

FEED-FORWARD COMPENSATION FOR NONLINEARITY OF VIBRATING PLATE AS THE SOUND SOURCE FOR ACTIVE NOISE CONTROL

SUMMARY

Active Noise Control (ANC) systems are usually designed in the feed-forward structure with adaptive linear control filters. However, performance of such systems, when a vibrating plate is used as the secondary source, may be poor due to significant non-linearity of the plate response. The linear systems are then unable to cope with higher harmonics generated by the nonlinearity. One solution to this problem is to apply a nonlinear ANC algorithm. However, it adds additional complexity to this layer. It is particularly severe for multichannel systems, where the algorithms are complex by themselves, and making them nonlinear may significantly reduce their scalability. In this paper, another approach is proposed. Multiple actuators are mounted on a single plate, in order to effectively excite more vibration modes and generate a higher acoustic power, than in case of a single actuator. The response of the vibrating plate as the sound source is then linearized with a set of nonlinear finite impulse response filters, operating individually for each actuator. The Filtered-x LMS algorithm is adopted to update parameters of the nonlinear filters. The control system is experimentally verified and obtained results are reported.

Keywords: active noise control, active structural acoustic control, vibrating plate, sound radiation, adaptation, feed-forward, equalization, nonlinear control

KOMPENSACJA NIELINIOWOŚCI PŁYTY DRGAJĄCEJ JAKO ŹRÓDŁA DŹWIĘKU DLA SYSTEMÓW AKTYWNEJ REDUKCJI HAŁASU

Systemy aktywnej redukcji hałasu zwykle projektowane są w strukturze kompensacyjnej z adaptacyjnymi liniowymi filtrami sterującymi. Wówczas, w przypadku zastosowania płyty drgającej jako źródła dźwięku, skuteczność działania może być zmniejszona z powodu występujących nieliniowości w torze wtórnym. Liniowy układ kompensacji nie jest w stanie poradzić sobie z wyższymi harmonicznymi generowanymi przez nieliniowości. Jednym z rozwiązań jest zastosowanie nieliniowego algorytmu aktywnej redukcji hałasu. Zwiększa to jednak złożoność układu w tej warstwie sterowania. W przypadku wielokanałowych systemów aktywnej redukcji hałasu, które same w sobie są złożone, wprowadzenie nieliniowości w algorytmie znacząco zmniejsza skalowalność całego systemu. W niniejszym artykule zaproponowane jest inne podejście. Aby zapewnić efektywne pobudzenie większej liczby mod i uzyskanie większej mocy akustycznej jak w przypadku pojedynczego elementu wzbudzającego, zastosowano wiele elementów wykonawczych wzbudzających drgania. Odpowiedź płyty drgającej pełniącej rolę złożonego źródła dźwięku jest linearyzowana przez zbiór nieliniowych filtrów o skończonej odpowiedzi impulsowej działających niezależnie na poszczególne elementy wykonawcze. Do aktualizacji parametrów filtrów nieliniowych zastosowany jest algorytm Filtered-x LMS. Zaproponowany system sterowania został poddany weryfikacji eksperymentalnej, a uzyskane wyniki zostały zaprezentowane i skomentowane.

Słowa kluczowe: aktywna redukcja hałasu, płyta drgająca, nieliniowe sterowanie, kompensacja, emisja dźwięku, adaptacja, ewaluacja

1. INTRODUCTION

Vibrating plates have been found to have a potential to be used as sound sources for Active Noise Control (ANC) applications (Hansen 1997, Fahy 2007, Elliot 2001, Pietrzko 2009). ANC systems are usually designed in the feed-forward structure with adaptive linear control filters. However, performance of such systems, when a vibrating plate is used as the secondary source, may be poor due to significant non-linearity of the plate response (El Kadiri *et al.* 1999, Saha *et al.* 2005, Stuebner *et al.* 2009). The linear systems are then unable to cope with higher harmonics generated by the nonlinearity.

One solution to this problem is to apply a nonlinear ANC algorithm. However, it adds additional complexity to this

layer. It is particularly severe for multichannel systems, where the algorithms are complex by themselves, and making them nonlinear may significantly reduce their scalability.

In this paper, another approach is proposed. Multiple actuators are mounted on a single plate, in order to effectively excite more vibration modes and generate a higher acoustic power, than in case of a single actuator (Wiciak 2008). The response of the vibrating plate as the sound source is then linearized with a set of nonlinear finite impulse response filters, operating individually for each actuator. The Filtered-x LMS algorithm is adopted to update parameters of the nonlinear filters. The control system is experimentally verified and obtained results are reported.

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2. NONLINEAR CONTROL

A linear feed-forward control system operating for a nonlinear secondary path is unable to cope with harmonics generated by the nonlinearity. Because of this even an adaptive filter, which has an inherited simple nonlinearity (bilinearity for LMS), is unable to properly issue appropriate secondary sound to effectively reduce the primary noise.

For feed-forward systems this problem can be solved by using a nonlinear filter. When a model of the secondary path is known and it is sufficiently simple, an appropriate nonlinear control filter can be designed by inverting the secondary path model and adding a delay to guarantee a causal solution. If the model is unknown or it is too complex, a nonlinear black-box plant model can be used to design the control filter. Parameters of the black-box model should be estimated, and the control filters computed accordingly. This approach is inefficient for adaptive control because computation of control filter parameters can require too many mathematical operations. A better solution for adaptive control is a direct estimation of optimal control filter parameters. This approach is commonly used for ANC. However, usually a linear control filter model is then assumed.

One of the most frequently used nonlinear black-box models is based on an artificial neural network. The artificial neural network can effectively model most of static nonlinear functions. A dynamic nonlinear model can be created by adding previous samples of input and output signals as additional network inputs. Such model for the feed-forward ANC with a vibrating plate was successfully used in (Hansen 1997). However the proper structure of the artificial neural network is sometimes hard to find. The artificial neural network is also computationally demanding.

In this paper another approach is used. The poor performance of a linear feed-forward system is caused by lack of higher harmonics in the input signal. However, these harmonics can be generated by using a nonlinear function prior to the linear control filter, as in the Hammerstein model:

$$c(i) = \sum_{j=0}^m h_j f(x(i-j)) \quad (1)$$

where i is the sample number, $c(i)$ is the model output, $x(i)$ is the model input, $f(x)$ is a nonlinear static function, and h_j are parameters of the impulse response of the linear path. The $f(x)$ function is modelled as a linear combination of functions:

$$f(x) = \sum_{k=0}^n h_k f_k(x) \quad (2)$$

The output of the Hammerstein model is then equal to:

$$c(i) = \sum_{j=0}^m h_j \sum_{k=0}^n c_k f_k(x(i-j)) \quad (3)$$

To effectively estimate parameters of the model (3) can be rewritten as:

$$c(i) = \sum_{j=0}^m \sum_{k=0}^n w_{j,k} f_k(x(i-j)) \quad (4)$$

where $w_{j,k} = h_j c_k$. This model is linear with respect to $w_{j,k}$ parameters. Instead of estimating the Hammerstein model parameters it is much easier to estimate some functions of the parameters. This model is also more general, because a different linear dynamics can be used for each nonlinear function.

The proposed resulting model is presented in the form of a block diagram in Figure 1. It can be interpreted as a sum of Hammerstein models with arbitrary $f_k(x)$ functions. This feed-forward controller model is proposed to be used for each actuator on the vibrating plate.

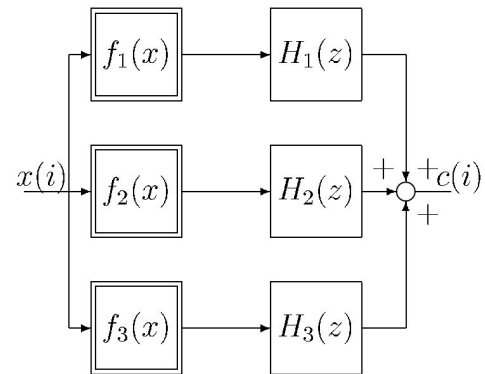


Fig. 1. Control filter based on the Hammerstein model

3. ESTIMATION OF CONTROLLER PARAMETERS

The linear part of the control filter is a multiple input – single output FIR filter. It can be interpreted as a linear adaptive Wiener filter with multiple reference inputs, and the FXLMS algorithm can be used for estimating its parameters.

For control of sound radiation from the vibrating plate a single input – multiple output adaptive feed-forward compensator is chosen as presented in Figure 2. The notation is as follows: $x(i)$ – input signal, $d(i)$ – desired signal at the position of the microphone, $e(i)$ – error signal being the difference between desired signal and sound acquired by the microphone, W_1 – W_3 – nonlinear control filters (see Fig. 1) used to drive the selected MFC patches, B – model of the desired response, S_j – secondary paths due to different MFC actuators, \hat{S}_j – models of respective secondary paths. Parameters of the j -th control filter for the k -th nonlinear function, stored in vector $\underline{w}_{j,k}(i) = [w_{j,k,0}(i), w_{j,k,1}(i), \dots, w_{j,k,N-1}(i)]^T$, are updated with the Normalized Leaky LMS algorithm (Pa-welczyk 2005):

$$\underline{w}_{j,k}(i+1) = \alpha \underline{w}_{j,k}(i) - \mu \frac{r_{j,k}(i)}{r_{j,k}^T(i)r_{j,k}(i) + \zeta} e(i) \quad (5)$$

where $0 \ll \alpha < 1$ is the leakage coefficient, μ is the convergence coefficient, and ζ is a parameter protecting against division by zero in case of lack of excitation. In this equation $r_{j,k}(i) = [r_{j,k}(i), r_{j,k}(i-1), \dots, r_{j,k}(i-(N-1))]^T$ is a vector of regressors of the filtered-reference signal obtained as:

$$r_{j,k}(i) = \hat{S}_j(i)^T x_k(i) \quad (6)$$

where $\hat{S}_j(i) = [\hat{S}_{j,0}(i), \hat{S}_{j,1}(i), \dots, \hat{S}_{j,M-1}(i)]^T$ is the j -th secondary path filter impulse response model, $x_k(i) = [f_k(x(i)), f_k(x(i-1)), \dots, f_k(x(i-M+1))]^T$ is a vector of regressors of the k -th function of input signal.

The j -th control signal u_j is calculated as:

$$u_j(i+1) = \sum_{k=0}^n w_{j,k}(i)^T x_{u,k}(i) \quad (7)$$

where $u_j(i+1)$ is j -th control signal value, and $x_{u,k}(i) = [f_k(x(i)), f_k(x(i-1)), \dots, f_k(x(i-(N-1)))]^T$ is a vector of functions f_k of last N input signal values.

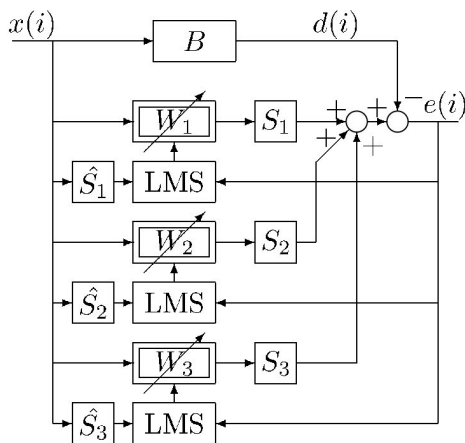


Fig. 2. Control system diagram

4. SELECTION OF THE NONLINEAR FUNCTIONS

Input signals for the multichannel FXLMS algorithm should be uncorrelated to yield the unique solution (Tu 2000). This is a challenge because all inputs are functions of the same common input signal $x(i)$.

Since $f_k(x(i))$ is calculated based on $x(i)$ only, correlation for the same time could only be checked, and $f_k(x(i))$ should be made uncorrelated to $f_l(x(i))$:

$$\forall_{k \neq l} E \{ f_k(x(i)) f_l(x(i)) \} = 0 \quad (7)$$

The simplest idea is to use polynomials as the nonlinear functions. However, for instance, x^n used to generate the n -th harmonic will not satisfy the above condition. This is demonstrated in Figure 3, where x^3 generates not only the

3rd harmonic but also the fundamental frequency. In turn, x^4 generates not only the 4-th harmonic but also the 2-nd harmonic and a constant. Generally, x^n generates the n -th harmonic and also $(n-2)$ -nd, $(n-4)$ -th, ... harmonics.

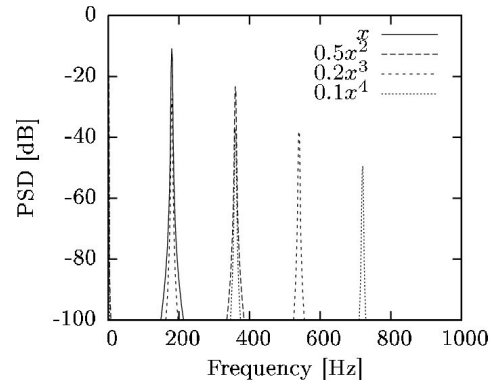


Fig. 3. Power Spectral Density of 180 Hz tone passed through polynomial

For tonal signals this problem can be avoided by using Chebyshev polynomials. Such polynomials generate one harmonic only (Fig. 4). Therefore, using such polynomials allows for applying an individual linear filter for each harmonic. Unfortunately, such property of the Chebyshev polynomials is not valid for signals other than tones.

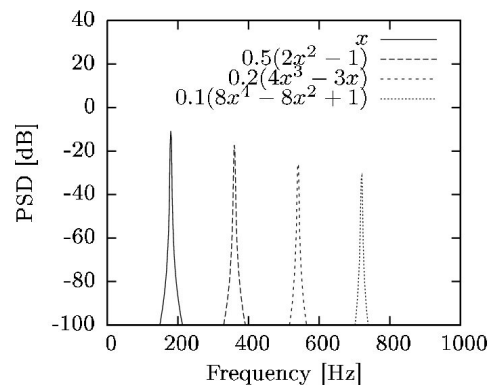


Fig. 4. Power Spectral Density of a 180 Hz tone passed through the Chebyshev polynomial

For other types of input signals, other polynomials that generate uncorrelated outputs can be found. For example, for a normally distributed random signal the Hermite polynomials can be used.

However, generation of multiple harmonics by a single polynomial can also be useful for simple signals. For example, polynomials x^4 and x^5 can be used for generating first five harmonics.

In the general case, correlation between outputs of the nonlinear part can be removed with an additional adaptive decorrelation filter (Tu 2000). However, it would be achieved with the expense of a higher computational load and possible slow-down of convergence of the whole algorithm.

5. EXPERIMENTAL VERIFICATION

A clamped rectangular aluminium vibrating plate of dimensions $400 \times 500 \times 1$ mm is mounted at a wall of a laboratory room. The sound radiated from the plate is measured by a microphone located 1.2 m away from the plate at the centre line in the laboratory room.

Three Macro-Fiber Composite (MFC) patches are attached to the plate as presented in Figure 5. Locations of the patches were chosen experimentally with the highest sound radiation criterion. The MFC patches work in the bending mode, i.e. they bend when appropriate voltage is applied (Smart Material 2010). MFC patches using the d33 effect are used. This type of elements has a higher power-per-size ratio, but very high voltage (-500 V to 1500 V) is needed.

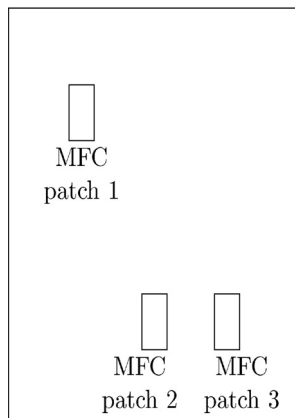


Fig. 5. Positions of MFC elements on the plate

For all experiments the sampling frequency was set to 2 kHz and 8th order Butterworth lowpass analogue filters

with 600 Hz cut-off frequency were used as antialiasing and reconstruction filters. Additional high-pass digital filter with transfer function equal to $(1 + z^{-1}) / (1 - 0.999z^{-1})$ is used for microphone inputs to filter out DC offset. The order of the FIR path models is $M = 256$ for all experiments. This value was chosen based on impulse response analysis. The order of FIR control filters is $N = 256$ for all experiments.

An exemplary result for a 180 Hz tone as the desired signal is shown in Figure 6. In case of the linear feed-forward controller (dashed line, using $x(i)$ signal only) it is observed that unwanted higher harmonics are also generated due to plate nonlinearity. The linear feed-forward control system is unable to reduce them because they are not present in the input signal. For the same reference signal and two input signals for FXLMS – $f_0(x) = x^4$, $f_1(x) = x^5$ the controller is able to reduce the second harmonic (360 Hz) by 34 dB, third harmonic (540 Hz) by 28 dB and fourth harmonic (720 Hz) by 6 dB (solid line). For first four harmonics the Total Harmonic Distortion (THDR) is reduced from 10% to 0.2% (Shmilovitz 2005). For comparison, results obtained with the Internal Model Control (IMC) feed-back controller having the potential to reduce effects of non-linearity are also included. In this case the second harmonic is reduced by 15 dB and third harmonic is reduced by 5 dB, as compared to the linear feed-forward controller. The THDR_R for IMC is equal to 2%. It should, however, be emphasised that the IMC controller generally exhibits poorer performance than the feed-forward controller operating under causal conditions, particularly for signals more complex than a single tone.

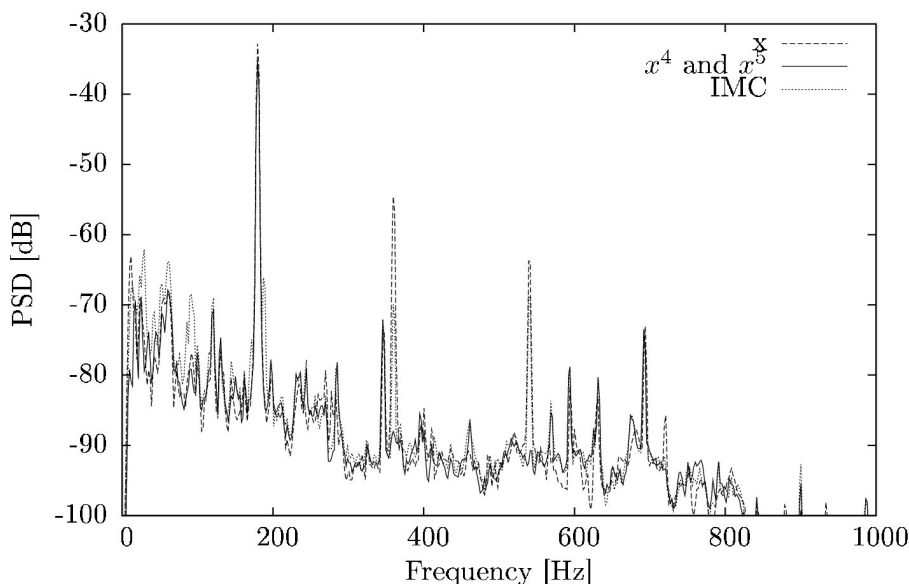


Fig. 6. Power Spectral Density of a 180 Hz tone generated by the plate under different control

6. CONCLUSIONS

In this paper the problem of harmonics generation due to vibrating plate non-linearity has been considered. It can significantly degrade performance of the ANC system. The fundamental frequencies interfere with primary disturbance and they are reduced even to the floor level. However, higher harmonics are left and the total noise reduction level is very poor or the noise can even be reinforced, particularly when taking into account that the human hearing system is much more sensitive to higher harmonics as compared to the fundamental ones (Bauer 1966). Therefore, for vibrating plates to be found suitable as sound sources and more useful for ANC, they responses should be linearised. In this paper an efficient non-linear adaptive feed-forward compensation system has been proposed. It has proven to effectively compensate the higher harmonics generated by the plate, much better than with inherited features of linear feedback. The proposed approach has a potential to significantly improve performance of an Active Noise Control or Active Structural Acoustic Control system.

Acknowledgements

The research reported in this paper has been partially supported by the Ministry of Higher Education and Science, Poland, under the grant no. N N514 232037 performed in 2009–2012.

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