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CRITERION FOR ANGLE PREDICTION FOR THE CRACK IN MATERIALS WITH RANDOM STRUCTURE

SUMMARY

Presented paper contains results of fracture analysis of brittle composite materials with a random distribution of grains. The composite structure has been modelled as an isotropic matrix that surrounds circular grains with random diameters and space position. Analyses were preformed for the rectangular "numerical sample" by finite element method. FE mesh for the examples were generated using the authors' computer program RandomGrain. Fracture analyses were accomplished with the authors' computer program CrackPath3 executing the "fine mesh window" technique. Calculations were preformed in 2D space assuming the plane stress state. Current efforts focus on brittle materials such as rocks or concrete.

***Keywords**: numerical analysis, fracture mechanics, cracks, anisotropy, composites, concrete, rock*

KRYTERIUM OKREŚLANIA KĄTA PROPAGACJI SZCZELINY W MATERIAŁACH O LOSOWEJ STRUKTURZE

Prezentowana praca zawiera wyniki analizy pękania kruchego kompozytu o losowo rozmieszczonych ziarnach. Struktura kompozytu zawiera koliste ziarna o losowo dobranych średnicach i położeniu, otoczone izotropową matrycą. Analizy numeryczne zostały wykonane dla prostokątnej rozciąganej „numerycznej próbki" przy użyciu metody elementów skończonych. Losowy model próbki wygenerowany został przez autorski program RandomGrain, a analiza pękania wykonana została za pomocą innego autorskiego programu CrackPath3, który wykorzystuje technikę przesuwającego się okna o zagęszczonej siatce (fine mesh window). Obliczenia wykonano dla modelu 2D przy założeniu płaskiego stanu naprężenia. Charakterystyki materiału analizowanego modelu zbliżają go do betonu, skał i innych geomateriałów.

***Słowa kluczowe**: analiza numeryczna, mechanika zniszczenia, szczeliny, anizotropia, kompozyty, beton, skała*

1. GENERATING THE RANDOM STRUCTURE OF THE MODEL

For generating the geometry of the model containing randomly spread inclusions surrounded with matrix material, authors propose the *Grains Neighbourhood Areas* algorithm (GNA) (Podgórski *et al.* 2006) which creates models of the material in the way similar to the algorithm "larger first", proposed by Van Mier and Van Vliet (2003), however GNA works much more quickly. In the proposed method three random numbers generators based on probability distribution function are used: uniform, normal (Gauss) and Fuller. The generator of the Fuller distribution was obtained from the cumulative function for Fuller sieve curve. Diameters of grains which are located in the space of the model are calculated by the Fuller generator. The generator of the uniform distribution is used for receiving the angle in the polar coordinate system which describes direction of grain location. The generator of the uniform distribution is used also for determining the distance of next grains in the case of A-type samples and Gauss generator in case of B-type samples.

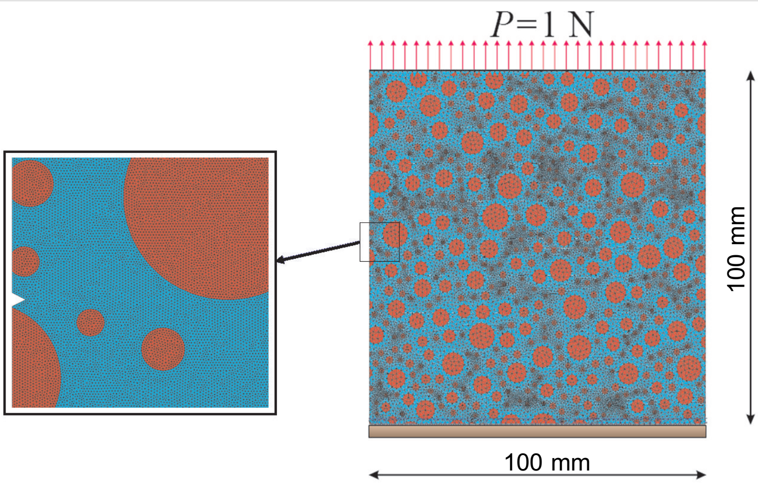

Fig. 1. Boundary conditions and the random distribution of grains in the model sample

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Every new grain is located in the neighbourhood of the previous grain. The area of the neighbourhood is defined as the circle with the set radius, divided in 6 sectors. In every vacant sector location of next random grains are tried. The process of positioning the grain assumes that polar coordinates in every sector are changing in the interval (α, r): $0^{o} < \alpha = 60^{o}$, $R_{min} = r = R_{max}$. If the generated grain location, are not colliding with the grain already existing in the model, the attempt is recognized as successful otherwise a next attempt is taken. The number of attempts N is one of parameters of the algorithm and it decides on the degree of packing of material. The structure received in this way is discretized in order to receive FE mesh.

2. ANALYSIS OF CRACKING

Analysis of cracking was performed using the authors' computer program *CrackPath3*, in which the technique of moving windows with the high density of the FE mesh was applied. This technique assumes the high density of the FE mesh in surroundings of the crack tip and the coarse mesh in area away from the crack.

Inside the window with fine mesh, material of composite is modeled as precisely as it is possible, while outside this window the composite is modeled as the homogeneous material with elastic characteristics determined in homogenizations procedures. The window with the fine FE mesh is moved with the top of the crack in every computational step or after a few steps (what shortens the computation time), in which position of the crack tip is being estimated (Fig. 3). The point in which the crack is initiated is determined at each calculation step using PJ failure criterion described in earlier papers of the author (Podgórski 1984, 1985, 2002). The shape of the limit surface associated with this condition is shown on Figure 2.

2.1. PJ failure criterion

The criterion was proposed in 1984 in the form:

$$\sigma_0 - C_0 + C_1 P(J)\tau_0 + C_2\tau_0^2 = 0 \tag{1}$$

where:

$P(J) = \cos\left(\frac{1}{3}\arccos \alpha J - \beta\right)$ – function describing the shape of limit surface in deviatoric plane,

$\sigma_0 = \frac{1}{3} I_1$ – mean stress,

$\tau_0 = \sqrt{\frac{2}{3} J_2}$ – octahedral shear stress,

I_1 – first invariant of the stress tensor,

J_2, J_3 – second and third invariant of the stress deviator,

$J = \dfrac{3\sqrt{3} J_3}{2 J_2^{3/2}}$ – alternative invariant of the stress deviator,

$\alpha, \beta, C_0, C_1, C_2$ – material constants.

Classical failure criteria, like Huber-Mises, Tresca, Drucker-Prager, Coulomb-Mohr as well as some new ones proposed by Lade, Matsuoka, Ottosen, are particular cases of the general form (1) PJ criterion (Podgórski 1984).

Material constants can be evaluated on the basis of some simple material test results like:

f_c – failure stress in uniaxial compression,
f_t – failure stress in uniaxial tension,
f_{cc} – failure stress in biaxial compression at $\sigma_1/\sigma_2 = 1$,
f_{0c} – failure stress in biaxial compression at $\sigma_1/\sigma_2 = 2$,
f_v – failure stress in triaxial tension at $\sigma_1/\sigma_2/\sigma_3 = 1/1/1$.

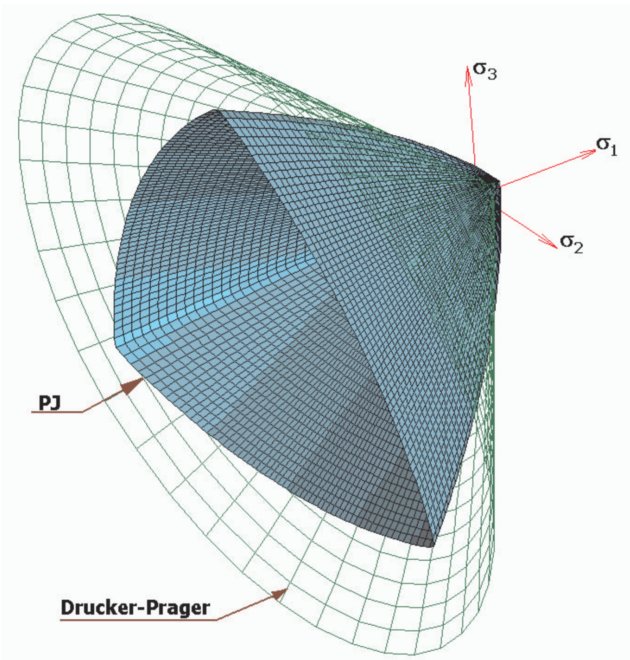

Fig. 2. The limit surface associated with PJ criterion

For concrete or rock-like materials some simplification can be taken on the basis of the Rankine - Haythornthwaite "tension cutoff" hypotesis: $f_v = f_t$.

Values of the material constants C_0, C_1, C_2 can be calculated from following equations:

$$C_0 = f_t$$

$$C_1 = \frac{\sqrt{2}}{P_0}\left(1 - \frac{3}{2}\,\frac{f_t/f_{cc}}{f_{cc}/f_t - 1}\right) \tag{2}$$

$$C_2 = \frac{9}{2}\,\frac{f_t/f_{cc}}{f_{cc} - f_t}$$

where $P_0 = \cos\left(\frac{1}{3}\arccos\alpha - \beta\right)$.

Values of the α and β parameters can by calculated from the author iterative folmulas (Podgórski 1985) or from equations proposed by P. Lewiński (Lewiński 1996):

$$\alpha_0 = \arccos\left[\frac{\theta}{2}\left(1 + \frac{1}{\lambda}\right)\right]$$

$$\beta = \frac{\pi}{6} - \arctan\left[\frac{\theta(1-\lambda)}{2\lambda\sin\alpha_0}\right] \tag{3}$$

$$\alpha = \sin 3\alpha_0$$

where:

$$\lambda = \frac{f_{cc}}{f_t}\,\frac{\dfrac{1}{3} + \dfrac{f_t}{f_c} - \dfrac{f_t\,f_c}{(1 - f_t/f_{cc})\,f_{cc}^{\,2}}}{1 + \dfrac{2f_{cc}}{3f_t} - \dfrac{1}{1 - f_t/f_{cc}}},$$

$$\theta = \frac{\sqrt{3}f_{0c}}{2f_t}\,\frac{\dfrac{1}{3} + \dfrac{f_t}{f_c} - \dfrac{f_t\,f_c}{(1 - f_t/f_{cc})\,f_{cc}^2}}{1 + \dfrac{f_{0c}}{2f_t} - \dfrac{3f_{0c}^2}{4(1 - f_t/f_{cc})\,f_{cc}^2}}.$$

2.2. Crack propagation analysis

The technique of the moving window with fine mesh was presented in previous author papers (Podgórski *et al.* 2007, 2008). This simple re-meshing procedure considerably reduces (3–4 of times) the numerical problem to solve what is related to reduction of the number of nodes in FE model.

Inside the window with fine mesh, material of composite is modeled as precisely as it is possible, while outside this window the composite is modeled as the homogeneous material with elastic characteristics determined in homogenizations procedures. The window with the fine FE mesh is moved with the top of the crack in every computational step or after a few steps (what shortens the computation time), in which position of the crack tip is being estimated (Fig. 3).

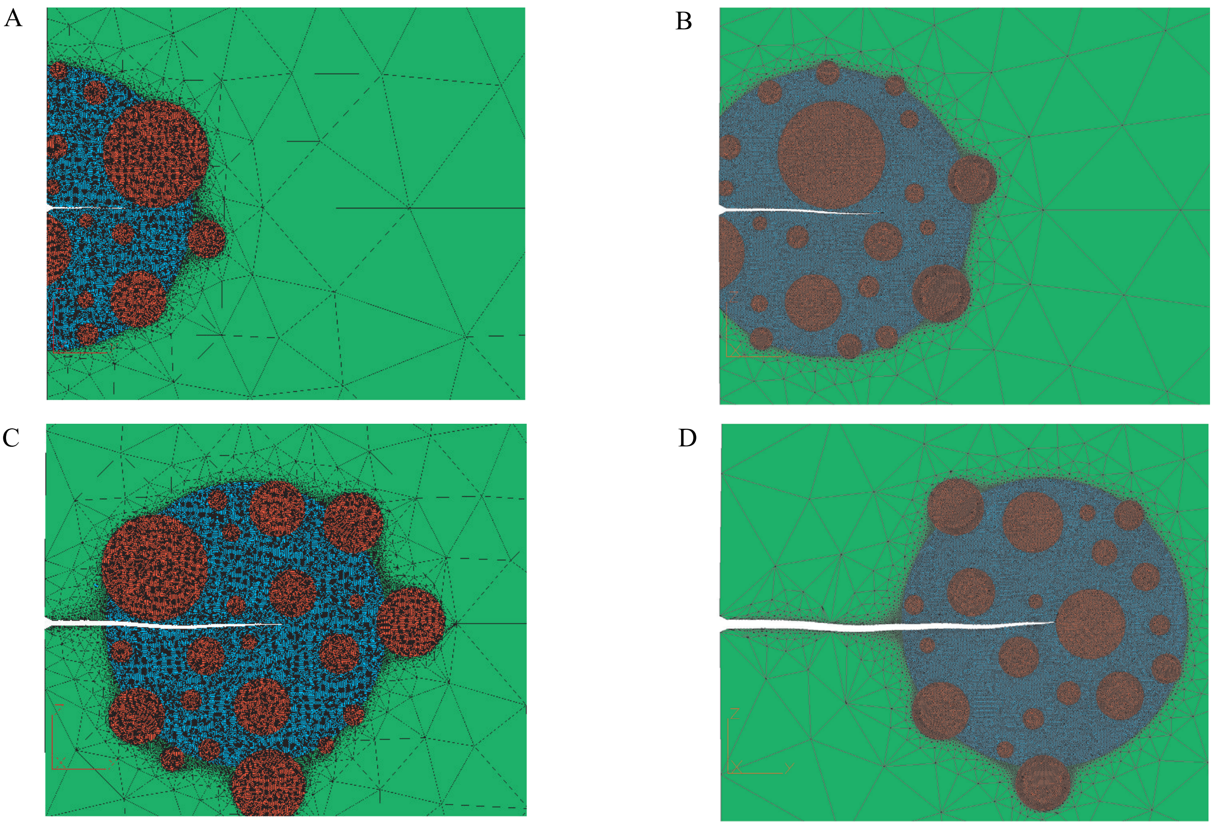

Fig. 3. The view of crack propagation in the case of 4 windows with fine FE mesh

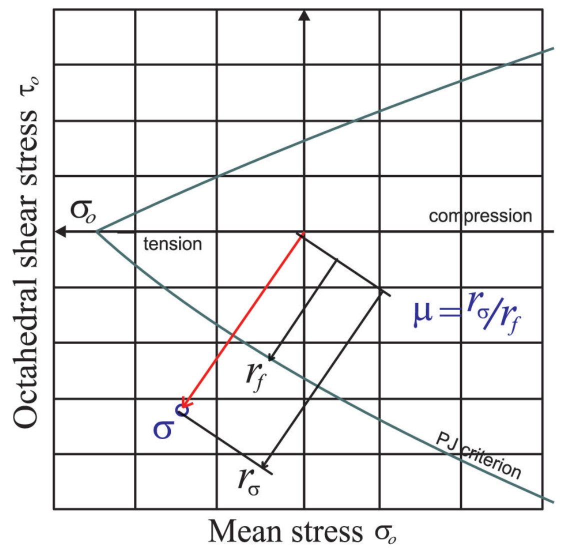

Fig. 4. The Definition of the material effort ratio μ

The point in which the crack is initiated is determined at each calculation step using PJ failure criterion.

Figures 3 are showing the result of calculations of the crack propagation paths with applying 4 windows (marked with letters A, B, C, D) of fine FE mesh. The mesh with this density allows making ca 80 calculation steps of the crack propagation without changing the window position.

In each crack step *CrackPath3* program calculates the stress field using finite elements methods and then it seeks the point of the crack initiation on the basis of the PJ criterion. This is the point of the highest value of the material effort (μ). The value of the material effort ratio μ is calculated based on the formula containing stress tensor components and material constants according to the PJ failure criterion.

$$\mu = r_\sigma(\sigma)/r_f(\sigma) \tag{4}$$

where r_σ and r_f are radii in the stress space:

$$r_{\sigma,f} = \sqrt{\tau_O{}^2 + \sigma_O{}^2} \quad \text{(see Fig. 4).}$$

The the crack is assumed to continue in direction in which the gradient of μ ratio get the highest value (Fig. 5).

After finding the direction of the crack propagation, a FE mesh is modified in surroundings of the crack tip in order to add the next crack segment with the length equal to the size of the cracked element. The procedure is carried on until the demanded number of steps is achieved or the crack stops propagating (Podgórski *et al.* 2007, 2008) (Fig. 6).

The propagation of the analyzed crack was performed on FE mesh consisted of 20498 (window A) up to 42326 (window C) nodes. For comparison purpose calculations for models without the windows were also performed: Model 2 – 16032 and Model 3 – 31311 nodes. In the last two cases, paths of the crack are less stable and the calculations times are comparable to the time needed for the Model 1. The hypothetical model 4, with mesh density comparable to the model 1, would require execution time 20 times lon-ger to calculate 10 steps of the crack.

Windows with the fine FE mesh presented in this paper were generated as a circle with the radius $r \cong 10$ mm, created around of the crack tip. Grains lying on the border of the circle were included in this domain in order to make impossible creation of artificial effects of the stress concentration on the border of homogenized material. Model shown on Figure 3 (Model 1 with windows A, B, C, D) was created assuming material constants given in the Table 1, where: E – Young modulus, ν – Poisson ratio, f_c, f_{cc}, f_{0c} – failure stresses in 2D stress state, f_t – tension strength.

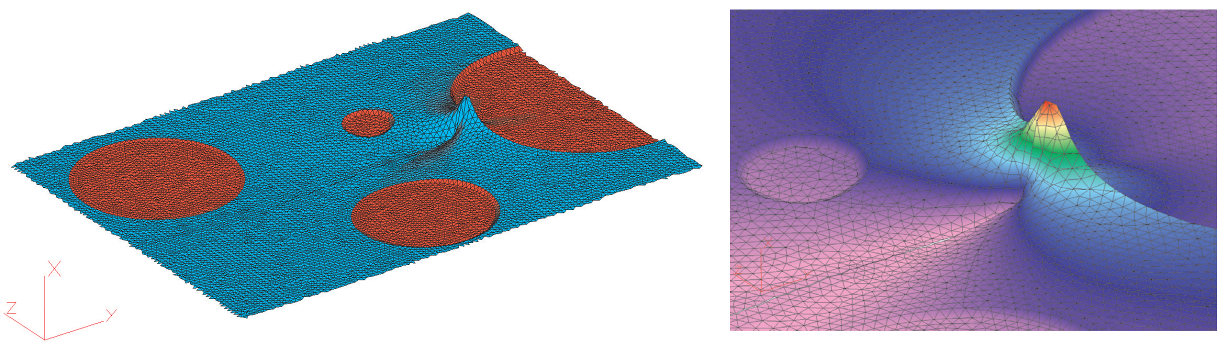

Fig. 5. Values of the material effort ratio μ near the crack tip and grain border

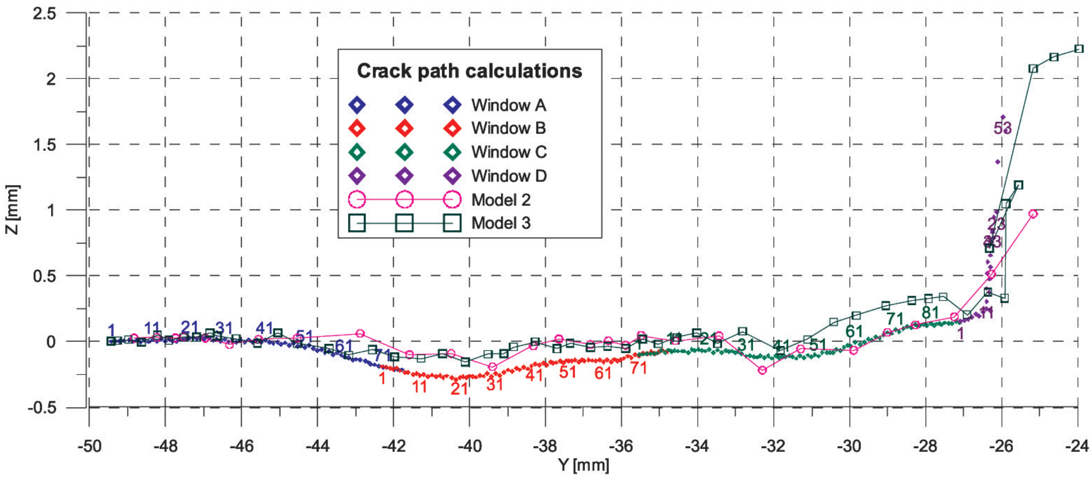

Fig. 6. The path of the crack propagation

Table 1

Material constants

Material type	E [GPa]	ν	f_c [MPa]	f_t [MPa]	f_{cc} [MPa]	f_{0c} [MPa]
Inclusions	36	0.2	40	4	44	50
Matrix	27	0.2	20	2	22	25
Homogen	29	0.2				

Other methods of analysis of crack propagation in the heterogeneous materials were described e.g. in papers: Bažant (2002), Carpinteri and others. (2003), Mishnaevsky (2007). Other method of determining the direction of the crack propagation in polycrystalline material was described in paper of Sukumar and Srolovitz (2004).

3. CONCLUSIONS

Simulation of the crack propagation for composite materials by FE method requires precise remeshing technique and very fine element mesh. The method of movable window with high mesh density seems to be a promising solution technique for problems requiring a high discretization level at a local scale. Cracking analyses of geomaterials with random structures fit naturally in this group. The *CrackPath3* computer code uses the new criterion for prediction of the crack propagation direction which is simpler than suggested for polycrystalline materials by Sukumar and Srolovitz. The new strategy exploits the condition of the minimum energy of cracking material calculated on the basis of the author's failure criterion for brittle materials.

Certainly would be interesting testing the behaviour of crack propagation in three-dimensional models. Analysis of this types of FE models is planned as the subject of next works of the author.

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