prediction horizon. For small prediction horizons, less than the period corresponding to natural frequency, the performance is dissatisfied. In order to obtain the good performance the prediction horizon should be equal to a few periods.

The influence of MR fluid on forced vibration of the beam is very weak. Therefore, the noticeable decrease of beam vibration can be observed only for the frequencies in the narrow interval around the resonance.

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