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A MECHANICAL LAYERED MODEL OF A VIBRATORY MILL

SUMMARY

Vibratory mills are applied in processes of mechanical production of powders, micropowders, nanopowders and nanostructures. The paper comprises a description of tube mills operation: vibratory and rotary-vibration ones. It also presents their application and construction description including two industrial mills elaborated at the AGH University of Science and Technology in Krakow. There is also a brief analysis of existing models of mechanical vibratory mills. The main part of the paper deals with a model of mechanical vibratory mill known as a layered model. The most important element is a quantitative view of the load movement (grinding media with material) in the chamber. The model also takes into account the whole mechanical system of the mill i.e. inertial vibrator, vibro-isolation system and drive. The model was elaborated for the purposes of the mill computer simulation. During the simulation there were accepted the same technological (grinding media, ground material, grinding process) and kinematic parameters of the mill as the ones that occur in a real object i.e. the laboratory rotary-vibration LAMOW-B-5/2 mill. The paper includes results of a computer simulation of the model. The final part of the work verifies the computer simulation on the same mill and at the same kinematic and technological parameters of the mill and grinding process. Positive results of the verification indicate some possibilities of its application in designing of industrial and laboratory vibratory and rotary-vibration mills.

Keywords: *vibratory mill, model of mill, layered model of a vibratory mill*

MODEL MECHANICZNY WARSTWOWY MŁYNA WIBRACYJNEGO

Młyny wibracyjne stosuje się w procesach mechanicznego wytwarzania proszków, mikroproszków oraz nanoproszków i nanostruktur. W pracy zamieszczono opis działania rurowych młynów: wibracyjnych oraz obrotowo-wibracyjnych. Podano także ich zastosowanie oraz opis budowy, w tym dwóch młynów przemysłowych opracowanych w Akademii Górniczo-Hutniczej. Podano krótką analizę znanych modeli mechanicznych młynów wibracyjnych. Zasadniczą część pracy stanowi model mechaniczny młyna wibracyjnego, zwany modelem warstwowym, którego najważniejszą częścią jest ilościowe ujęcie ruchu ładunku (mielników z materiałem) w komorze. Model ten uwzględnia także cały ustrój mechaniczny młyna, to jest wibrator bezwładnościowy, układ wibroizolacji oraz napęd. Model opracowano dla potrzeb symulacji komputerowej młyna. W symulacji przyjęto identyczne parametry technologiczne (mielników, mielonego materiału, procesu mielenia) i kinematyczne młyna które występują w rzeczywistym obiekcie, czyli laboratoryjnym młynie obrotowo-wibracyjnym LAMOW-B-5/2. Praca zawiera wyniki symulacji komputerowej tego modelu. Finalną część pracy jest weryfikacja symulacji komputerowej na tym samym młynie i przy tych samych parametrach kinematycznych i technologicznych młyna oraz procesu mielenia. Pozytywne wyniki weryfikacji modelu warstwowego wskazują na możliwość jego zastosowania w projektowaniu przemysłowych i laboratoryjnych młynów wibracyjnych i obrotowo-wibracyjnych.

Słowa kluczowe: *młyn wibracyjny, model młyna, model warstwowy młyna wibracyjnego*

1. INTRODUCTION

Powders, i.e. materials created from solid phase of graining below 100 μm are one of the foundations of our civilisation. Currently, there are several billions Mg powders produced in the world of materials like: mineral binders, carbon fuels, ores, mineral resources and fertilizers, chemicals, pharmaceutics, cosmetics, foodstuff and many others. Annually, the number of produced micropowders (graining below 1÷5 μm) and nanopowders (graining below 50÷300 nm) increases. Most of powders are produced from solid phase with the use of a mechanical method during the process commonly known as powdering and at powder production it is called grinding. The applied processing machines are called mills. The method bases on producing powders by mechanical application of the mills working elements onto

the material grains resulting in creating in them normal and shear loads exceeding their strength limits. That leads to disintegration of their solidity and structure followed by creation of higher number of grains of lower sizes.

Differentiated physical and chemical properties of materials used for powders production, technological requirements determined by their application, method of providing energy for the grinding process and the scale of the process resulted in creation of hundreds of mills constructions (Chichelnicky 2006, Drzymała *et al.* 1994, Fritsch 2010, Gämmerler 1973, Gock *et al.* 1996, 1998, Höffl 1985, Lowrison 1974, Metsominerals 2010, Ritec 2010, Sidor 2005, 2007, 2009, 2010).

Accepting the method of energy transfer for the grinding process as the main criterion the mills were divided into four groups (Sidor 2005):

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– mills with floating grinding medium with energy transfer through a movable chamber;
– mills with floating grinding medium with energy transfer through a movable impeller;
– percussive mills with energy transfer through movable working elements;
– jet mills with energy transfer through gas medium put into motion.

The first group is the dominating one and its biggest subgroup consists of gravitational mills in which the grinding media are fed from the rotating chamber. The second subgroup due to the number of produced powders comprises vibratory mills. In such mills the energy is received from the vibrating chamber (Höffl 1985, Lowrison 1974, Schubert 1989, Sidor 2005).

In comparison to gravitational mills, the vibratory mills of the same technological indexes are characterised by the following (Lowrison 1974, Sidor 2005):

– higher technological capabilities, including a possibility to obtain a product of much more fine graining, narrower graining class, sharp grain edges and ability to grind materials of lamellar;
– lower energy consumption by about 4÷10 times for grinding process;
– mill weight lower by about 4÷5 and the cost of grinding media by approximately 6÷8 times;
– lower mounting area (3÷4 times) and the mill cubage by 4÷20 times;
– simplicity of connecting the mill with feeder and a device for collecting the ground product.

In relation to agitating and jet mills, the vibratory mills are characterised by:

– lower energy intake by 6÷10 times for grinding process;
– maximal size of feed grain lower by 10÷20 times;
– less complicated milling system and easier operation.

The main drawback of classical vibratory mills is their negative influence on the environment – generation of noise of 95÷120 dB (Höffl 1985, Schubert 1989) and transmission of remarkable dynamic loads onto the ground increasing costs of foundations and sound absorbing and isolating housings. Significant dynamic loads occurring in the working units of vibratory mills require applying materials of high strength parameters in their housing and monitoring temperature of bearings. However, technological advantages and economic issues decide about wider and wider application of the mills and undertaking research works on new constructions more friendly for the environment.

The mills are the subject of research and development studies of vibratory grinding technologies realized in many centres: Technische Universität in Clausthal (Gock *et al.* 1996, 1998), Vibrocom and Activator companies in Russia (Activator 2010, Vibrocom 2010) and the AGH University of Science and Technology in Cracow (Sidor 2005, 2007, 2009, 2010).

The main direction of the works is focused on elaboration of new construction of vibratory mills which would be less harmful for the environment while keeping good technological parameters.

The Cracow University works concern mills of lowered parameters of vibratory movement of vibration frequency (10÷16 Hz), acceleration of vibratory movement of the mill working unit (below 80 m/s^2) and are performed in two directions (Drzymała *et al.* 1994, Sidor 2005, 2007, 2009, 2010):

– rotary-vibration mills i.e. mills of complex chamber movement;
– vibratory mills of only chamber vibrating movement.

An increase of demands that new constructions must meet refers also to other important issues, i.e.:

– lowering the costs of their design and manufacture i.e. investment costs;
– lowering their exploitation costs;
– shortening the time of their design and manufacture.

Shortening the time of vibratory mills designing can be obtained by applying new techniques basing on modelling. These are process models (Sidor 2005), models of load move in the mill chamber (Sidor 2005), models of grinding media movement (Michalczyk *et al.* 1999, 2010, Sidor 2005), mechanical models of the mill structure (Sidor 2005) and process models of the mill installation.

The work refers to the model of a mechanical rotary-vibration mill used in processes of designing and constructing the mills and also applicable in designing vibratory mills of low vibration frequency.

2. OPERATION, APPLICATION AND CONSTRUCTION OF VIBRATORY MILLS

The grinding process in vibratory and rotary-vibration mills takes place in a container called referred to as a chamber. In most commonly applied vibratory mills (tube ones) the chamber is a tube closed on both sides by covers. In mills grinding temporarily these are discs whereas in continuously grinding mills they are perforated discs called riddle baffler which keep the grinding media in the chamber and provide free flow of the ground material. The working-grinding elements are grinding media, most frequently of a ball shape being fed from a vibrating chamber. The vibration frequency in classical mills amounts at 17÷25 Hz and the acceleration of vibrating chamber movement at 60÷300 m/s^2. The method of tube vibratory and rotary-vibration mills operation is presented in Figure 1.
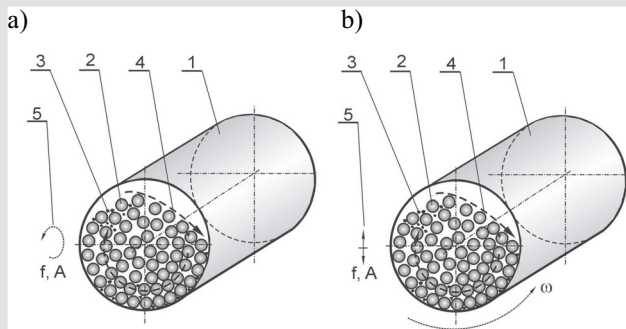
Fig.1. Operation of vibratory mills: a) tube; b) rotary-vibration:
1 – chamber, 2 – grinding media, 3 – ground material,
4 – direction of media movement, 5 – direction of chamber
movement, f – vibration frequency, A – vibration amplitude,
ω – angular velocity

Vibratory mills have been in use for dozens of years in most technologies of powders production of materials characterised by different physical properties in all kinds of grinding processes. Their main application concerns following grinding processes: fine – for graining of the ground material below $50 \div 100\,\mu m$, very fine – below $10 \div 20\,\mu m$, colloidal – below $1 \div 5\,\mu m$ and for production of nanopowders and metallic nanostructures (Chichelnicky 2006, Sidor 2007). They are applied in industry in technologies of the following scale (Gock *et al.* 1998, Höffl 1985, Lowrison 1974, Schubert 1989, Sidor 2005, 2010):

– mass efficiency of $20 \div 60$ Mg/h – in processes of continuous grinding in the air environment;

– medium efficiency of $0.5 \div 20$ Mg/h and low efficiency of $0.01 \div 2$ Mg/h in processes of continuous grinding in the air or fluid environment;

– special technologies and microtechnologies of efficiency at $0.1 \div 20$ kg/h, realized temporarily in the air or fluid environment;

– processes of nanostructures production, mechanical activation and preparation through grinding and homogenization of samples of weight of $0.1 \div 500$ g for analytical purposes.

Most commonly applied are two-chamber tube mills with chambers located one above the other with single mass vibrator located in the centre of gravity of the mill vibrating unit. Such a solution ensures application of the mill in three options of chambers operation: serial, parallel and at feed in the middle of chambers – the middle-parallel one. A constructional solution of such a mill is presented in Figure 2.

Tube mills are built with different number of chambers i.e. one-, two-, three-, four-, and six-chamber mills. The chamber lengths are the multiple of the diameter within the range of $1.5 \div 5$. The mills chambers are always positioned horizontally. The working unit of a vibratory mill consists of chambers connected by a support construction in which a vibrator is mounted of shaft geometric axis parallel to chambers geometric axis. Mills are most frequently equipped with single mass vibrator located in the centre of gravi-

ty of the mill working unit. The working unit is mounted on elastic elements, usually steel coil springs. There are also used springs made from rubber or elastomer. Tube one-chamber mills are typically constructed with dual mass inertial vibrators with a sync gear with chamber located in the centre of gravity of the working unit (Metsominerals 2010) which ensures quasi-circular trajectory of vibration amplitude (Fig. 3a). Two vibration shafts are located on both sides of the chamber and their rotation is synchronized by a belt transmission with cogbelt. The latest construction of a one-chamber vibratory mill features a vibrator located on one side of the chamber whereas on the opposite chamber side there is inert mass found. In this solution the chamber implements movement of elliptical vibration trajectory (Fig. 3b). The mill is constructed in a module version (Gock *et al.* 1996, 1998, Sidor 2005, 2010).

Most vibratory mills are constructed as two-chamber ones, with chambers located one above the other – Figure 3c (Drzymała *et al.* 1994, Ritec 2010, Sidor 2005, 2010). The vibration shaft is driven usually from one side by a Cardan shaft directly from the engine. A compact drive known as central one in which the engine drives a driven belt wheel located on a stationary frame where two shafts ends of the wheel drive two vibrator shafts through short Cardan shafts – Figure 3d is used by only one company (Gämmerler 1973, Sidor 2005, 2010) in two-, four-, and six-chamber mills. The highest power among mills of inertial vibrating movement enforcement belongs to a six-chamber mill – 200 kW (Gämmerler 1973).

Figure 4 presents a one-chamber rotary-vibration mill MOW-H-200 of temporary operation elaborated at the AGH University of Science and Technology and designed by the author (Sidor 2005, 2010). The mill features a dual mass vibrator, chamber capacity of 200 dm^3 and power of 12.6 kW. It was used for very fine grinding of high purity aluminium oxide for graining of 90% below 8 μm.
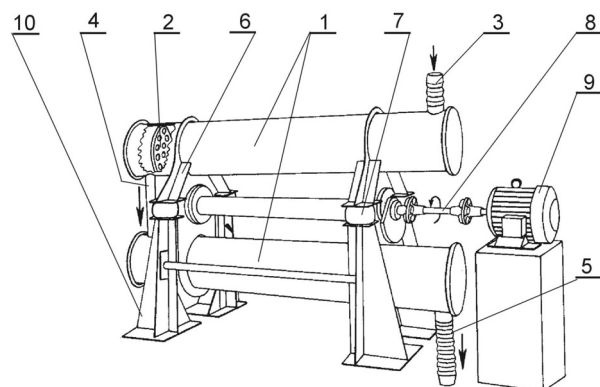

Fig. 2. Two-chamber tube vibratory mill made by Humboldt Palla 35U with serial variant of chamber operation (Schubert 1989): 1 – mill chambers, 2 – riddle baffler, 3 – feed intake, 4 – ground material outlet from upper chamber, 5 – ground material outlet from lower chamber, 6 – support construction, 7 – elastic elements, 8 – vibrator shaft, 9 – engine, 10 – mill frame
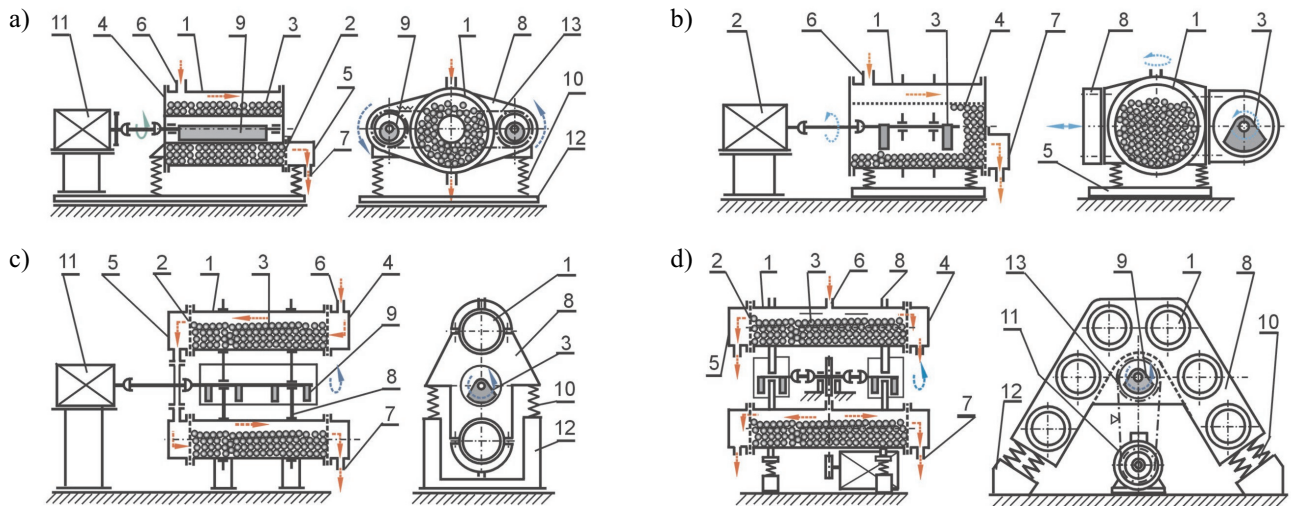
Fig. 3. Basic types of tube vibratory mills: a) one-chamber with dual mass vibrator; b) one-chamber with single mass vibrator;
c) two-chamber; d) six-chamber. 1 – chamber, 2 – riddle baffler, 3 – load (grinding media and ground material), 4 – inlet cover,
5 – outlet cover, 6 – feed inlet, 7 – outlet of the grinding product, 8 – support construction, 9 – inertial vibrator, 10 – elastic support,
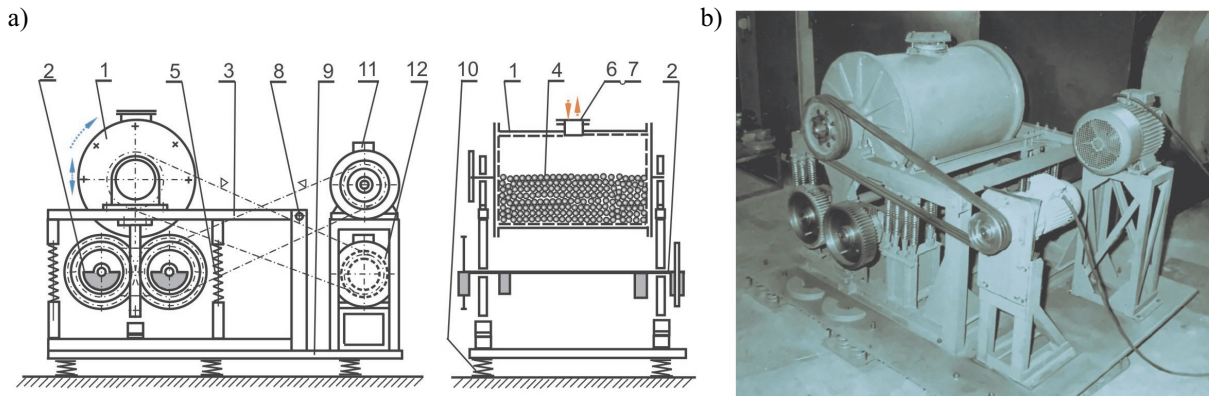11 – engine, 12 – frame, 13 – transmission

Fig. 4. Industrial rotary-vibration mill MOW-H-200: a) construction diagram; b) mill without housing. 1 – chamber,
2 – vibrator, 3 – rocker, 4 – load, 5 – elastic support, 6 – cover, 7 – feed inlet and grinding product outlet, 8 – joint, 9 – base,
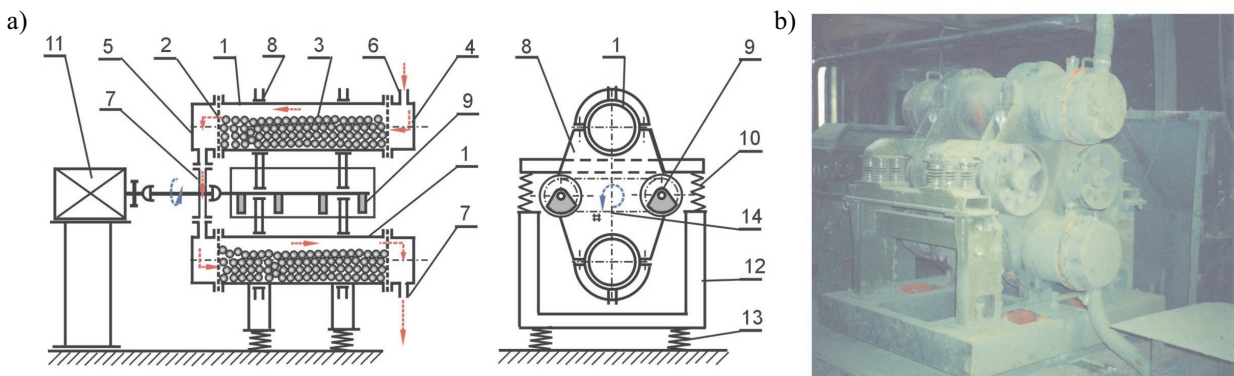10 – vibro-isolator, 11 – vibrator drive, 12 – chamber drive

Fig. 5. Industrial vibration mill MW-A-150m: a) construction diagram; b) mill without housing. 1 – chamber, 2 – riddle baffler,
3 – load, 4 – inlet cover, 5 – outlet, 6 – feed inlet, 7 – grinding product outlet, 8 – support construction, 9 – inertial vibrator,
10 – elastic support, 11 – engine, 12 – frame, 13 – vibro-isolator, 14 – sync transmission

Figure 5 presents a two-chamber vibratory mill MW-A 150m of continuous operation elaborated at the AGH University of Science and Technology and designed by the author (Sidor 2002, 2005, 2010). The mill features a dual mass vibrator with ability operating one vibrator shaft – after removing the chain from the sync gear. The mill as a total capacity of both chambers of 150 dm^3. It was applied for very find grinding of high purity chromium oxide of 0.05% sieve

residue on 0.063 mm riddle. The mill power is 2×15 kW. The level of sound pressure generated by it amounts below 84 dB (Sidor 2005).

The University has elaborated and applied in the industry four vibratory mills and eight rotary-vibration mills. The laboratory rotary-vibration mills of low vibration frequency were the most commonly applied ones. They are used in a few universities, research units of Polish Academy of Sciences, government institutes, laboratories of industrial plants and private companies. To illustrate it – the AGH University of Science and Technology uses 12 mills, The Silesian University of Technology – 3 mills, other universities – 2 mills, Polish Academy of Sciences – 2 mills and the others are found in laboratories of plants. Between 1986 and 2010 there were applied 32 mills of that type of chambers capacity from 0.1 to 5 dm³ of different construction adapted for particular laboratory requirements.

3. ASSESSMENT OF EXISTING MODELS OF MECHANICAL VIBRATORY MILLS

The analysis of existing models of mechanical vibratory mills was presented in the following works: (Kurer 1986, Sidor 2005). Those are models of Bachmann, Liebold, Rose and Sullivan, Raasch, Bauman, Klušancev and Martynov, Kaczmarczyk and Bayer and Höffl. The paper comprises models of: Tokarski and Beenken (Sidor 2005).

The most commonly applied are models of Bauman, Klušancev and Martynov and Bayer and Höffl. The mechanical model of Bauman, Klušancev and Martynov treats a mill as a vibrating machine of one degree of freedom with support incorporating an elastic element and a viscose damper. The free load influence on the mill system was considered by the inert impact coefficient. The Bayer and Höffl model features a two degree freedom. The second degree is generated by the free load mass affecting the mill system by the viscose damping coefficient.

On the basis of the two models some basic design calculations of vibratory mills can be performed, including a simplified selection of elastic support elements for the mill working unit and the engine power.

4. A MECHANICAL LAYERED MODEL OF A ROTARY-VIBRATION MILL

An advance in the field of bodies collision modelling that is the effect of J. Michalczyk and G. Cieplok works (Michalczyk *et al.*1999, 2001) and observations of load movement in the chambers of vibratory and rotary-vibration mills conducted by the author (Sidor 2005) enabled elaboration of a new model of a vibratory mill. The scientists (Michalczyk *et al.*1999, 2001) worked out a model for simulation of free material grains movement on a vibratory table. Sets of free material grains were treated as several layers of the same weight affected by the table vibrating chute. The layer lying on the table collide with the table and next with the layer above it which transfers the energy to subsequent layers. The collision processes were described by the following coefficients: material dissipation, slide friction, rolling friction and elasticity of the material that is collided.

For the purposes of a mechanical rotary-vibration mill also known as layered model, the author modified it significantly i.e. considered real influence of the chamber on the load from its bottom to top zones and differentiated weights of particular load layers. The modification included:

- disregarding part of power taken to implement of the chamber rotary movement due to its small participation in the power consumed – the measurements results are to be found in Table 1;
- assuming that the chamber performs a vibratory movement of vertical sectional trajectory;
- division of the free grinding media load into six layers of different weights, assessed based on visualisation of load movement in the mill chambers;
- considering two impacts on the top load layer – from the bottom of the grinding media and from the top of the top chamber zone;
- accepting the weight of the first layer of grinding media counting from the bottom at 25.2% of the total weight of grinding media, the second layer at 18%, the remaining four layers at 14.2% each; the weight of sand was divided proportionally to the weight of grinding media in layers;
- considering two kinds of grinding media material – steel and corundum;
- considering grinding media and chamber material through coefficients: material dispersion, slide and rolling friction and elasticity;
- the coefficient of material dissipation was calculated from the restoration coefficient determined empirically for vertical impulses from given dependencies (Michalczyk *et al.*1999, 2001);
- accepting two cases of interaction between grinding media and between the media and chamber, i.e. at the absence of the grinding material and with the grinding material; they were distinguished by the material dissipation coefficient defined experimentally at the same variants of interaction of the same grinding media between each other and with the clean steel base and ground material (Tab. 2).

Technological parameters of the grinding process and the mill construction for power consumption (Tab. 1) were determined from the process models of a rotary-vibration mill (Sidor 2005). The estimation of means was performed accepting the confidence level $\alpha = 0.05$. The model is more complex in comparison to existing ones (Kurrer 1986, Sidor 2005) but it more precisely describes the whole construction of the mill – including the engine. The model structure also comprises dynamic impact of the chamber working unit, conditions of vibro-isolation and the power demand.

Table 1

Tests results of power measurement in the rotary-vibration mill LAMOW-B-5/2

Filling level, [–]	Balls material	Balls diameter mm	Balls mass, kg	Sand mass, kg	Power of rotary-vibration movement – MOW, W	Power of vibratory movement – MW, W	Power difference MOW–MW	
							W	%
0.40	corundum	Φ19	4.2	0.0	482.1±6.2	463.8±6.4	18.3±6	3.8
0.40	corundum	Φ19	4.2	1.20	484.7±6.7	467.2±6.1	17.5±7	3.7
0.60	corundum	Φ19	6.3	0.0	493.1±6.9	473.1±6.7	19.7±7	4.0
0.60	corundum	Φ19	6.3	1.80	499.2±6.7	479.1±7.1	21.0±7	4.2
0.40	steel	Φ17.2	9.2	0.0	502.9±6.5	493.5±6.1	9.4±7	1.9
0.40	steel	Φ17.2	9.2	1.20	517.9±6.8	508.1±6.7	9.8±7	1.9
0.60	steel	Φ17.2	13.8	0.0	529.0±7.2	517.0±6.9	12.0±7	2.3
0.60	steel	Φ17.2	13.8	1.80	535.2±7.4	523.1±7.4	12.1±7	2.3
Power with empty chambers					464.5±5.4	453.3±5.5	11.2±6	2.4

Table 2

Restoration and material dissipation experimentally determined coefficients for accepted balls material and collisions conditions

Balls diameter mm	Balls material	Chamber material	Material among balls	Restitution coefficient, R	Dissipation coefficient, ψ
19.0	corundum	corundum	without material	0.73	0.93 ± 0.03
25.0	corundum	corundum	without material	0.65	1.16 ± 0.04
13.5	corundum	corundum	ground sand	0.36	1.74 ± 0.05
13.5	steel	steel	without material	0.54	1.32 ± 0.04
13.5	steel	steel	ground sand	0.15	1.95 ± 0.06
13.0	corundum	steel	without material	0.54	1.42 ± 0.05
13.5	corundum	steel	ground sand	0.32	1.76 ± 0.05
19.0	corundum	steel	ground sand	0.29	1.83 ± 0.06

A layered mill model also known as a mechanical model includes:

– main working unit consisting of the chamber and vibrator connected by a support construction (Fig. 6);
– base plate to which the frame of mill with the engine is fixed;
– elastic system connecting the working unit with the base;
– elastic system connecting the base with the ground i.e. vibro-isolation system;
– housing mounted on the base, only for laboratory mills and small industrial mills.

A general mechanical model of a rotary-vibration mill elaborated according to the above mentioned establishments is presented in Figure 6 with a layered view of the load movement in the chamber of a rotary-vibration mill.

Equations of the physical system movement found in Figure 6 have the following form:

$$(m_k + m_o)\ddot{x}_k + m_o e\ddot{\phi}\cos\phi - m_o e\dot{\phi}^2\sin\phi +$$
$$+ k_k(x_k - x_w) + b_k(\dot{x}_k - \dot{x}_w) = \tag{1}$$
$$= -F_{1k} - m_k g + F_{k6}$$

$$(m_w)\ddot{x}_w + k_k(x_w - x_k) + b_k(\dot{x}_w - \dot{x}_k) +$$
$$+ k_w x_w + b_w \dot{x}_w = -m_w g \tag{2}$$

$$J_{zr}\ddot{\phi} + m_o e\ddot{x}_k\cos\phi = M_{elzr} - m_o e\dot{\phi}^2 r_1\mu \tag{3}$$

where:

m_k – mass of the chamber with body, kg,
m_w – mass of the base, kg,
m_o – mass of the bobs set, kg,
x_k – chamber displacement, m,
x_w – base displacement, m,
k_k – coefficient of the chamber springs elasticity, N/m,
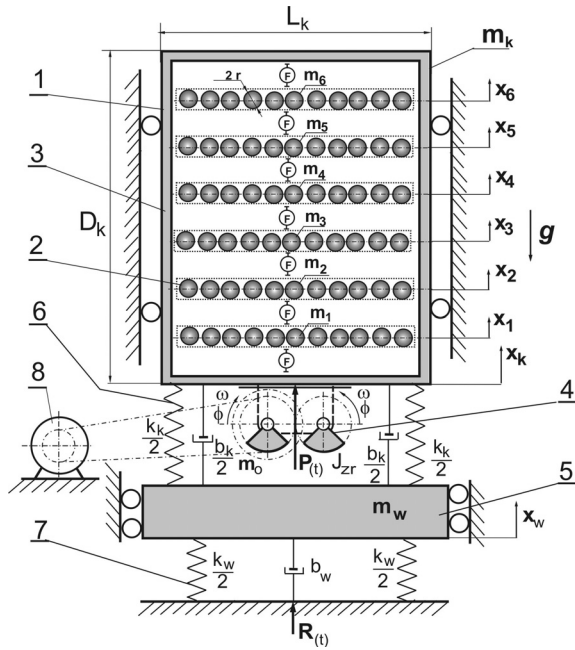
Fig. 6. Model of mechanical-layered rotary-vibration mill.
1 – working unit, 2 – chamber, 3 – layer of grinding media,
4 – vibrator, 5 – support system of working unit, 6 – base,
7 – mill support system, 8 – engine, r – ball radius – grinding
medium, D_k – chamber diameter, m_k – chamber weight,
$m_{1 \div 6}$ – weight of media layers, m_w – weight of base,
m_o – weight of bobs, $x_{1 \div 6}$ – media layers displacement,
x_k – chamber displacement, x_w – base displacement,
k_k – chamber elasticity coefficient, k_w – vibro-isolation
elasticity coefficient, b_k – chamber damping coefficient,
b_w – vibro-isolation damping coefficient, $P(t)$ – vibrator force,
J_{zr} – moment of inertia of rotating elements reduced on the
vibrator shaft, $R(t)$ – force affecting the ground

k_w – coefficient of the vibro-isolation springs elasticity, N/m,
b_k – chamber damping coefficient, Ns/m,
b_w – vibro-isolation damping coefficient, Ns/m,
F_{1k} – impact force of the grinding media first layer on the chamber, N,
F_{k6} – impact force of the sixth grinding media layer on the chamber, N,
e – vibrator eccentric, m,
J_{zr} – vibrator, engine and chamber subunit moment of inertia reduced on the vibrator shaft, kg·m^2,
M_{elzr} – engine electromagnetic moment reduced on the vibrator shaft, N·m,
r_1 – friction radius in the vibrator bearings, m,
μ – friction coefficient in the vibrator bearings,
g – acceleration of gravity, 9.81 m/s^2.

Equation of the first bottom layer movement:

$$m_1 \ddot{x}_1 = F_{(1k)} - F_{(21)} - m_1 g \qquad (4)$$

Equations of the (2), (3), (4) and (6) layers movement:

$$m_i \ddot{x}_i = F_{(i,i-1)} - F_{(j+1, i)} - m_j g \qquad (5)$$

$$m_6 \ddot{x}_6 = F_{(65)} - F_{(k6)} - m_6 g \qquad (6)$$

where:

  i – i-layer of the grinding media, $i = 1 \div 6$
  j – j-layer of the grinding media, $j = i+1$,
  x_{1-6} – displacements of the grinding media, m,
  m_{1-6} – mass of the media layers, kg,
  F_{ij} – forces of collision of i-layer of the grinding media with j-layer, N.

If

$$\left(x_k - x_1 + \frac{h_1}{2} \right) \geq 0 \qquad (7)$$

so

$$F_{1k} = \left(x_k - x_1 + \frac{h_1}{2} \right) k_{1k} \left[1 - \frac{\psi_{1k}}{4} + \frac{\psi_{1k}}{4} sign(\dot{x}_1 - \dot{x}_k) \right] \qquad (8)$$

(Michalczyk *et al.* 2010)

on the contrary $F_{1k} = 0$.

If

$$\left(x_1 - x_2 + \frac{h_1 + h_2}{2} \right) \geq 0 \qquad (9)$$

so

$$F_{21} = \left(x_1 - x_2 + \frac{h_1 + h_2}{2} \right) k_{12} \cdot$$
$$\cdot \left[1 - \frac{\psi_{12}}{4} + \frac{\psi_{12}}{4} sign(\dot{x}_2 - \dot{x}_1) \right] \qquad (10)$$

or: $F_{21} = 0$.

Because particular load layers have the same physical properties, they have the same coefficient of material dissipation ψ_1 and the same elasticity coefficient k_{12}. That's why in subsequent relations it was assumed:

$$\psi_{12} = \psi_{ij} , \quad k_{12} = k_{ij} \qquad (11)$$

If

$$\left(x_2 - x_3 + \frac{h_2 + h_3}{2} \right) \geq 0 \qquad (12)$$

so

$$F_{32} = \left(x_2 - x_3 + \frac{h_2 + h_3}{2} \right) k_{ij} \cdot$$
$$\cdot \left[1 - \frac{\psi_{ij}}{4} + \frac{\psi_{ij}}{4} sign(\dot{x}_3 - \dot{x}_2) \right] \qquad (13)$$

or: $F_{32} = 0$.

If

$$\left(x_3 - x_4 + \frac{h_3 + h_4}{2} \right) \geq 0 \qquad (14)$$

so

$$F_{43} = \left(x_3 - x_4 + \frac{h_3 + h_4}{2} \right) k_{ij} \cdot$$
$$\cdot \left[1 - \frac{\Psi_{ij}}{4} + \frac{\Psi_{ij}}{4} \operatorname{sign}(\dot{x}_4 - \dot{x}_3) \right] \tag{15}$$

or: $F_{43} = 0$.

If

$$\left(x_4 - x_5 + \frac{h_4 + h_5}{2} \right) \geq 0 \tag{16}$$

so

$$F_{54} = \left(x_4 - x_5 + \frac{h_4 + h_5}{2} \right) k_{ij} \cdot$$
$$\cdot \left[1 - \frac{\Psi_{ij}}{4} + \frac{\Psi_{ij}}{4} \operatorname{sign}(\dot{x}_5 - \dot{x}_4) \right] \tag{17}$$

or: $F_{54} = 0$.

If

$$\left(x_5 - x_6 + \frac{h_5 + h_6}{2} \right) \geq 0 \tag{18}$$

so

$$F_{65} = \left(x_5 - x_6 + \frac{h_5 + h_6}{2} \right) k_{ij} \cdot$$
$$\cdot \left[1 - \frac{\Psi_{ij}}{4} + \frac{\Psi_{ij}}{4} \operatorname{sign}(\dot{x}_6 - \dot{x}_5) \right] \tag{19}$$

or: $F_{65} = 0$.

If

$$\left[x_6 - (x_k + D_k) + \frac{h_6}{2} \right] \geq 0 \tag{20}$$

so

$$F_{k6} = \left[x_6 - (x_k + D_k) + \frac{h_6}{2} \right] k_{ij} \cdot$$
$$\cdot \left[1 - \frac{\Psi_{1k}}{4} + \frac{\Psi_{1k}}{4} \operatorname{sign}(\dot{x}_k - \dot{x}_6) \right] \tag{21}$$

or: $F_{k6} = 0$.

where:

h_{1-6} – thickness of the grinding media layers, m,

D_k – chamber diameter, m,

Ψ_{ij} – coefficient of material dissipation between payers, dimensionless.

Work transmitted by the engine onto the whole physical system of the mill is described by the equation:

$$W_{si \ln} = \int_0^t M_{el} \, \dot{\phi}_s \, dt \tag{22}$$

where:

M_{el} – electric torque of engine, N m,

$\dot{\phi}_s$ – velocity speed of engine, rad/s.

Whereas works performed by the chamber W_k used for movement of grinding media with material i.e. process of grinding and mixing material is expressed by:

$$W_k = \int_0^t (F_{1k} - F_{k6}) \dot{x}_k \, dt \tag{23}$$

Dynamic force transmitted onto the ground is defined by:

$$R(t) = k_w x_w + b_w \dot{x}_w \tag{24}$$

Assuming harmonic run of x_w, the maximal value of dynamic force transmitted onto the ground amounts at:

$$\left| R(t) \right| = \sqrt{k_w^2 + (b_w \omega)^2} \cdot A_w \tag{25}$$

where:

$R(t)$ – force transmitted onto the ground, N,

W_{siln} – work done by the engine, J,

A_w – amplitude of the base vibrations displacements, m,

t – time, s.

5. VERIFICATION OF THE MECHANICAL-LAYERED MODEL

The elaborated model was the base for conducting a computer simulation. For the simulation purposes the real parameters of a medium size laboratory rotary-vibration mill LAMOW-B-5/2 – Figure 7 (Sidor 2005) were accepted. The mill has been used in the Department of Advanced Ceramics at the AGH University of Science and Technology in Krakow since 1986. It was worked out for colloidal grinding of silicon carbide. The construction of the mill is quite close to the one of the MW-H-200 one – Figure 4. The differences cover the size and capacity of chambers, common vibrator and chamber drive and a removable chamber.

The simulation results were verified by comparison of the simulation results and the measurements results of the LAMOW-B-5/2 mill obtained at the same technological and kinematic parameters of the mill (Sidor 2005) in a chamber of 5 dm^3 capacity.

The mechanical-layered model verification was preceded by an identification of its all important parameters and parameters of the grinding media movement.
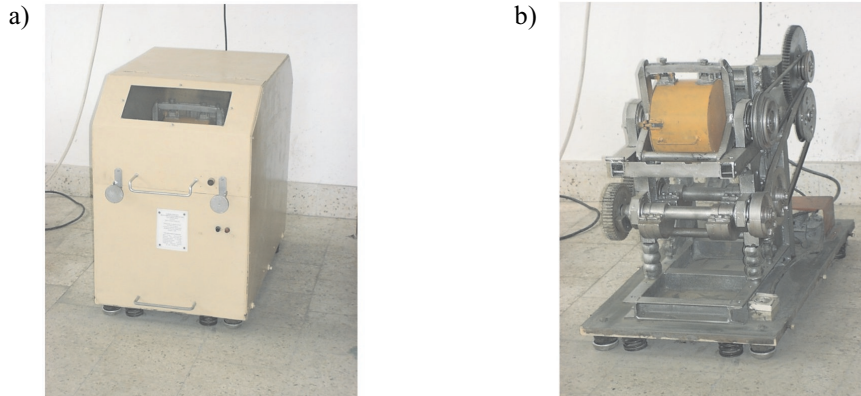
a)   b)

Fig. 7. Laboratory rotary-vibration mill LAMOW-B-5/2 of low vibration frequency:
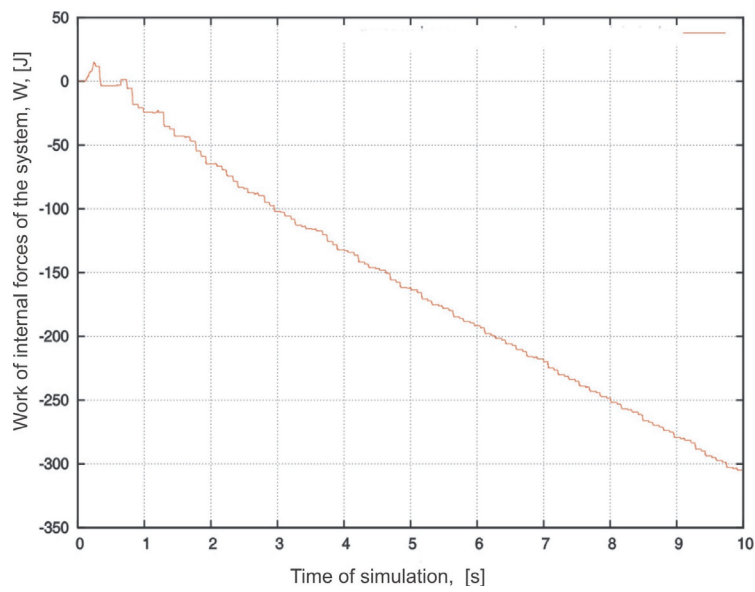a) mill in soundproof and isolating housing; b) mill without housing (Sidor 2005)

Fig. 8. Action of inner forces of the system (W < 0), (performed by chamber on movement and collisions of balls):
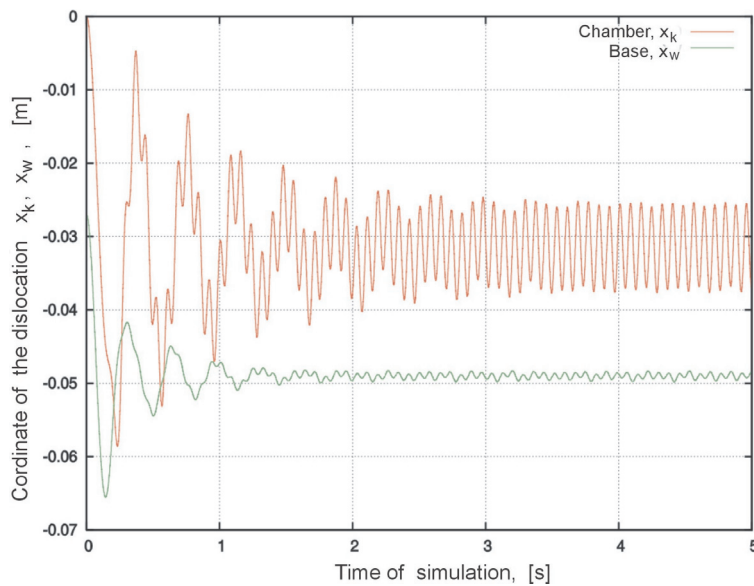filling level 0.40, steel balls with material

Fig. 9. Coordinates of the working unit displacement-chamber − x_k and base plate − x_w at filling level 0.40,
steel balls with material

Tests of power consumption by the mill were performed at the two types of grinding media (steel and corundum), three levels of the chamber filling (0.20; 0.40 and 0.60) and three corresponding variants of the grinding media weight. For each media material there were conducted measurement with and without the grinding media movement – high-silica sand. The tests also took into account two variants of the mill operation i.e. working as a rotary-vibration mill and as a vibratory mill (with the chamber transmission belt taken off). The power intake at neutral run was also measured for both variants of the mill operation. In the mill both movements of the working unit: rotary and vibratory, are driven from one engine of 0.75 kW power. The measurements of power were conducted with the NanoVip Plus wattmeter and registered on a computer disc.

Some results of the simulation are presented in Figures 8 and 9. Figure 8 shows a simulation concerning action of the system inner forces obtained at nominal parameters of the mill operation: vibration frequency of 12 Hz, vibration amplitude of 6.7 mm, chamber angular velocity of 8,4 rad/s. In the chamber filled at 0.40 there was a load of 10.4 kg weight, i.e. steel media – balls of 17.2 mm diameter – 9.2 kg and the sand – 1.2 kg. Figure 9 presents one of the simulation results concerning displacements of the working unit with the chamber and the base plate.

Figure 10 shows results of acceleration measurements of the working unit vibratory movement. The measurement was performed with a low-pass filter 100 Hz. The vibration amplitude calculated from the vibration measurement amounts at 6.7 mm, and from the computer simulation 6.1 mm. A relative difference between the measurement and simulation results is 8.9%.

Table 3 shows a comparison of the mill computer simulation results with the results of measurements performed on a real object – the LAMOW-B-5/2 mill.

The LAMOW-B-5/2 mill is driven only from one engine (Fig. 7). Its dual mass vibrator is driven directly from the engine and the chamber is driven through two transmissions: belt and toothed one, and the driving wheel of the toothed transmission is located on the engine shaft. Such a solution of the drive decreased its cost but caused unfavourable work of both transmissions deteriorating their efficiency. Vibratory movement of the working unit upward caused an increase of the vibrator belt tension and downward movement rose the chamber belt tension. It resulted in difficulties in assessing the drive efficiency – for the purposes of the model verification it was accepted at 0.70. The engine power defined from the simulation is 448 W while measured for the same parameters of the mill operation is 518 W – Table 1, line 4 from the bottom. The difference is 70 W i.e. 13.5%.
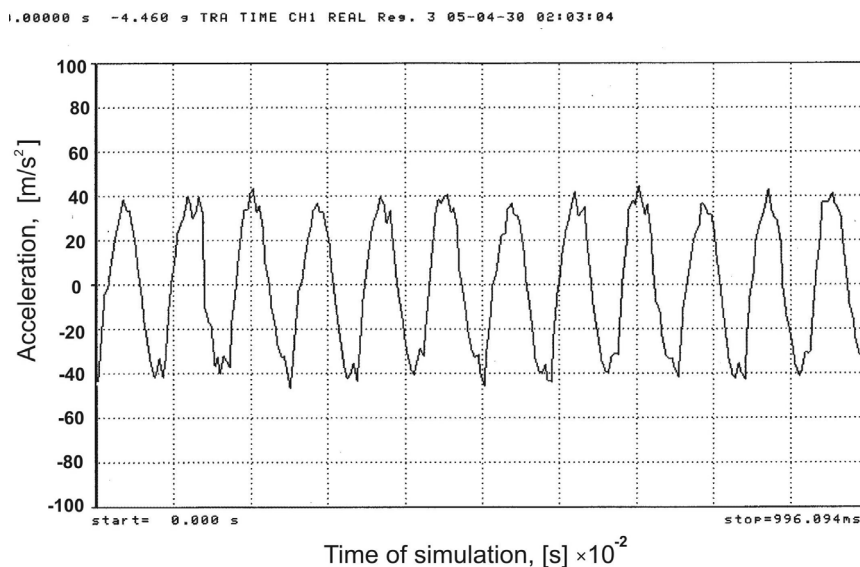

Fig. 10. Acceleration of vibrating movement of the mill working unit, filling level 0.40 [–], steel balls with material

Table 3

Verification results of a mechanical model and a real object

Filling level, [–]	Balls material	Balls diameter mm	Balls mass, kg	Sand mass, kg	Balls power, experiment, W	Balls power simulation, W	Difference between experiment – simulation	
							W	%
0.20	steel	Φ17.2	4.6	0.60	24.7	16.5	8.2	33.2
0.40	steel	Φ17.2	9.2	1.20	53.4	36.3	17.1	32.0
0.60	steel	Φ17.2	13.8	1.80	70.7	56.0	14.7	20.8
0.20	corundum	Φ19.0	2.1	–	8.1	6.5	1.6	19.8
0.40	steel	Φ17.2	9.2	–	38.4	30.0	8.4	21.9
0.60	steel	Φ17.2	13.8	–	65.5	53.8	11.7	17.9

6. CONCLUSIONS

The mechanical-layered model presented in the paper was worked out for a rotary-vibration mill, with omission of the chamber rotary movement influence, i.e. for a vibratory mill of sectional trajectory of vibration amplitude. The rotary movement of the chamber could be omitted due to its low significance as it was indicated by tests results presented in Table 1. The movement increases power consumption by only about 2% with steel grinding media to about 4% with corundum media.

In case of the vibration amplitude the simulation error was slightly below 9%, whereas in case of power intake it was 13.5% which could result from inaccurate assessment of the mill drive efficiency.

The simulation results of power demand for load movement carried much bigger errors i.e. from 18 to 22% at simulation without ground material and from 21 to 33.2% with the material. Because the free load in case of the LAMOW-B-5/2 mill made about 20÷25% of the working unit height, the real model error was about four times smaller.

The layered model does not reflect a real movement of the load in the rotary-vibration mill chamber but it sufficiently precisely describes the movement.

The model was positively verified on a real object of a laboratory scale and it can be applied in designing of laboratory and industrial vibratory mills of the chambers capacity up to 50 dm^3. In case of industrial mills o higher chamber capacity, the model requires verification of an industrial object of the chamber capacity higher than 50 dm^3. A positive result of the verification will provide designers and constructors with tools for fast designing of large vibratory and rotary-vibration mills.

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