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Tomasz Bernat*, Robert Ruszkiewicz**

Hedging the investment portfolio with derivatives present on the Polish market

1. Introduction

If we were to try to describe the capital market of the second and third decades of the 21st century, one of its distinguishing features would undoubtedly be its mass character. The popularization of investments, whether purely speculative or not, is clearly discernible to the naked eye. As a result, the capital market has evolved in a direction where it is not the exclusive domain of financial institutions but a space in which a single individual investor also finds themself. These investors, driven by the lust for profit and often needing the appropriate knowledge, make investment decisions that do not comply with the accepted standards, seem devoid of common sense, break the basic principles of investment hygiene, and ultimately bring difficult losses.

Apart from the previously mentioned bravado, several options on the market allow one to minimize these losses. We are talking about simple tools and techniques, such as the skillful use of stop loss orders and complex strategies for which derivatives are used. Although the indicated methods of hedging positions seem to be an effective solution to the problem of incurring losses, they are not so everywhere. Investors wishing to invest their capital in the Polish capital market have to contend with a shortage of derivatives, which, in contrast to more developed foreign markets, is present there. This state of affairs significantly ties your hands and complicates securing positions accepted on the market. This is an essential issue as it contributes to the inhibition of investments made on the

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Polish stock exchange, thus contributing to the inhibition of the Polish economy. Therefore, the purpose of this publication is to indicate how to minimize the risk on the capital markets in a practical way, using the financial opportunities and instruments present in the Polish realities.

The article presents considerations regarding models and strategies for optimizing investment processes. For the study, an investment portfolio was constructed and tested, the sensitivity of which to unfavorable price movements is minimal. The construction of the portfolio mentioned above is based on the theory of investment portfolio construction presented by H. Markowitz, while the attempt to hedge it was carried out by the strategy indicated by J. Hull.

The main research hypothesis of the presented article assumes that, in Polish conditions and with the use of available investment tools, it is possible to construct an investment portfolio to a large extent resistant to unfavorable price movements on the market.

2. Literature review

According to the commonly accepted definition, investment is the renunciation of current consumption in favor of increased consumption in the future. Its implementation is held to be possible thanks to the capital increase over time. Therefore, investments are also defined as investing capital in ventures or assets designed to multiply it. This multiplication of capital is accounted for as a success/profit. Its decrease is interpreted as a failure/loss. The specified success or failure is usually measured by the rate of return, the forms, and the types of which there are many variations in the literature (Reilly, Brown, 2001). While the rate of return and the effects of the investments made are intuitive and easy to define, a problem arises when explaining investment risk. It can be interpreted as the probability of harm or uncertainty about the effects of the investments made (Knight, 1921). Risk can also be perceived as subjective and objective. The first approach refers to the predispositions, skills, and disposition of people implementing investments, and the second to the nature of financial instruments and factors influencing their prices (Podlewski, 2021). In the context of the subject of this publication, the approach within which investment risk should be described is an objective approach, with tools helpful in the investment portfolio management process (Markowitz, 1952). The given tools include such indicators as, for example, the variance of the rate of return or the standard deviation of the rate of return (Hayes, 2021).

One of the precursors of the diversification process and portfolio analysis was H. Markowitz, who developed a model optimizing the diversification process, where his task was to determine the appropriate proportions of assets in the portfolio so as to minimize its exposure to risk measured by the standard deviation (Markowitz, 1959). The effectiveness of this model has been tested many times over the years, and its use indeed reduces the returns of the portfolio compared to alternative portfolios of a similar class, and the application of a given model is relatively simple, and its concept is easy to understand (Amu, Millegård, 2009). Other studies, however, indicate that the reduced standard deviation of the rate of return is not large enough compared to alternative portfolios to significantly translate into investment results. In addition, the expected rate of return of the research portfolio shows disproportionately lower values than those characterizing alternative portfolios with a higher reading of the standard deviation of the rate of return (Širůček, Křen, 2015). Researchers also note that the standard deviation, variance, and covariance of the rate of return are not the only factors determining the risk of the investment portfolio, and the model proposed by H. Markowitz does not take into account extraordinary situations (Marling, Emanuelsson, 2012). What follows, and what has also been empirically demonstrated, is that the optimal construction of the investment portfolio is insufficient in the face of the threats present on the market to effectively protect it against profound fluctuations in the capital markets, which can be leveled with derivatives (Castellano, Giacometti, 2001).

Derivatives allow us to effectively reduce the total risk of a portfolio, regardless of whether the losses stem from a systematic or specific risk. Precisely because of this, it is reasonable to use derivatives to hedge positions taken on the capital market (Hull, 1999). When choosing specific instruments to hedge positions, it is worth paying particular attention to options that, using appropriate speculative strategies, such as *strangles* or *straddles*, can bring outstanding results at a relatively small cost (Kownatzki et al. 2021). However, apart from the issues of stock market speculation, one of the basic strategies protecting the portfolio against price collapses is the so-called **protective put** consisting of the purchase of put options for assets requiring protection, which generates positive cash flows when the price of the protected assets falls below predetermined values, compensating for the losses (McMillan, 2002). **Protective put**, otherwise known as **hold stock, buy put**. However, it does not work in speculative strategies aimed at achieving above-average profits, and ensures portfolio stability in terms of protecting assets (Dash et al. 2008).

Considering all of the above, an attempt to build a portfolio with minimal risk will be carried out by constructing an effective portfolio with the lowest possible standard deviation of the rate of return and hedging it with derivatives, using the protective put strategy.

3. Research methodology

The purpose of this paper is to attempt to build a portfolio with minimal risk, which is to verify the hypothesis that it is possible in Polish conditions to construct an investment portfolio which is to a large extent resistant to unfavorable price movements on the market and that hedging an investment portfolio with the use of the derivatives market is compelling. As one can easily guess, building an investment portfolio is a complicated process with many successive stages. One of them is the tactical allocation of assets, finally ending with portfolio analysis, serving the purpose of appropriately diversifying the constructed portfolio. The person who was one of the first to notice the importance of diversification in the investment process was H. Markowitz, who not only made the observation above but also developed a model that allows the diversification of selected assets in an optimal way. A given model is the foundation of portfolio analysis.

The study's main hypothesis assumes that it is possible to construct an investment portfolio in Polish conditions which is to a large extent resistant to unfavorable price movements on the market. Therefore the construction of the research portfolio will be based on the theories of H. Markowitz, the application of which will increase the probability of the assumption adopted. The aim of one of the first stages of the study was, therefore, an attempt to construct an effective portfolio, which is such a portfolio, which, with the expected rate of return, is characterized by the lowest possible risk or, for the selected risk level, maximizes the potential rate of return. The constructed portfolio had a one-year investment horizon and included equity instruments issued by five selected companies whose shares are included in the WIG20 index. The selection consisted of companies with the highest capitalization on February 1, 2022, whose stock exchange debut took place in 2007 at the latest.

The conditions mentioned above were met by the following companies: **Powszechna Kasa Oszczędności Bank Polski, Santander Bank Polski, Bank Polska Kasa Opieki, Polskie Górnictwo Naftowe i Gazownictwo, KGHM Miedź Polska.**

The next step after the selection was the estimated annual rates of return achieved by the companies on February 1, starting from the first year in which the shares of all selected companies were traded and ending with the current year. What follows the research period of the conducted study was seventeen years. If February 1 fell on a non-trading day, i.e., Saturday or Sunday, the closing prices from the last session were used to estimate. On the basis of the calculated returns, the variance and standard deviation of the rates of return of selected instruments were estimated because the risk of the examined portfolio is defined by the value of the standard deviation it assumed.

The next step was to examine the interdependence of companies, measured by the correlation of rates of return. And all this is to use the given ratios and values to carry out a portfolio analysis to build an effective portfolio according to H. Markowitz's multi-company portfolio theory, with an initial value of one million zlotys.

After constructing the portfolio, its expected rate of return and standard deviation were determined. Then, it was assessed by comparing it to other six portfolios with the same expected rate of return but different, randomly selected shares of assets forming it. The comparison consisted in juxtaposing the standard deviation, the Sharpe index, and the coefficient of volatility characterizing the described portfolios.

Unfortunately, obtaining a portfolio with the minimum standard deviation value while maintaining the highest investment potential measured by the rate of return does not exhaust the topic of building a portfolio with minimal risk. The design of an efficient portfolio reduces specific risks with little or no impact on systematic risk. Fortunately, the development of financial markets, especially the derivatives market, allows one to effectively reduce the total risk of a portfolio, regardless of whether the losses resulting from the impact of systematic or specific risk. Therefore, the next research stage focused on using derivatives on the Polish stock exchange, designed to protect the portfolio against the loss of its value in the event of unfavorable price movements on the market. This procedure verified the study's hypothesis, indicating that hedging the investment portfolio using derivatives present in the Polish market is effective.

As described earlier, the portfolio was hedged against impairment using derivatives listed on the Polish market, specifically options. Since the only options existing in Poland are those based on the WIG20 index, the constructed portfolio was hedged with these options after examining the relationship between the concerned portfolio and the WIG20 index. After the protection was made, a simulation showed its method and effectiveness.

The methodology of the entire study is presented in the Figure 1.

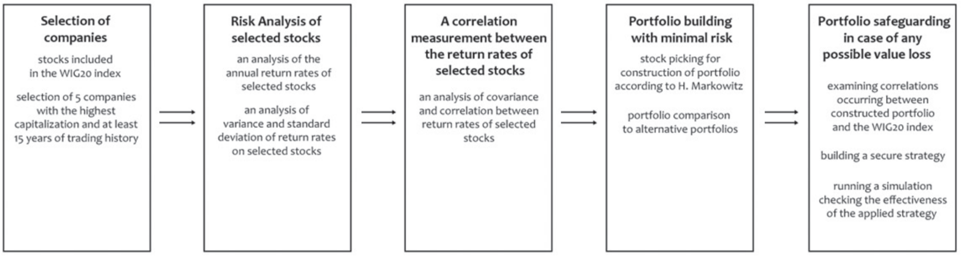

Figure 1. Scheme presenting the methods of the conducted research

Source: own elaboration

4. Research outcome

To start constructing a portfolio with minimal risk, it is essential to determine the risk of the assets included in the portfolio at the beginning. According to H. Markowitz's multi-company portfolio theory, the risk of investment instruments that make up a portfolio is determined by their standard deviations. To know these, one must make a series of calculations enabling a given estimate. Therefore, the arithmetic average of the rates of return of all instruments included in the portfolio from February 1, 2006, to February 1, 2022 should be calculated first, and then the variances characterizing the rates of return. The obtained results are worth interpreting. The following tables (Tabs 1, 2) present the results of the estimates and their interpretation.

Table 1

Risk of selected investment instruments

No.	Symbol	Arithmetic mean rate of return [%]	Return variance [%]	The standard deviation of the rate of return [%]
1	PKO	6.474	8.846	29.741
2	SPL	13.765	19.365	44.006
3	PEO	4.162	14.330	37.854
4	PGN	5.891	6.024	24.544
5	KGH	23.043	54.981	74.149

Source: own elaboration

Table 2

Research outcome interpretation

No.	Symbol	Lowest rate of return expected [%]	Expected highest rate of return [%]
1	PKO	−23.267	36.216
2	SPL	−30.241	57.770
3	PEO	−33.692	42.017
4	PGN	−18.653	30.435
5	KGH	−51.106	97.192

Source: own elaboration

The next stage of the conducted research is the recognition of the interdependence of the rates of return of selected investment instruments. This is important because the given instruments will create one portfolio which will be determined by the price reaction of some assets to the change in the prices of others. Therefore, the estimation of the level and type of interdependence will allow the selection of individual shares to minimize the risk of the optimally constructed portfolio.

The determination of the relationship described above will be carried out by using the indicator, which is the Pearson linear correlation. The calculation results for all pairs of companies in the portfolio are presented in Table 3.

Table 3

Measurement of the relationship between the rates of return of selected stocks

No.	Pair	Covariance	Correlation
1	PKO & SPL	0.117	0.892
2	PKO & PEO	0.096	0.849
3	PKO & PGN	0.001	0.016
4	PKO & KGH	0.071	0.324
5	SPL & PEO	0.148	0.885
6	SPL & PGN	−0.008	−0.075
7	SPL & KGH	0.140	0.428
8	PEO & PGN	−0.016	−0.176
9	PEO & KGH	0.052	0.186
10	PGN & KGH	0.050	0.277

Source: own elaboration

After calculating the correlation coefficient, it must be interpreted. The interpretation is two-part. The sign before the result is evaluated first, then its absolute value. For example, the result obtained for the PKO/SPL pair shows a very high correlation between the two companies share prices. This allows us to conclude that the given assets show almost identical price movements. Table 4 presents the interpretation for all pairs.

Table 4

Interpretation of the obtained results

No.	Pair	Correlation	Interpretation of the sign	Value interpretation
1	PKO & SPL	0.892	Positive sign. As one company grows, the other also grows. When one goes down, the other goes down as well	Very strong relation, price movements of the examined assets are almost identical
2	PKO & PEO	0.849	Positive sign. As one company grows, the other also grows. When one goes down, the other goes down as well	Very strong relation, price movements of the examined assets are almost identical
3	PKO & PGN	0.016	Positive sign. As one company grows, the other also grows. When one goes down, the other goes down as well	Very weak relation, price movements of the examined assets are practically unrelated
4	PKO & KGH	0.324	Positive sign. As one company grows, the other also grows. When one goes down, the other goes down as well	Weak relation, price movements of the examined assets show little connection
5	SPL & PEO	0.885	Positive sign. As one company grows, the other also grows. When one goes down, the other goes down as well	Very strong relation, price movements of the examined assets are almost identical
6	SPL & PGN	−0.075	Negative sign. When one company goes up, the other goes down. When the first decreases, the second increases	Very weak relation, price movements of the examined assets are practically unrelated
7	SPL & KGH	0.428	Negative sign. When one company goes up, the other goes down. When the first decreases, the second increases	The relation is moderate, the price movements of the examined assets are insignificantly related to each other

Table 4 cont.

8	PEO & PGN	−0.176	Negative sign. When one company goes up, the other goes down. When the first decreases, the second increases	Very weak relation, price movements of the examined assets are practically unrelated
9	PEO & KGH	0.186	Positive sign. As one company grows, the other also grows. When one goes down, the other goes down as well	Very weak relation, price movements of the examined assets are practically unrelated
10	PGN & KGH	0.277	Positive sign. As one company grows, the other also grows. When one goes down, the other goes down as well	Weak relation, price movements of the examined assets show little connection

Source: own elaboration

The obtained results show that the level of dependence between the share prices of selected companies is ambiguous. The portfolio will be built of both assets closely related to each other and those for which there is no correlation.

Based on the results obtained, an attempt will be made to construct an effective portfolio with minimal risk. All the ratios calculated so far will be merged to create a kind of "resultant" of their characteristics, allowing to minimize the risk of investing in selected assets while maximizing the investment potential measured by the rate of return. The merge above will be obtained by combining the estimated data in the matrix proposed by H. Markowitz. Its transcript is presented below:

	PKO	**SPL**	**PEO**	**PGN**	**KGH**	
PKO	0.177	0.233	0.191	0.002	0.143	1
SPL	0.233	0.387	0.295	−0.016	0.279	1
PEO	0.191	0.295	0.287	−0.033	0.105	1
PGN	0.002	−0.016	−0.033	0.120	0.101	1
KGH	0.143	0.279	0.105	0.101	1.100	1
	0.177	0.233	0.191	0.002	0.143	1

After constructing the matrix above, create an inverse matrix to multiply it by a vector with $n+1$ components, where the first n components are zero, and the last one is unity. This will create a vector with dimension $n+1$, where the first n components are shared in the constructed portfolio. In the case of the wallet, the construction of which is the subject of this article, the vector appears as follows:

$$\begin{vmatrix} 0.377 \\ -0.310 \\ 0.356 \\ 0.576 \\ 0.001 \\ -0.064 \end{vmatrix}$$

Taking into account the fact that the initial value of the portfolio assumed in the assumptions should be 1 million PLN, the shares of the companies selected for its construction, both in percentage and amount terms, are presented in the Table 5.

Table 5

Shares of selected companies in the investment portfolio

No.	Symbol	Percentage share [%]	Amount Share [zł]
1	PKO	37.7	377 194.08
2	SPL	−31.0	−310 312.39
3	PEO	35.5	355 826.33
4	PGN	57.6	576 286.77
5	KGH	0.1	1 005.21
Sum		100.000	1 000 000.00

Source: own elaboration

The obtained results may be difficult to interpret due to the negative values appearing with the shares of Santander Bank Polska. The outstanding reading should be interpreted so that it is recommended to make the so-called short sale in the case of the indicated assets.

It is also worth explaining that the above shares are only theoretical values that may differ from the actual shares. The given situation's determinants are selected companies' share prices on February 1, 2022. To estimate the number of shares of a specific company that should be purchased for the constructed portfolio, the share shown above should be divided by their price. The result obtained may cause consternation because with a high degree of probability, it will not be an integer. Therefore, it should be rounded down, which will result in determining the previously searched value. In the case of the considered portfolio, the actual shares are as presented in the Table 6.

Table 6

The actual shares of selected companies in the investment portfolio

No.	Symbol	Number of shares	Real share [zł]	Percentage share [%]
1	PKO	7858	377 184.00	37.718
2	SPL	−880	−310024.00	−31.002
3	PEO	2598	355 796.10	35.580
4	PGN	108 324	576 283.68	57.628
5	KGH	7	999.25	0.100
Total:		117 907	1 000 239.03	100.024

Source: own elaboration

To evaluate the constructed portfolio, four indicators characterizing it should be calculated and compared to other portfolios with similar investment potential. Indicator data include the expected rate of return, standard deviation, coefficient of volatility, and Sharpe ratio. The Table 7 presents a list of six portfolios with a given expected rate of return of 3.074% (the expected rate of return obtained by the research portfolio), with randomly selected shares of selected companies totaling nearly 100%. It allows you to evaluate the constructed portfolio against others of the same class.

As one can see, the Table 7 clearly shows that the constructed portfolio is of the highest quality compared to the other six portfolios. This is supported by the lowest value of the standard deviation, the coefficient of variation, and the Sharpe index's highest value.

Table 7

Comparison and evaluation of portfolios of the same class

No.	Portfolio under review	Share of PKO [%]	Share of SPL [%]	Share of PEO [%]	Share of PGN [%]	Share of KGH [%]	Expected rate of return [%]	Standard deviation [%]	Coef-ficient of variation	Sharpe index
1	Research portfolio	37.718	−31.002	35.580	57.628	0.100	3.074	17.861	5.810	0.064
2	Portfolio A	29.124	35.874	27.978	36.754	−30.724	3.074	32.259	10.500	0.035
3	Portfolio B	39.965	−77.783	56.846	55.890	24.016	3.074	23.477	7.640	0.049
4	Portfolio C	7.890	−46.869	38.876	91.611	8.675	3.074	24.406	7.940	0.047
5	Portfolio D	6.069	43.875	44.967	35.899	−31.876	3.074	35.174	11.440	0.033
6	Portfolio E	−19.876	22.872	99.097	8.234	−14.745	3.074	40.070	13.040	0.029
7	Portfolio F	22.334	35.765	39.340	30.879	−29.301	3.074	34.039	11.070	0.034
8	WIG20						1.084	21.899	20.200	−0.039

Source: own elaboration

According to the theory of H. Markowitz, a constructed portfolio is a portfolio for which it is impossible to create another one, which with the same risk could achieve a higher expected rate of return or lower risk for a given rate of return. In addition, a portfolio built with the same assets but with a more minor standard deviation will have a disproportionately low rate of return compared to a constructed portfolio. Thus, its investment potential would be zero and devoid of any attractiveness.

Despite achieving the set goal, the range in which the rate of return on investment in a given portfolio may be found oscillates between −14.788% and 20.935%. It is worth noting that the mentioned range is only a theoretical outline because it is impossible to predict the market behavior that may affect selected assets so that the actual annual rate of return will reach a result outside the established limits. The situation would be acceptable if it were a result higher than 20.935%, and it would be different if the ongoing market slump led to losses significantly exceeding the designated range. To prevent this, the portfolio will be hedged with derivatives.

As one can see, the table above clearly shows that the constructed portfolio is of the highest quality compared to the other six portfolios. This is supported by the lowest value of the standard deviation, the coefficient of variation, and the Sharpe index's highest value. According to the theory of H. Markowitz, a constructed portfolio is a portfolio for which it is impossible to create another one, which with the same risk could achieve a higher expected rate of return or lower risk for a given rate of return. In addition, a portfolio built with the same assets but with a more minor standard deviation will have a disproportionately low rate of return compared to a constructed portfolio. Thus, its investment potential would be zero and devoid of any attractiveness.

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Considering the prevailing conditions in the Polish market, options are the best derivatives used to protect the portfolio against the loss of value. This hedging can be performed using the protective put strategy, a typical portfolio hedging strategy that takes a long position in put options for the indicated assets. Unfortunately, the problem with the Polish capital market is that it has a significantly limited derivatives market, which allows you to take positions only in options based on the WIG20 index. This situation makes it much more difficult to hedge, as taking any position in options that directly protect the assets used to create the portfolio is impossible. As a result, hedging should be done indirectly, using options on the WIG20 index. In a given situation, the WIG20 index should be considered a market portfolio in which the hypothetical portfolio constructed in this publication operates. This is necessary to determine the mutual influence of the instruments studied, which will allow us to build an appropriate strategy. After estimation, the impact was measured using the beta coefficient, a final value of −1.352.

A negative value of the obtained result indicates that the rate of return of the examined portfolio reacts inversely to the rate of return resulting from changes in the WIG20 index price. This means that when the index's return increases, the portfolio's return decreases, and vice versa. In addition, the absolute value of the beta coefficient higher than one means that changes in the value of the examined portfolio over time are more rapid than those resulting from the change in the WIG20 index price.

Having all the information gathered so far, it is possible to establish a portfolio hedging strategy. To do this, you need to perform a few simple mathematical operations. Because this strategy will be an indirect strategy, you should start with an attempt to hedge a market portfolio with an initial value of one million zlotys. The collateral will be exercised with options whose redemption date will be closest on February 1, 2022, to the end of the investment horizon of the hypothetical portfolio, i.e., February 1, 2023. Options meet this requirement with a redemption date of December 2022. Taking into account the fact that on February 1, 2022, the closing price of the WIG20 index was 2,224.28, the "closest" out-of-money option that can be used to implement a hedging strategy is OW20X222100 with the strike price 2100. Notably, the options using which hedging strategy should be implemented should be out of the money or at the money at the time of constructing the strategy, as it is associated with a negative and zero cash flow from the exercise of given options, and thus the amount the premium for their acquisition is small. The phrase "closest" option used earlier means that its exercise price should be close to the current price of the hedged asset so that the hedge starts to work even with tiny fluctuations in stock exchange rates.

Another crucial step in constructing a strategy for hedging the hypothetical portfolio is determining the multiplier that characterizes the options on the Polish market. In the case of options on the WIG20 index, the multiplier is 10. Thus, each decrease or increase of the index by one point changes the value of the contract by 10 PLN. When a given value is known, it should be multiplied by the WIG20 index price from 01/02/2022 to determine the value of one option contract. In the case of the WIG20 index, the value of one contract is 22,242.8 PLN.

Consequently, entering into a single contract protects such a portfolio value. To find out how many options to hedge the entire portfolio, divide the portfolio's initial value by the result obtained above. In the case of the examined portfolio, the estimated effect is close to 45. As a result, to hedge a market portfolio with an initial value of 1,000,000 PLN, one should take a long position in 45 put options with an exercise price of 2,100.

Knowing how many options contracts are needed to hedge the market portfolio, this knowledge can be translated into a previously constructed hypothetical portfolio. To see the number of options that can hedge a given portfolio, one should multiply the number obtained above by the estimated beta coefficient of the portfolio. This practice is based on the assumption that each change in the value of the market portfolio by one point affects the change in the value of the hypothetical portfolio precisely by the value of its beta coefficient, in this case, by −1.352. The mathematical operation described above should give a result of −61.

So, to hedge the hypothetical portfolio, you need to take positions in 61 options. Due to the negative sign before the obtained result, it is known that the position taken should be opposite to the originally intended one. So it would be best to take a long position in call options. Revising the above assumptions, the option that will be used to achieve the set goal will be the call option OW20L222300, with an exercise price of 2300. It was selected for hedging because it is an out-the-money option, the exercise price is close to the current one on a given day, and the level of the WIG20 index. The premium for a given option is 101.36 PLN. Thus, taking a position in 61 options will incur costs of 6,161.66 PLN.

Is the developed strategy effective? The answer to this question is provided by the simulation presented below in the Table 8, which, depending on the level of the WIG20 index price, illustrates the behavior of the hypothetical portfolio, cash inflows from taking a position on the derivatives market, and the consequences of given inflows on the final value of the portfolio under the study of this article.

As shown in the table above, the applied strategy brings the expected results, reducing the risk of the hypothetical portfolio from the desired level of −14.788% to the maximum possible loss of −5.219%. It is also worth noting that implementing this strategy lowers the rate of return when the prices of purchased shares increase and the portfolio's value increases. This is the execution price of the option hedge, related to incurring costs generated by paying a premium for taking a position in 61 call options.

Table 8

Simulation of the effectiveness of the applied strategy

Simulation of payments related to taking positions in 61 options on the WIG20 with an exercise price of 2300							
An exemplary value of the WIG20 index	2100	2200	2300	2400	2500	2600	2700
Profit/loss on options [zł]	−6 161.66	−6 161.66	−6 161.66	54 838.34	115 838.34	176 838.34	237 838.34
Simulation of the change in the value of the portfolio after applying the option strategy							
The value of the portfolio resulting from changes in stock prices in the market [zł]	1 075 549.60	1 014 759.77	953 969.94	893 180.11	832 390.28	771 600.45	710 810.62
Rate of return before using the option strategy [%]	7.555	1.476	−4.603	−10.682	−16.761	−22.840	−28.919
Value of the portfolio after using the option strategy [zł]	1 069 387.94	1 008 598.11	947 808.28	948 018.45	948 228.62	948 438.79	948 648.96
The rate of return after using the option strategy [%]	6.939	0.860	−5.219	−5.198	−5.177	−5.156	−5.135

Source: own elaboration

5. Summary

This paper aims to answer whether an individual investor operating in Polish conditions can construct an investment portfolio, optimizing the investment process while reducing the total risk to a minimum. The main emphasis in creating the publication was on pragmatism and practice so that the tools available in the Polish market; could be used most effectively. As the study results showed, the goal set at the beginning was achieved, and the implementation itself leaves no illusions that the portfolio constructed during the research fulfills its tasks and the options strategy securing it. Diversified according to the model proposed by H. Markowitz, the portfolio has the highest possible investment potential, measured by the rate of return, which makes investments optimal in generating profits and defending against current market slumps. To summarize the above argument, this work is a model that allows for the implementation of stock exchange investments in a responsible, safe, and conscious manner, presented step by step.

Despite the achievement of the adopted objectives, it is worth explaining a few things that took place during the study and may need clarification about the reliability of its conduct. The first of these issues is selecting companies that comprise the research portfolio. Building an investment portfolio is a complicated process consisting of consecutive steps. The choice of investment instruments needed to construct a specific portfolio begins with an attempt to determine the individual profile of the investor. It ends with the tactical allocation of assets. Focusing on the subject of this work and not wanting to lose it, the method of selecting investment instruments has been simplified and reduced to selecting shares of five companies included in the WIG20 index, with the highest capitalization on February 1, 2022, whose stock exchange debut took place in 2007 at the latest.

An issue that raises doubts may be the period selected for the selection, assuming at least fifteen years of trading in equity instruments on the capital market. Why is it at least fifteen years and not ten or twenty? A longer time series provides a more extensive series of statistical data to validate the results of the calculations. Therefore, the history of stock trading must be well-spent. On the other hand, a given interval can only set the selection border a little in the past, as this would narrow down the pool of instruments available for testing. This is due to the relatively short period of operation of the Warsaw Stock Exchange. Therefore, the fifteen-year interval is a compromise, giving credibility to the results obtained in the study.

Another element worth explaining is why the portfolio was built with only five stocks. The number of selected companies must be more relevant for creating and securing the portfolio and its effectiveness. The transparency of estimates and calculations dictates the decision to build a portfolio using a limited number of companies shares.

The last issue needing clarification is the choice of derivatives to implement the hedging strategy. Why were options for this purpose, not futures contracts available on the Polish capital market, for assets building the portfolio? Well, futures contracts would compensate for the losses incurred by the portfolio while generating identical ones when its value increases. As a result, the portfolio would be deprived of its earning potential, maintaining a value close to its original value throughout the investment period, regardless of the prices of the instruments that make it up over time. As a result, there are better choices than the use of futures contracts. It is much more reasonable to use options that, apart from the costs associated with taking the position, will not generate any other expenses while compensating for the potential decline in the portfolio's value at critical moments.

Finally, it is worth noting that, due to the limited choice of derivatives, the portfolio was hedged with options for assets other than those used to construct the portfolio. As a result, the security used in the study is also adequate outside the capital market. It can increase optimization in securing transactions in metals, energy resources, agricultural products, and other goods traded on stock exchanges. Therefore, it makes the analyzed strategy universal and can be used by many recipients.

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Summary

This paper explores the evolving nature of the capital market in the second and third decades of the 21st century, characterized by its widespread accessibility to individual investors. However, the emergence of inexperienced investors driven by profit-seeking motives has led to non-conventional investment decisions resulting in significant losses. To address this challenge, the study investigates practical methods to minimize risk, focusing on the limited availability of derivatives in the Polish capital market. The research develops and tests an investment portfolio, drawing on Markowitz's portfolio construction theory and Hull's hedging strategy. The findings support the hypothesis that resilient investment portfolios can be constructed under Polish conditions using available tools and strategies.

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A statistical interpretation of a market demand curve for a commodity obeying the law of demand

1. Introduction

It is well known that in demand theory "budget constrained utility maximization" implies the weak axiom of revealed preference. Kihlstrom, Mas-Colell and Sonnenschein (1976) investigate demand theory based on the assumption that the demand function satisfies the weak axiom of revealed preference. Kihlstrom, Mas-Colell and Sonnenschein show that satisfaction of the weak axiom of revealed preference implies the "law of compensated demand", i.e., quantity demanded of a commodity, changes in a direction that is opposite to the direction of a price change, provided the initial consumption bundle is exactly affordable (i.e. budget-balance for the initial consumption bundle) at the new price. This follows, since if the original bundle is affordable but not chosen at new prices, then by the weak axiom of revealed preference, at original prices the new consumption bundle is required to be unavailable. We provide an independent derivation of this result in Appendix A of this paper. The converse of this result holds provided the Marshallian demand function satisfies "budget balance". For a normal good, the "law of compensated demand" applied to the Slutsky equation for own prices, implies the "law of demand", i.e., quantity demanded of a commodity, changes in a direction that is opposite to the direction of a price change. Thus, the weak axiom of revealed preference implies the "law of demand" for normal goods. A very lucid account of the related demand theory is available in chapter 2 of Mas-Colell, Whinston and Green (1995). A comprehensive discussion of demand theory, using money as a numeraire, and the related welfare economic theory of consumer surplus when the demand function satisfies the "law of demand" is

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rigorously presented in Lahiri (2022a, 2022b), the genesis of which is available in Lahiri (2020).

In this note we provide a statistical interpretation of the Marshallian market demand curve of a commodity which obeys the "law of demand" and whose unit of consumption is fixed (e.g., single units of refrigerator, air conditioner or full tank capacity of fuel oil of a Maruti 800) and which has a finite and positive level of satiation. Alternatively, we could consider the demand function to be a "compensated market demand function", except in section 5, where we discuss the Marshallian demand curve generated by "budget constrained linear utility maximization". The "finite and positive level of satiation" means that allowing for free disposability of the good, there is a finite positive amount of the good after which the consumers are not willing to pay anything for incremental units of the good. We refer to this positive level of satiation as "market size". The market size may depend on the tastes and preferences and income distribution of the consumers as well as the prevailing prices of other goods and services. Further, suppose that the unit of consumption is infinitesimally small compared to the market size. This latter assumption allows the good to be viewed as "homogeneous and infinitely divisible", as far as the demand analysis of the good is concerned. An interesting consequence of the statistical approach that we adopt in this paper is that in the context of two goods, we are able to obtain demand functions which are very similar to those obtained by "budget-constrained Cobb-Douglas utility maximization", but now as a result of a "budget-constrained linear utility maximization" exercise, although our budget constraint is "slightly different" from the one that would be used for the former optimization problem. This is discussed in section 5 of this paper.

A standard reference for what follows is Peterson and Lewis (1999). For alternative and more advanced demand theory one may refer to Katzner (1970, 2008), Lahiri (2022a, 2022b).

2. The model

Suppose $\bar{q} > 0$ is the "market size". In what follows, we can assume without loss of generality that $\bar{q} = 1$.

A "reservation price" for a particular unit of the good in the market is the maximum amount that some consumer (in the market) would be willing to pay for that particular unit of the good. We allow for the same buyer to have different reservation prices for successive units of the good. Let $F:[0, +\infty)\rightarrow[0, 1]$ be a function, with at most a finite number of points of discontinuity, denoting the "distribution function of the reservation prices" for the good in the market, where F may depend on the tastes, preferences and income distribution of the consumers as well as the prevailing prices of other goods and services. Thus for $p \in [0, +\infty)$, the reservation prices for first $1 - F(p)$ units of the good – and not more – are greater than or equal to 'p'. Thus at price $p \in [0, +\infty)$, consumers would be willing to buy at most $q(p) = 1 - F(p)$ units of the good.

In what follows, we assume the following, which is known as the "uncompensated" Law of Demand: F is strictly increasing on the set $\{p \in [0, +\infty) \mid 0 < F(p) < 1\}$.

Thus the function $1 - F(\cdot)$ is strictly decreasing on the set $\{p \in [0, +\infty) \mid 0 < F(p) < 1\}$.

Clearly the function F is invertible with the inverse function $F^{-1}:[0, 1) \rightarrow [0, +\infty)$ being strictly increasing on (0, 1) and satisfying $F^{-1}(0) = 0$.

Consider the function $P:(0, 1] \rightarrow [0, +\infty)$ defined by $P(q) = F^{-1}(1 - q)$.

The function P (which may depend on prices of other goods and services as well as the distribution of income of the consumers) is the "(Marshallian) statistical demand curve" for the good. In what follows we will refer to P as the "demand curve" for the good.

For $q \in (0, 1]$, P(q) is the maximum price the consumers are willing to pay for q units of the good. This is possible if and only if P(q) is the "lowest" reservation price when q units – and no more – of the good is bought in the market. Units of the good in excess of the first q units have reservation prices less than P(q) and hence, are not bought, at price P(q).

Let $W: [0, 1] \rightarrow [0, +\infty)$ with $W(0) = 0$ be a function such that for all $q \in (0, 1]$, W(q) is the total amount of the good that consumers are willing to pay for q units of the good. W is the "**willingness to pay function**".

Clearly W may depend on tastes, preferences and the distribution of income of the consumers as well as the prices of other goods and services.

At price $p \geq 0$ and quantity of the good $q \in (0, 1]$ the "consumers' surplus" is given by $W(q) - pq$.

The consumers are said to be "surplus maximizers" if for all $p > 0$, such that $0 < F(p) < 1$, $q(p) = [1 - F(p)]$ solves

Maximize $W(q) - pq$

s.t. $0 < q < 1$.

We know that at price p, only those units of the good will be bought whose reservation prices are greater than or equal to 'p' – no more and no less.

In the following sections we compute the willingness to pay functions for linear and piece wise linear demand curves for surplus maximizing consumers.

3. Linear Demand Curves

Recall that if the consumers are surplus maximizers, then for all $p > 0$, such that $0 < F(p) < 1$, $q(p) = 1 - F(p)$ solves

Maximize $W(q) - pq$

s.t. $0 < q < \bar{q}$.

For a real number $a > 0$, let $F(p) = \frac{p}{a}$ for all $p \in [0, a]$ and $F(p) = 1$ for all $p > a$.

Then $q(p) = 1 - \frac{p}{a}$ for all $p \in [0, a]$, and $q(p) = 0$ for all $p > a$.

Hence, $P(q) = a - aq$ for $q \in (0, 1]$.

Let q belong to the open interval (0, 1).

The market's total willingness to pay for q units denoted by $W(q) = aq - \frac{a}{2} q^2$, for $q \in (0, 1]$.

A derivation of this result without using calculus is provided in Appendix B of this paper.

It is easy to see that for $q \in (0, 1]$, W(q) is the area under the straight-line $P(q') = a - aq'$ from '0' to 'q'.

Let $p > 0$ be the price of the good. We know that for $p \geq a$, the quantity demanded is zero. Hence suppose $p < a$.

The surplus obtained from consuming q units of the good, where $q \in (0, 1)$ is given by $(a - p)q - \frac{a}{2} q^2 = -\frac{a}{2}(q^2 - 2(1 - \frac{p}{a})q) = -\frac{1}{2}[(q - (1 - \frac{p}{a}))^2 - (1 - —)^2]$.

The surplus is maximized for the value of q that minimizes $(q - (1 - \frac{p}{a}))^2 - (1 - \frac{p}{a})^2$ subject to $q \in (0, 1)$. Thus, the surplus is maximized at $q = 1 - \frac{p}{a}$, i.e., the point $q \in (0, 1)$ satisfying $P(q) = p$.

The interesting thing to note is that for $q \in (0, 1]$, $W(q) = aq - \frac{a}{2} q^2 = aq - aq^2 + \frac{a}{2} q^2 = \frac{(a + (a - aq))q}{2}$ = area of trapezium below the demand curve from 0 to q.

Note: It is easy to see that for the linear demand curve $W(q) = \int_0^q (a - aq')dq' = aq - \frac{a}{2} q^2$ for $q \in (0, 1]$.

4. Piece-wise Linear Demand Curves

For a positive integer $n \geq 2$, let $a_0 > a_1 > \ldots > a_n = 0$ (the entire array of prices possibly depending on the income distribution of the consumers as well as the prevailing prices of other goods and services) and let $0 = q_0 < q_1 < \ldots < q_n = 1$ (the entire array possibly depending on the tastes, preferences and income distribution of the consumers as well as the prevailing prices of other goods and services) be such that for all $j \in \{0, \ldots, n\}$, q_j is the quantity demanded at price a_j. Let $G(p) = 0$ for all $p \geq a_0$, $G(p) = (a_0 - p)q_1$ for all $p \in (a_1, a_0]$, and for all $j \in \{1, \ldots, n - 1\}$

let G(p) = $G(a_j) + \frac{(a_j - p)}{(a_j - a_{j+1})}(q_{j+1} - q_{(j)})$ for all $p \in (a_j, a_{j+1}]$. Let F(p) = 1 – G(p) for all $p \in (0, a_0]$.

Thus, for $q \in (0, 1]$, the reservation prices for units corresponding to the quantity demanded q are greater than or equal to $P(q) = a_j - \frac{(q - q_j)}{(q_{j+1} - q_j)}(a_j - a_{j+1})$ if $q \in (q_j, q_{j+1}]$.

By an argument similar to the one used for linear demand curves, we get that the market's willingness to pay for 'q' units of the good is measured by "the area of the polygon below the demand curve from 0 to q".

Thus, $W(q) = \frac{1}{2} q(2a_0 - \frac{a_0 - a_1}{q_1} q)$ if $q \in (q_0, q_1]$

and for $q \in (q_j, q_{j+1}]$ with $j \in \{1, \ldots, n-1\}$, $W(q) = \sum_{(k=1)}^{j-0} \frac{(q_{k+1} - q_{(k)})(a_{k+1} + a_k)}{2}$ +

$+ \frac{1}{2}(q - q_j)(2a_j - \frac{(q - q_j)}{(q_{j+1} - q_j)}(a_j - a_{j+1}))$.

5. Linear utility maximization

Of considerable interest is the demand function obtained by budget constrained "linear" utility maximization subject to a satiation constraint. In order to present the result in its full generality, in this section we will relax the assumption the market size, $\bar{q}$ is 'one' and allow it to be any positive real number. Let $\mu > 0$ be the money available to the consumers who are willing to pay a maximum price $u > 0$ for the commodity. The commodity has the feature of a "quasi-essential" good, so that up to the market size $\bar{q}$, a strictly positive share $\alpha < 1$ of the entire amount of money that is available, is spent on the good, after which if there is any money left, that is used for the consumption of other goods and services. We assume that $u > \frac{\alpha\mu}{\bar{q}}$ and the distribution of reservation prices has a discontinuity at u. Thus, $F:[0, +\infty) \to [0, 1]$ is defined as follows:

For $p \in (u, +\infty)$, F(p) = 1; for $p \in \left[\frac{\alpha\mu}{\bar{q}}, u\right]$, $F(p) = 1 - \frac{\alpha\mu}{p\bar{q}}$; and for $p \in \left[0, \frac{\alpha\mu}{\bar{q}}\right)$, F(p) = 0.

The associated demand function, which is very similar to the one generated by the Cobb-Douglas utility function within the price range $\left[u, \frac{\alpha\mu}{\bar{q}}\right]$, is obtained as an optimal solution to the following maximization problem:

Choose 'q' to

Maximize $u(\min\{q, \bar{q}\}) + y$

s.t. $y + pq \leq \alpha\mu$,

$q \geq 0, y \geq 0$.

The interval on which the demand functions generated by the above maximization exercise coincides with those generated by "budget-constrained Cobb-Douglas utility maximization" expands as u and/or $\bar{q}$ increases.

6. Surplus maximization using calculus

In this section we use (Newtonian) calculus to obtain the relationship between the demand curve and the willingness to pay function. Clearly, the results in this section are not applicable for piecewise linear demand curves.

Assumption 1: $\int_0^q P(q')dq'$ exists and $0 < \int_0^q P(q')dq' < +\infty$ for all $q \in (0, \bar{q}]$.

Assumption 2: There exists a differentiable function $W: (0, 1] \to [0, +\infty)$ with $W':(0, 1) \to [0, +\infty)$ (where $W'(q) = \frac{dW(q)}{dq}$ for all $q \in (0, 1)$) positive valued and strictly decreasing, that gives for each $q \in (0,1]$ the buyers' willingness to pay for q units of the commodity, the latter possibly depending on the tastes, preferences, income distribution of the buyers as well as prices of other goods.

Theorem 1: The consumers are surplus maximizers if and only if for all $q \in (0, q)$, $W'(q) = \frac{dW(q)}{dq} = P(q)$.

Proof: It is easy to see that if consumers are surplus maximizers, then for all $q \in (0, 1)$, $W'(q) = \frac{dW(q)}{dq} = P(q)$.

Hence suppose that for all $q \in (0, 1)$, $W'(q) = \frac{dW(q)}{dq} = P(q)$.

Let $p > 0$, such that $0 < F(p) < 1$, and suppose $q^0 \in (0, 1)$ solves

Maximize $W(q) - pq$

s.t. $0 < q < 1$.

Then clearly, $W'(q^0) = \frac{dW(q^0)}{dq} = p$.

However, by assumption $W'(q^0) = \frac{dW(q^0)}{dq} = P(q^0)$.

Thus, $P(q^0) = p$.

Since P is strictly decreasing (follows from $W'(q) = \frac{dW(q)}{dq} = P(q)$ for all $q \in (0, 1)$ and W' is strictly decreasing) $p = P(q^0) = F^{-1}(1 - q^0)$, i.e. $q^0 = 1 - F(p)$.

This proves the theorem. Q.E.D.

An immediate corollary of the above is the following.

Corollary of theorem 1: The consumers are surplus maximizers if and only if for all $q \in (0, 1)$, $W(q) = \int_0^q P(q')dq'$.

Applying a change of variable theorem argument to the above, we get $W(q(p)) = \int_p^{+\infty} q(p')dp' + pq(p) = \bar{q}\int_p^{+\infty} 1 - F(p')]dp' + pq(p)$

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Appendix A

For $L \geq 2$, let D: $(\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++} \rightarrow \mathbb{R}^{L}_{+}$ be the demand function for L goods of which the first L – 1 goods are non-monetary goods and the L^{th} good is monetary savings for the future or monetary savings for non-monetary goods other than those considered in one of the first L – 1 goods. The price of money is fixed at '1'. At price vector $p \in (\mathbb{R}^{L-1}_{++} \times \{1\})$ and monetary value of wealth $\mu \in \mathbb{R}_{++}$, $D(p, \mu)$ is the vector denoting the quantities that are demanded for each of the L goods – monetary as well as non-monetary.

We assume that D satisfies the **Weak Axiom of Revealed Preference (WARP)**, i.e., for all $(p^0, \mu^0), (p^1, \mu^1) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++}$: $[D(p^0, \mu^0) \neq D(p^1, \mu^1)$ and $p^{0T}D(p^1, \mu^1) \leq \mu^0]$ implies $[p^{1T}D(p^0, \mu^0) > \mu^1]$.

Let H: $(\mathbb{R}^{L-1}_{++} \times \{1\}) \times (\mathbb{R}^{L}_{+} \setminus \{0\}) \rightarrow \mathbb{R}^{L}_{+}$ be the function defined by

$H(p, x) = D(p, p^T x)$ for all $(p, x) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times (\mathbb{R}^{L}_{+} \setminus \{0\})$.

H is said to be the **compensated demand function.**

Lemma 1: For all $(p, \mu) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++}$, $H(p, D(p, \mu)) = D(p, \mu)$.

Proof: Towards a contradiction, suppose they are not equal at some price income-pair. Note that, for all $(p, \mu) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++}$ we have $H(p, D(p, \mu)) =$ $= D(p, p^T D(p, \mu))$.

Hence at some $(p, \mu) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++}$ we have

$H(p, D(p, \mu)) = D(p, p^T D(p, \mu)) \neq D(p, \mu)$.

Thus, $p^T D(p, p^T D(p, \mu)) \leq p^T D(p, \mu) \leq \mu$, and $p^T D(p, \mu) \leq p^T D(p, \mu)$.

Thus, $D(p, p^T D(p, \mu))$ is available and not chosen when $D(p, \mu)$ is chosen and $D(p, \mu)$ is available and not chosen when $D(p, p^T D(p, \mu))$ is chosen.

This contradicts the Weak Axiom of Revealed Preference (WARP) and proves the lemma. Q.E.D.

Proposition 1: Suppose D satisfies WARP. Then, for all $p, p^0 \in (\mathbb{R}^{L-1}_{++} \times \{1\})$ and $\mu^0 \in \mathbb{R}_{++}$ it must be the case that $(p - p^0)^T(H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))) \leq 0$, with strict inequality if $H(p, D(p^0, \mu^0)) \neq H(p^0, D(p^0, \mu^0))$.

Proof: By definition of H, for all $(p, \mu) \in (\mathbb{R}_{++}^{L-1} \times \{1\}) \times \mathbb{R}_{++}$, $H(p, D(p^0, \mu^0)) = D(p, p^T D(p^0, \mu^0))$ and by Lemma 1, $H(p, D(p, \mu)) = D(p, \mu)$.

Hence,

$p^T[H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))] = p^T[D(p, p^T D(p^0, \mu^0)) - D(p^0, \mu^0)] \le 0$.

If $H(p, D(p^0, \mu^0)) = H(p^0, D(p^0, \mu^0))$, then Proposition 1 is obviously correct. Hence suppose $H(p, D(p^0, \mu^0)) \neq H(p^0, D(p^0, \mu^0)) = D(p^0, \mu^0)$

$p^T D(p^0, \mu^0) = p^T D(p^0, \mu^0)$ means $D(p^0, \mu^0)$ is available but not chosen when $D(p, p^T D(p^0, \mu^0))$ is chosen.

Thus, by WARP $p^{0T} D(p, p^T D(p^0, \mu^0)) > I^0 \ge p^{0T} D(p^0, \mu^0)$.

Thus, $p^{0T}[D(p, p^T D(p^0, \mu^0)) - D(p^0, \mu^0)] > 0$.

Hence, $-p^{0T}[D(p, p^T D(p^0, \mu^0)) - D(p^0, \mu^0)] > 0$, i.e.,

$-p^{0T}[[H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))] < 0$.

Combined with $p^T[H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))] \le 0$, we get

$(p - p^0)^T(H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))) < 0$.

This proves the lemma. Q.E.D.

Appendix B

Let $P(q) = a - aq$ for $q \in (0, 1]$.

Let q^0, q satisfying $0 \le q^0 < q$, belong to the open interval (0, 1).

Suppose that consumers are already consuming an amount q^0 and we want to find their willingness to pay for an additional amount of $q - q^0$.

For $m \in \mathbb{N}$, let us subdivide the interval (q^0, q) into 'm' equal and non-overlapping intervals of length $\frac{q-q^0}{m}$ each.

For the first $\frac{q-q^0}{m}$ units, the market's average willingness to pay for an incremental unit is less than equal to $a - aq^0$ and greater than or equal to $a - a\left(q^0 + \frac{q-q^0}{m}\right)$. Hence, for the first $\frac{q-q^0}{m}$ units, the market's total willingness to pay is greater than or equal to $\left[a - a\left(q^0 + \frac{q-q^0}{m}\right)\right]\frac{q-q^0}{m}$ and less than or equal to $[a - aq^0]\frac{q-q^0}{m}$.

For the j^{th} $\frac{q-q^0}{m}$ unit, with $j\in\{2, \ldots, m\}$, the market's total willingness to pay is greater than or equal to $\left[a-a\left(q^0+j\frac{q-q^0}{m}\right)\right]\frac{q-q^0}{m}$ and less than or equal to $\left[a-a\left(q^0+(j-1)\frac{q-q^0}{m}\right)\right]\frac{q-q^0}{m}$.

Hence the market's willingness to pay for the extra '$q - q^0$' units is greater than or equal to $\frac{1}{m}\sum_{j=1}^{m}\left[a-a\left(q^0+j\frac{q-q^0}{m}\right)\right]\frac{q-q^0}{m}$ and less than or equal to $\frac{1}{m}\sum_{j=1}^{m}\left[a-a\left(q^0+(j-1)\frac{q-q^0}{m}\right)\right]q-q^0$.

$$\frac{1}{m}\sum_{j=1}^{m}\left[a-a\left(q^0+j\frac{q-q^0}{m}\right)\right](q-q^0)=a(q-q^0)-aq^0(q-q^0)-a\left(\frac{q-q^0}{m}\right)^2\sum_{j=1}^{m}j=$$

$$=a(q-q^0)-aq^0(q-q^0)-a\left(\frac{q-q^0}{m}\right)^2\frac{m(m+1)}{2}=a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m+1}{2m}.$$

Similarly,

$$\frac{1}{m}\sum_{j=1}^{m}\left[a-a\left(q^0+(j-1)\frac{q-q^0}{m}\right)\right](q-q^0)=a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m-1}{2m}.$$

Hence, the market's willingness to pay for $q - q^0$ additional units is greater than or equal to $a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m+1}{2m}$ and less than or equal to $a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m-1}{2m}$ for all $m\in\mathbb{N}$.

Now,

$$\lim_{m\to\infty}\left[a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m+1}{2m}\right]=a(q-q^0)-aq^0(q-q^0)-$$
$$-\frac{a}{2}\left(q-q^0\right)^2=a[q-\frac{1}{2}q^2]-a[q^0-\frac{1}{2}q^{02}].$$

Similarly,

$$\lim_{m\to\infty}\left[a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\right]=a[q-\frac{1}{2}q^2]-a[q^0-\frac{1}{2}q^{02}].$$

Hence, the market's total willingness to pay for the additional $q - q^0$ units $W(q) - W(q^0) = a[q - \frac{1}{2}q^2] - a[q^0 - \frac{1}{2}q^{02}]$, where $W(q) = aq - \frac{a}{2}q^2$ for $q \in (0, 1]$, obtained by letting $q^0 = 0$, is the market's willingness to pay for the first q^{th} units of the good.

Summary

In this note we provide a statistical interpretation of the Marshallian market demand curve of a commodity that obeys the law of demand and which has a finite and positive level of satiation. A consequence of our approach is that in the context of two goods, we are able to obtain demand functions which are very similar to those obtained by "budget-constrained Cobb–Douglas utility maximization", but now as a result of a "budget-constrained linear utility maximization" exercise, although our budget constraint is "slightly different" from the one that would be used for the former optimization problem.

JEL Codes: C25, C44, C60, C61, D11.

Keywords: *market demand curve, statistical interpretation, reservation price, willingness to pay, consumers surplus*

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Expiration day effects of stock and index futures on the Warsaw Stock Exchange before and in the initial phase of the COVID-19 pandemic

1. Introduction

As important vehicles of hedging open positions on stock market, financial futures are constructed in a such manner that their prices and the prices of their underlying assets are strongly related. The stock and futures markets continually interact with each other so important events which significantly change stock prices indirectly can also influence prices of corresponding futures. Similarly, events which are related to the futures market may have an impact on stock market too, one of which is the expiration of derivatives.

Research on the influence of stock and index futures expiration on volume, prices, and the volatility of underlying assets have been conducted in reference to many spot markets around the world. Early studies on so called expiration day effects concerned U.S. market (Stoll, Whaley, 1986, 1987, 1990, 1991). Subsequently, such analyses have been extended to other markets such as Canadian, German, Spanish, Swedish, Australian, Korean, Polish, Taiwan, Indian or Vietnam (see e.g. Chamberlain et al., 1989; Stoll, Whaley, 1997; Aragó, Fernández, 2002; Park, Lim, 2004; Hsieh, Ma, 2009; Debasish, 2010; Chow et al., 2013; Xu, 2014; Wats, 2017; Gurgul, Suliga, 2020; Samineni et al., 2020; Suliga, 2021; Nguyen et al., 2022).

Many of these studies confirm that futures' expiration involves increased activity of investors on the spot market which can lead to a significant disturbance in

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the price formation process. The most frequently observed expiration day effects are abnormally high trading volume of underlying stocks and increased volatility of their prices (see e.g. Stoll, Whaley, 1987, 1990, 1991, 1997; Karolyi, 1996; Chay, Ryu 2006; Illueca, Lafuente, 2006; Hsieh, 2009; Debasish, 2010; Chay et al., 2013). On some markets, because of price pressure, stock prices fall unusually during expiration (e.g. Stoll, Whaley, 1986; Chow et al., 2003; Alkebäck, Hagelin, 2004; Vipul, 2005; Nguyen et al., 2022) or increase (Chamberlain et al., 1989; Chou et al., 2006; Bhaumik, Bose, 2007; Chuang et al., 2008; Narang, Vij, 2013; Suliga, 2020). Directly after the expiration, some researchers detected price reversal or so-called price shock (e.g. Stoll, Whaley, 1990, 1991; Schlag, 1996; Park, Lim, 2004; Vipul, 2005; Chay, Ryu, 2006; Hsieh, 2009; Chow et al., 2013) indicating that temporarily disturbed stock prices come back to the normal level, reflecting their intrinsic values.

The main cause of the occurrence of expiration day effects is cash settlement of financial futures. Such a procedure means that there is no physical delivery of the underlying asset from an issuer of a contract to a purchaser on expiration day, but appropriate cash flow is made between them. The direction and value of this cash flow depends on the final settlement price of a derivative which is calculated on the basis of the actual price of underlying stocks. It should be noted that in the case of index futures this is the only possible formula of final settlement but on most markets around the world stock futures are also cash settled (Dębski, 2010). This encourages investors with open positions on futures market to keep them until the expiration and to trade on the spot market on expiration day to influence or at least control the final course of a derivative.

One of two main groups of investors who can generate expiration day effects are speculators. Knowing that their loss or profit from expiring futures depends on underlying stock prices, they can try, by appropriate activity on a spot market, to not only reduce the risk of uncertainty of final settlement price of derivatives but also to designedly manipulate stock prices to bring about the final price of a contract at a level favorable from their point of view (Debasish, 2010; Narang, Vij, 2013). The leverage effect is an additional incentive to such activity which can be successful especially in case of stock futures with final course equal to a single value of underlying stock price (e.g. opening or closing price on expiration day). However, if the price does not reflect the real value of a stock, according to the Fama hypotheses of efficient markets, rational investors take advantage of this fact and their activity very quickly bring back the price to its normal level. Hence, such stock price distortion arising from futures expiration should only be temporary and significant price reversal or price shocks might occur just after the expiration day.

Anomalies on the stock market on the expiration days of futures can be also a result of an activity of arbitrageurs. If a mispricing between the underlying asset and its corresponding derivative occurs during contract's life, these investors exploit it and hold a short position in contract and a long position in its underlying asset or vice versa (this depends on the direction of a mispricing). Arbitrage strategies are often unwound on expiration day as no transaction on the futures market is needed in this case. It is only necessary to close a position on a stock market by selling (or buying previously short-sold) stocks. If arbitrageurs make up a relevant group on a market and if most of them unwind their positions in the same direction on expiration day, significant price effects might occur. This is especially true on markets where short selling of stocks in prohibited or severely restricted activity of arbitrageurs on expiration day can lead to a sharp fall of stock prices (Vipul, 2005), as only cash-and-carry arbitrage is possible to carry out while the opposite strategy (reverse cash-and carry) is limited by an inability to short sell stocks.

Expiration day effects, although temporary, are nevertheless undesirable. Artificially induced price changes misguide uninformed investors and distort the process of price discovery. For this reason, research on expiration day effects is still being conducted on many markets. If strong anomalies are detected, additional regulations are being introduced in an attempt to eliminate them. In the following years such studies are repeated to check if these regulations have had the desired effect (see e.g. Stoll, Whaley, 1991; Hsieh, Ma, 2009; Chay et al., 2013).

Especially on emerging market expiration day effects might be strong and adverse but as the market develops, anomalies should gradually diminish. On deep and liquid developed markets, successful manipulation as well as profitable arbitrage should be difficult to perform. Nevertheless, research on expiration day effects is being carried out for many markets in the world, differing in the degree of development.

On the Warsaw Stock Exchange, expiration day effects have been studied so far by Morawska (2004, 2007), Suliga (2017, 2020, 2021), Suliga and Wójtowicz (2019) as well as Gurgul and Suliga (2020). All these analyses confirmed the occurrence of undesirable anomalies on Polish stock market on days with the simultaneous expiration of stock futures, index futures and index options. However, in each of these articles, the research covers a period in which the Polish market was still perceived as emerging (i.e. period ending before or at most in 2017) while from 2018 one of the three biggest rating agencies – FTSE Russell – has classified Polish market as a developed one. Therefore, the question arises as to whether in recent years, which have not yet been covered by research, expiration day effects of futures have been still arising on the Warsaw Stock Exchange or if these anomalies completely disappeared with the development of the market?

Moreover, the COVID-19 pandemic broke out in 2020 and disturbed the stability of financial markets all over the world, leading to a sharp increase in stock price volatility around the world (Czech et al., 2020a), creating an unprecedented level of risk. The world's major indices fell sharply in March 2020 (e.g. Alam et. al., 2020; McKibbin, Fernando 2020; Dharani, 2022). Many companies experienced panic pressure to sell due to the growing fear of the spreading virus (Dharani, 2022). This certainly forced investors to search for new ways of hedging and thereby might have attracted more of them to the futures market (see e.g. Corbet et al., 2020). Therefore, the paper also considers whether the outbreak of the COVID-19 pandemic had a significant impact on the occurrence and strength of expiration day effects.

In this study we tackle the abovementioned questions and try to supply some answers. The paper extends the previous research on the impact of financial derivatives expiration on the Warsaw Stock Exchange, supplying a comprehensive analysis of the anomalies in the period 2018–2020. With the use of intraday data, all expiration day effects most mentioned in the literature are examined, that is: abnormally high trading volume and increased volatility of underlying stocks in expiration day as well as price reversal and price shock after the expiration. A comparison of the results obtained with those of previous research relating to Polish market allows us to assess the degree of the market's development. An important goal of the paper is to also answer the question of whether the strength of the effects significantly changed like-for-like in the initial period of COVID-19 pandemic. For this purpose, additional analysis of the anomalies is conducted in sub-periods (before and after March 2020).

The rest of the paper is organized as follows. Section 2 reviews existing literature. In section 3, the data and empirical methodology are presented. Section 4 describes and discusses the results of the research, while section 5 makes some concluding remarks.

2. Literature review

Most previous studies on expiration day effects concentrate on anomalies generated by the expiration of index futures which are the most popular derivatives on markets around the world (e.g. Herbst, Maberly 1990, 1991; Stoll, Whaley, 1997; Kan, 2001; Aragó, Fernández 2002; Chung, Hseu, 2008; Hsieh, 2009; Chow et al., 2013; Samineni et al., 2020; Batrinca et al., 2020; Nguyen et al., 2022). However, on many markets, a few different types of derivatives expire simultaneously and research related to them analyzes the cumulative impact of their expiration on a stock market (see e.g. Schlag, 1996; Chow et al., 2003; Alkebäck, Hagelin, 2004; Vipul, 2005; Chay, Ryu, 2006; Chuang et al., 2008; Xu, 2014; Mahalwala, 2016; Gurgul, Suliga, 2020). In the literature, the last hour of the day when index futures, index options and stock futures expire together is called the "triple witching hour". In this hour, anomalies on the stock market might be especially strong.

Potential expiration day effects of financial derivatives include increased trading volume of underlying asset, an unusual increase or decrease in price, abnormally high volatility and price reversal or price shock after the expiration. The existence of significant expiration day effects was confirmed for many markets. However, the type of the occurring anomalies, their duration and strength depend on many factors and thus vary between the markets. Importantly, some studies have shown that these anomalies can be very short-lived and their detection may require the use of high-frequency data, as stressed by Alkebäck and Hagelin (2004). Perhaps for this reason, in some of the analysis, which based on daily data, the occurrence of the expected expiration day effects was not confirmed (e.g. Kan, 2001; Samineni et al., 2020; Nguyen et al., 2022).

2.1. Volume effect

An unusual increase in investor activity on the spot market on expiration days of stock and index derivatives was detected in most earlier research. Firstly, it was confirmed by Stoll and Whaley (1986, 1987) in reference to the spot market in USA on the expiration days of futures on S&P 500 index and options on S&P 100 index in the 1980s. In following years, similar results were obtained for many other markets, e.g. Australian (Stoll, Whaley, 1997), German (Schlag, 1996), Indian (Vipul, 2005; Debasish, 2010; Mahalwala, 2016), Japan (Karolyi, 1996), Korean (Park, Lim, 2004; Chay, Ryu, 2006; Chay et al., 2013), Polish (Suliga, Wójtowicz, 2019; Gurgul, Suliga, 2020; Suliga 2021), Spanish (Illueca, Lafuente, 2006), Swedish (Alkebäck, Hagelin, 2004; Xu, 2014), Taiwan (Chou et al., 2006; Chuang et al., 2008). This anomaly was also detected in a wider, pan-European analysis conducted by Batrinca et al. (2020).

Some studies indicate that abnormally high trading volume may not only occur on the spot market on expiration days of derivatives but also on several days before the expiration. Such results were obtained e.g. by Alkebäck and Hagelin (2004) who studied expiration day effects of OMX index futures on Swedish market. They identify this effect with an activity of arbitrageurs who unwind their positions on stock market before the expiration to reduce the risk connected with the uncertainty about the final settlement price of derivatives. During the period under study (1988–1998), the final price of OMX futures was calculated as the weighted average of all index quotes from expiration day in which weights were equal to trading volumes of stocks.

Increased activity on the spot market directly before the expiration day, evinced in abnormally high trading volume, was also detected by Vipul (2005) and Debasish (2010) on the Indian market. These authors studied the anomalies generated by the simultaneous expiration of options and futures on Nifty index as well as option and futures on individual stocks. Final settlement price of these derivatives was equal to a weighted average price of underlying asset from the last 30 minutes of the last trading session. They also associate this early volume effect with the activity of arbitrageurs finalizing their strategies before the expiration day.

Agarwalla and Pandey (2013) claimed that in the case of final settlement procedures such as those mentioned above, it is impossible for arbitrageurs to assess in advance how to sell (buy) stocks during the trading session on expiration day to perfectly balance the open position in expiring futures. Thus, they often decide to close the position in stock market earlier in order to reduce the risk. This results in volume effects not only occurring on expiration day but also several days before this date.

Research on the volume effect indicates a close relationship between the duration of increased activity of investors on a spot market and a final settlement procedure. If final settlement price of futures is equal to a closing price of underling stock (level of an index at the close in case of index futures), investors' activity on the spot market usually increases during the last minutes of continuous trading as well as at the close (see e.g. Chay et al., 2013; Suliga, 2020). This is not surprising, as in such cases arbitrage strategies are mainly being finalized by holding open positions in futures until expiration and by placing an order to buy or sell stocks at the close on expiration day. Speculators who have open position in expiring futures also mainly trade on the stock market in the last minutes of a session. They certainly want to control the risk of an unfavorable final settlement price of derivatives but some authors (e.g. Alkebäck, Hagelin, 2004; Mahalwala, 2016) suggest that speculators may also deliberately manipulate stock prices to influence final price of futures. As this price depends on a single value of underlying asset, investor activity is concentrated in a very short period. A sudden, sharp increase in turnover at the end of the session may also result in unusual price changes.

On markets where the final settlement price of futures is calculated as an average price of an underlying asset from a short time interval, increased activity of investors on stock market can mainly be observed during the time when the final price of derivatives is being formed (e.g. Illueca, Lafuente, 2006; Fung, Yung, 2009; Gurgul, Suliga, 2020; Suliga, 2021). This also points to arbitrageurs' and speculators' attempts to control the final settlement price of expiring futures.

2.2. Price effects

The abnormal growth of investor activity on the spot market on expiration days often results in a significant increase in stock volatility. Many studies have confirmed that the volatility of the prices of underlying stocks (or indexes) of expiring futures is higher than normal on expiration days. Such results were obtained in reference to many markets i.a. Australian (Stoll, Whaley, 1997), Hongkong (Chow et al., 2003), Indian (e.g. Bhaumik, Bose, 2007; Narang, Vij, 2013; Wats, 2017), Korean (e.g. Chay, Ryu, 2006; Chay et al., 2013), Polish (e.g. Gurgul, Suliga, 2020; Suliga, 2020), Spanish (Illueca, Lafuente, 2006), Swedish (Alkebäck, Hagelin, 2004) or Taiwan (Chou et al., 2006).

In the case of the volatility effect, the research also indicates a close relationship between the procedure for calculating final prices of futures and the time and strength of the anomaly. The strongest changes may occur in the prices of stocks for which corresponding futures have a final price depending on a single value of an underlying asset (see e.g. Stoll, Whaley, 1987; Park, Lim 2004; Chay et al., 2013; Suliga, 2020; Suliga, 2021). There is then a high risk of market imbalance and another price effect – sharp fall or increase in stock prices. Such an anomaly is especially possible as a result of stock futures expiration. A significant change of single stock price, as an effect of arbitrageurs or speculators trading, is much more likely than unusual change of index level, which depends on the prices of many stocks.

If the short selling of stocks is prohibited on a market, this can be an additional factor which rise a risk of market imbalance and strong price anomalies (see e.g. Stoll, Whaley, 1986; Chow et al., 2003; Vipul, 2005; Debasish, 2010). The direction in which the price changes depends on which group of investors dominates on a spot market. If it is made up of arbitrageurs who finalize cash-and-carry strategies by placing orders to sell stocks on expiration day, the price can fall unusually. The activity of speculators can lead to an increase in stock prices as having long positions in expiring futures they can try to rise their final settlement prices by creating buying pressure on a stock market. The opposite actions of speculators with short positions in derivatives are limited by the number of stocks they hold and can sell. Vipul (2005) suggests that abolition of the restrictions on short selling would weaken price effects. This assumption was confirmed in reference to the Polish market by Suliga and Wójtowicz (2019) who examined the influence of the simultaneous expiration of stock and index futures on the Warsaw Stock Exchange during the period January 2001 – December 2016. They checked that after the introduction of new regulations abolishing most of the restrictions on short selling in May 2015, price effects of futures expirations significantly weakened.

In reference to price effects, the research confirms that the longer the time for final settlement of price formation, the less intense are the anomalies. When this price is calculated with the use of all underlying asset prices from the last trading session before the expiration, price effects either do not occur or are negligible: the volatility of prices can be slightly increased during all trading session but not lead to a significant market imbalance and sharp price changes (see e.g. Bollen, Whaley, 1999; Kan, 2001; Fung, Yung, 2009; Xu, 2014).

If the price of an underlying asset changes unusually on expiration day as an effect of arbitrageurs and speculators activity, such that it does not properly reflect the intrinsic value of an asset, this anomaly should only be temporal and disappear immediately after the expiration (Stoll, Whaley, 1986). The subsequent return of prices to a normal level can be observed as price reversal or price shock effect which are the last two of price effects of futures expiration.

A term price reversal was introduced by Stoll and Whaley (1986). They assume that if an underlying stock price (index level) is upset by futures expiration and unusually increased (or decreased), on the next day the price should change in the opposite direction, coming back to an equilibrium level. Thus, on expiration day and on the next day, rates of return should have opposite signs. Stoll and Whaley (1986, 1987) proposed a few measures of the effect. All of them are based on a comparison between the rate of return's sign on expiration day and the next day. The strength of the anomaly is measured by the magnitude of the rate of return from one of these two days (expiration day or the next day, depending on the measure).

The price reversal effect was not only detected on the USA market by Stoll and Whaley (1986, 1987, 1990, 1991) but also on other markets, e.g. German (Schlag, 1996), Korean (Park, Lim, 2004; Chay, Ryu, 2006; Chay et al., 2013) and Taiwan (Chou et al., 2006; Chuang et al., 2008; Hsieh, 2009; Chow et al., 2013). However, in many studies of this anomaly there have been no grounds for concluding that the direction in which underlying asset prices are changing, changes after the expiration of futures (see e.g. Karolyi, 1996; Stoll, Whaley, 1997; Kan, 2001; Chow et al., 2003; Alkebäck, Hagelin, 2004; Chung, Hseu, 2008; Fung, Yung, 2009; Hsieh, Ma, 2009; Debasish, 2010; Narang, Vij, 2013; Xu, 2014; Mahalwala, 2016; Samineni, 2020).

In research conducted on daily data, the reason for not detecting the anomaly might be their short-term nature. If the price of a stock returns to an equilibrium level at the opening of the first trading session after expiration day, the daily rate of return may not reflect this fact. What is more, Vipul (2005) notes that Stoll and Whaley's price reversal effect is insufficient to express all potential perturbations in the price formation process resulting from expiration of futures. For example, a strong upward (downward) trend in underlying asset price can be temporarily inhibited on expiration day (it does not necessarily have to be reversed). It would be an evident anomaly, but price reversal would not reflect it. Similarly, a weak trend in prices can be passingly strengthened on expiration day by arbitrageurs or speculators and price reversal would not capture this fact. Thereupon Vipul (2005) suggests measuring price anomalies by the price shock effect, which he defines as an unusually large difference between underlying asset returns on expiration day and the next day.

Vipul (2005) analyzed expiration day effects of Nifty index options and futures as well as individual stock options and futures on the Indian market during the period November 2001 – May 2004. He showed that impact of derivative expirations on stock returns was significant. This impact was reflected in price shock measures but was not detected in the research on the price reversal effect. On average, stock prices fell the day before expiration, stayed at this level on expiration day and rose after the expiration. The price reversal effect was not detected due to the lack of a specific pattern of price behavior on expiration day (systematic increases or systematic decreases in prices).

The price shock effect defined by Vipul (2005) was analyzed in the research conducted by Xu (2014), who studied expiration day effects of OMX index options and futures on the Swedish market. However, Xu (2014) slightly modified the measure of price shock proposed by Vipul (2005), as instead of the difference between stock return on expiration day and the next day, she used the absolute value of this difference. Such a modification is crucial if we assume that unusual price changes on expiration days generated by the activity of speculator and arbitrageurs do not have the same direction every time and if we draw our conclusions based on the average value of the measure.

The results of the study conducted by Suliga (2020) can be seen as a confirmation that price shock measures are more effective than price reversal measures in detecting price anomalies occurring after futures expiration. Using intraday data from January 2011 to March 2017, Suliga (2020) checked the existence of the effects in individual stocks quotes on Polish market after the expiration of stock and index futures. She calculated the measures proposed by Stoll and Whaley (1986) and Xu (2014) with the use of hourly stock returns: return from the last hour of trading session on expiration day and return from the first hour of the next trading session. On average, the effect of price shock was visible in the quotations of six out of fourteen shares included in the study in the period under study, while price reversal was only detected in case of three of them.

2.3. Earlier studies on expiration day effects on the WSE

In reference to the Polish equity market, research on expiration day effects was conducted by Morawska (2004, 2007). In recent years, such analyses were also carried out by Suliga (2017, 2020, 2021), Suliga and Wójtowicz (2019) as well as Gurgul and Suliga (2020).

Morawska (2002, 2007) studied expiration day effects in a very initial period of futures market existence on the WSE. She examined the influence of stock and index futures expiration on returns of WIG20 index as well as on trading volume of stocks from the index. In the first piece of research, Morawska (2004) analyzed each of the expiration days between December 2002 and March 2004 separately. She drew attention to arbitrage opportunities on the market and to signs indicating that arbitrage strategies were being finalized in expiration days. She also listed examples of the occurrence of price reversal effect in WIG20 index markings, increase in volatility of index level at the begining of the triple withing hour and large sell orders of all stocks from the index during this time. She concluded that during the period under study, expiration day effects were intensifying.

In the second mentioned piece of research, Morawska (2007) studied anomalies in WIG20 index markings in the period December 2002 – June 2006. On expiration days she detected significant growth in the trading volume of stocks from the index as well as abnormal increase in volatility of intraday returns of the index. The results did not confirm the existence of price reversal effect. Although during the period under study futures on individual stocks and MIDWIG index futures expired on the same days as WIG20 futures, Morawska (2007) only focused on anomalies in WIG20 markings, as futures on WIG20 were the most popular derivatives in those years and their turnover accounted for over 97% of the entire futures' market turnover.

Suliga (2017) only studied the price reversal effect during the period January 2001 – December 2016. She examined the existence of this anomaly in WIG20, mWIG40 as well as individual stocks returns after simultaneous expiration of stock and index derivatives (WIG20 futures and options, mWIG40 futures and futures on individual stocks). Using daily data, she employed three different measures of price reversal. Each of them confirmed that this effect occurs in individual stock returns after derivatives expirations. However, none of the measures support the thesis about the occurrence of price reversal effect in WIG20 or mWIG40 returns.

The same research period was adopted by Suliga and Wójtowicz (2019), who used daily data to examine three expiration day effects of futures, namely: volume effect, volatility effect and price reversal. They employed two different methodologies. The first one was based on a comparison of the measures of effects between expiration days and control days, while the second was an event study. Their results confirmed that the trading volume of stocks from the WIG20 index and the mWIG40 index was abnormally high on expiration days but did not give any presumptions about a significant increase in the volatility of the returns of indexes or about the price reversal effect in index markings. In the case of individual stock, all three anomalies were detected. Additionally, Suliga and Wójtowicz (2019) divided the research period into two parts – before and after the introduction of new short-selling regulations in May 2015. Their results revealed that after the lifting of the substantial restrictions on short selling, expiration day effects lessened.

Gurgul and Suliga (2020) studied expiration day effects on WSE basing the research on high frequency data from the period January 2011 – March 2017. Using event study analysis, they checked that during the period under study on quarterly expiration Fridays trading volume of underling assets of stock and index futures, as well as volatility of their prices was abnormally high. However, an unusual rise in trading volume and volatility was mostly visible during the last hours of continuous trading and at the close. This observation confirmed that investor activity on the stock market increased during a crucial time for the final settlement prices of expiring futures. Dividing the research period into three parts, Gurgul and Suliga (2020) tried to check the impact of two events on the occurrence and strength of expiration day effects. The events were: introduction of new transaction system on April 15, 2013 and changes in short-selling rules on May 31, 2015. The strongest anomalies were detected in the second subperiod, which was interpreted as showing that expiration day effects intensified after the first event but were attenuated by the second one (which is consistent with the results obtained in this area by Suliga and Wójtowicz (2019)).

In the last two articles Suliga (2020, 2021) studied the impact of futures expirations on the marking of individual stocks which are underlying assets of stock futures. Most of them are also in one of the two indexes: WIG20 or mWIG40. As well as Gurgul and Suliga (2020) she used high frequency data and analyzed the anomalies in the same research period, namely January 2011 – March 2017. In the first research (Suliga 2020) all potential expiration day effects were analyzed, namely the increase of trading volume and volatility of stocks, unusual fall/rise in prices, price reversal and price shock. Suliga (2020) compared distributions of appropriate expiration day effects measures between expiration days and control days (the third Fridays of the months in which stock and index futures do not expire). The results revealed that anomalies (abnormal trading volume and abnormal volatility) were visible in the marking of all stocks under study during the time when transactions made on the stock market had a decisive influence on final settlement prices of derivatives. For one company (Asseco Poland), it was found that stock prices had a steady tendency to rise on expiration days during the last hour of continuous trading. Price reversal and price shocks were only detected in the case of the shares of a few companies.

In the second piece of research, Suliga (2021) employed linear models to identify factors which determine the strength of expiration day effects occurring in individual stock markings. The results suggest that the stronger the anomalies, the more positions in futures are opened before expiration. There is also a positive relationship between the strength of anomalies and the number of shares per contact: the larger the contact multiplier, the stronger the expiration day effects. The study also allowed us to check the impact of some events on expiration day effects. It partially confirmed the results obtained earlier by Gurgul and Suliga (2020) as well as by Suliga and Wójtowicz (2019). The estimated coefficients of liner models indicate that the implementation of new short-selling rules only weakened the anomalies occurring during the continuous trading phase but did not have any impact on the strength of the effects at the close. The last conclusion from the research was that the change of the transaction system WARSET to the faster and more efficient UTP system contributed to the weakening of trading volume effects and to the intensification of price reversal and price shock, which could be related to the development of high-frequency trading strategies.

One of the important goals of this article is to check the impact of another event, namely the outbreak of the COVID-19 pandemic on the occurrence and strength of expiration day effects on the Polish stock market.

3. Data and methodology

In this research we examine the existence of four expiration day effects, namely: the unusual increase in volume and in volatility, price reversal and price shock. The data set used in the study included tick-by-tick data from the period January 1, 2018 – December 31, 2020 containing price and trading volume of each transaction and the time of its conclusion (with an accuracy of 1 second). The data was used to calculate intraday trading volume and 5-minute returns of stocks which were underlying assets of stock futures in the research period. As stock and index futures expire quarterly, on the third Friday of March, June, September and December, there were 12 expiration days within the given period. Table 1 presents an alphabetical list of companies included in the research. It contains the full name of each company, an abbreviation of its name as well as information about the number of expiration days included in the research. Most of the analyzed stock futures were launched before 2018, therefore this number is 12. However, some of them were introduced to the market during the research period. In such cases, the number of expiration days assigned to an underlying stock is less than 12.

Table 1

Alphabetical list of companies included in the research

Name of the company and its abbreviation	Number of expiration days	Name of the company and its abbreviation	Number of expiration days
Alior Bank (ALR)	12	Kruk (KRU)	12
Asseco Poland (ACP)	12	LiveChat Software (LVC)	9
Bank Millennium (MIL)	12	LPP (LPP)	12
Bank Polska Kasa Opieki (PEO)	12	Lubelski Węgiel Bogdanka (LWB)	12
Biomed-Lublin (BML)	1	MBank Spółka Akcyjna (MBK)	12
CCC	12	Mercator Medical (MRC)	1
CD Projekt (CDR)	12	Orange Polska (OPL)	12
Ciech (CIE)	12	PGE Polska Grupa Energetyczna (PGE)	12
Cyfrowy Polsat (CPS)	12	PlayWay (PLW)	8
CI Games (CIG)	7	Polimex Mostostal (PXM)	12
Dino Polska (DNP)	12	Polskie Górnictwo Naftowe i Gazownictwo (PGN)	12
Enea (ENA)	12	Polski Koncern Naftowy Orlen (PKN)	12
Giełda Papierów Wartościowych w Warszawie (GPW)	12	Powszechna Kasa Oszczędności Bank Polski (PKO)	12
Grupa Azoty (ATT)	12	Powszechny Zakład Ubezpieczeń (PZU)	12
Grupa Eurocash (EUH)	12	Santander Bank Polska (SPL)	9
Grupa Lotos (LTS)	12	Tauron Polska Energia (TPE)	12
ING Bank Śląski (ING)	12	Ten Square Games	9
Jastrzębska Spółka Węglowa (JSW)	12	X-Trade Brokers (XTB)	1
KGHM Polska Miedź (KGH)	12	11 bit studio (11B)	4

This table contains basic information about companies which stocks are included in the study as underlying assets of stock futures. Full name of each company is presented with its abbreviation and information about the number of expiration days of corresponding stock futures which were included in the study.

To study expiration day effects, Stoll and Whaley (1986, 1987), pioneers of research in this area, defined appropriate measures of the anomalies and compared distributions of these values between expiration days and control days. In the following years this stream of research has been used by many authors (i.a. by Schlag, 1996; Chow et al., 2003; Alkebäck, Hagelin, 2004; Debasish, 2010; Chay et al., 2013; Xu, 2014; Suliga 2020). However, in recent years, various researchers have also proposed other methods of detecting anomalies generated on stock market by futures expiration. In reference to the Polish market, Suliga (2017) was first, who studied the anomalies employing event study methodology to classical expiration day effect measures. This research method was then used by Suliga and Wójtowicz (2019), as well as Gurgul and Suliga (2020) and is also applied in this article.

3.1. Measures of expiration day effects

In terms of earlier studies, expiration day effects mostly occur on the market during the time when spot transactions have a direct impact on final settlement prices of derivatives. On the WSE, the final prices of stock futures are calculated as the closing price of the underlying stock on expiration day. Such a settlement procedure might result in significant anomalies during the last minutes of continuous trading as well as at the close of the trading session on expiration day. However, stock futures expire simultaneously with WIG20 futures, mWIG40 futures and WIG20 options and most of the stocks which are underlying assets of stock futures are also in one of the two indexes: WIG20 or mWIG40. This should be kept in mind as it means that not only expiration of individual stock futures but also expiration of index derivatives may contribute to anomalies in stocks' quotations. On the Polish market, the final settlement prices of index futures and options are calculated as the arithmetic means of all index values from the last four of continuous trading and the value at the close on expiration day (after removing the five highest and five lowest). Thus, anomalies may also occur during the whole last hour of trading session on expiration days.

To identify the relationship between expiration effects and the settlement procedure of stock and index derivatives, the trading session was divided into three parts: before the last hour of continuous trading, in the last hour of continuous trading, at the close and during overtime. Expiration day effect measures were calculated separately in each of these intervals.

As a measure of the abnormal trading volume effect, the natural logarithm of turnover value $\ln(V_t)$ was used. The measure was calculated in three subperiods of a trading day separately: $\ln(V_{t,1})$ denotes log-volume from before the last hour of the continuous trading, $\ln(V_{t,2})$ – from the last hour of continuous trading and $\ln(V_{t,3})$ – at the close and during overtime.

Measures of the effect of increased volatility were also calculated for the three time intervals mentioned above. Before the last hour of continuous trading, as well as in the last hour of the continuous trading phase, the mean absolute deviation of the 5-min returns was employed as an indicator of stock price volatility (designated as $d_{t,1}$ and $d_{t,2}$ respectively), where:

$$d_{t,i} = \frac{1}{n_i}\sum_{k=1}^{n_i}\left|r_{t,k} - \overline{r}\right| \tag{1}$$

In equation (1) $r_{t,k}$ stands for 5-min returns from a given time interval, $\overline{r}$ is their arithmetic mean and n_i is their number.

Volatility at the close was estimated as the absolute value of a return at the close $|R_t^c|$, calculated as:

$$\left|R_t^c\right| = \left|\ln\left(P_{t,close}\right) - \ln\left(P_{t,close-5}\right)\right| \tag{2}$$

were $P_{t,close}$ is a closing price of a stock on day t and $P_{t,close-5}$ is the price of the last transaction concluded on day t in the continuous trading phase.

Previous research on expiration day effects on the Polish stock market have shown that the anomalies mainly occur on expiration days during the time when the final settlement prices of futures are being calculated, that is in the last hour of trading session and at the close. Therefore, to detect potential price reversal or price shock effect, data from this short time interval should be employed. In this paper we follow Suliga (2020) and use for the last to effects measures calculated with the use of one-hour returns:

$$REV_t^1 = \begin{cases} R_{t+1}^{fh} & if \quad R_t^{lh} < 0 \\ -R_{t+1}^{fh} & if \quad R_t^{lh} \geq 0 \end{cases} \tag{3a}$$

$$PS_t^1 = \left|R_{t+1}^{fh} - R_t^{lh}\right| \tag{3b}$$

$R_t^{lh} = \ln(P_{t,close}) - \ln(P_{t,close-60})$ is a logarithmic rate of return from the last hour of trading session on day t. $P_{t,close-60}$ is a price of the last transaction conducted on day t before the beginning of the last hour of continuous phase.
$R_{t+1}^{lh} = \ln(P_{t+1,open+60}) - \ln(P_{t,close})$ is a logarithmic rate of return from the first hour of trading on day $t+1$, where $P_{t+1,open+60}$ is a price of the last transaction concluded in the first hour of continuous trading on this day.

Using the measures described above, Suliga (2020) only detected price anomalies in the case of a few of the 14 stocks under study. To check if it is possible that these anomalies may be very short-lived and can already be observed at the opening of the trading session following the expiration of contracts, two additional measures were used, based on a comparison between the rate of return from the last hour of trading in expiration day R_t^{lh} and the rate of return realized between the close of the trading session on the expiration day and opening on the next trading day R_{t+1}^{co}:

$$REV_t^2 = \begin{cases} R_{t+1}^{co} & if \quad R_t^{lh} < 0 \\ -R_{t+1}^{co} & if \quad R_t^{lh} \geq 0 \end{cases} \tag{4a}$$

$$PS_t^2 = \left| R_{t+1}^{co} - R_t^{lh} \right| \tag{4b}$$

where $R_{t+1}^{co} = \ln(P_{t+1,open}) - \ln(P_{t,close})$

3.2. Event study analysis

As already mentioned, event study methodology was applied to study expiration day effects. The main assumptions and basics of this methodology can be found, among others, in Gurgul (2006) or Kothari and Warner (2006). In the research based on event study, four basic stages can be distinguished:

- defining the event,
- time frame determination (construction of pre-event and event window),
- the choose of the model which is then applied to estimate expected values of the analyzed variable in the event window,
- assessment of the effect of the event with the use of an appropriate test.

The main goal of this research is to assess the impact of derivatives expirations on trading volume, intraday volatility, and the prices of individual stocks which set underlying assets of expiring stock futures. Thus, the event is identified with an expiration of a single series of stock futures. As during the period under study (January 2018 – December 2020) there were 12 expiration days (the third Fridays of March, June, September and December) and more than 30 series of stock futures expired on each of these days, the total number of events is $N = 397$, as it comes from the data in Table 1.

As mentioned in section 2, some earlier studies revealed that anomalies in trading volume and volatility of stocks can not only be observed on the market on expiration days (denoted by $t = 0$) but also within a week before expiration.

However, Suliga and Wójtowicz (2019) detected some distortion in the turnover value of underlying stocks of futures also within two days following expiration day. Given these results, in the study of the first two anomalies, that is abnormal trading volume and abnormal volatility effect, an event window which begins on the fifth day before expiration and ends two days after it ($t = -5, \ldots, 2$) is used. As price reversal and price shocks may only occur after the expiration, in case of these effects event window is a lot shorter and only covers the expiration day and the previous day ($t = -1, 0$).

The estimation window should be as wide as possible but should not contain any confounding event and must not overlap the previous event window. As stocks and index derivatives expire on the WSE quarterly, we define an estimation window in the same way as Suliga and Wójtowicz (2019) as well as Gurgul and Suliga (2020), namely, it covers 45 days directly before the event window ($t = -50, \ldots, -6$). It also enabled us to compare our results with those obtained by the abovementioned authors.

Measures of expiration day effects defined in section 3.1 are treated as variables X_t, used in the event study. Thus:

$$X_t \in \left\{ \ln\left(V_{t,1}\right), \ln\left(V_{t,2}\right), \ln\left(V_{t,3}\right), d_{t,1}, d_{t,2}, \left|R_t^c\right|, REV_t^1, REV_t^2, PS_t^1, PS_t^2 \right\}$$

The event study analysis consists in testing whether values of the variables in the event window differ from their "normal" values. Thus AX_{it} is abnormal variable defined for the i-th event as the difference between X_{it} and its expected value $E(X_{it} \mid \Omega_{-6})$, conditional to the data set from the pre-event window (the set of the data Ω_{-6} available on day ($t = -6$):

$$AX_{it} = X_{it} - E(X_{it} \mid \Omega_{-6}) \tag{5}$$

Approximation of an expected value $E(X_{it} \mid \Omega_{-6})$ is calculated as mean value in the estimation window. Testing of the occurrence of the expiration effect on day $t = t_0$ in the event window is indetified with testing the hypotheses:

$$\begin{aligned} H_0 &: E\left(AX_{it_0}\right) = 0 \\ H_1 &: E\left(AX_{it_0}\right) \neq 0 \end{aligned} \tag{6a}$$

or

$$\begin{aligned} H_0 &: E\left(AX_{it_0}\right) = 0 \\ H_1 &: E\left(AX_{it_0}\right) > 0 \end{aligned} \tag{6a}$$

For this purpose, the Kolari–Pynnönen test is applied (see Kolari and Pynnönen 2011). Two-sided alternative hypothesis is used in the case of abnormal trading volume and the abnormal volatility effect. Earlier studies on these anomalies in relation to various markets revealed that during the period covered by the event window not only significant increase but also that a significant decrease of the analyzed variables is possible (see e.g. Stoll and Whaley 1986, Mahalwala 2016, Suliga and Wójtowicz 2019).

In the analysis of price reversal and price shock effects, a one-sided test with hypotheses (6b) is applied. This time, the aim of the test is to check if the price reversal effect is stronger after futures expiration than after other days, and if the changes in underlying stock prices are unusually large (which would mean that price shock occurs after expiration). In both cases, therefore, the interest is to check the occurrence of significantly positive abnormal values of the analyzed variables.

The construction of the test statistic and the procedure used to test the expiration effects with the Kolari-Pynnönen test will be described below, assuming an eight-day event window ($t = -5, \ldots, 2$), i.e. the window applied to study volume effect and volatility effect.

After calculating the values of abnormal variables AX_{it} for each event i and each day t in the pre-event window and the event window, these values are divided by their standard deviation from the pre-event window $S(AX_i)$. This brings to standardized abnormal variables SAX_{it} defined by the formula:

$$SAX_{it} = \frac{AX_{it}}{S(AX_i)} \tag{7}$$

where

$$S(AX_i) = \sqrt{\frac{1}{44}\sum_{t=-50}^{-6}\left(AX_{it} - \overline{AX_i}\right)^2} \tag{8}$$

is the standard deviation of forecast errors in the applied constant mean model and:

$$\overline{AX_i} = \frac{1}{45}\sum_{t=-50}^{-6} AX_{it} \tag{9}$$

To take account of the possibility of an event-induced increase in volatility, standardized abnormal variables SAX_{it} in the event window are divided by their cross-sectional standar deviation and thus adjusted standardized abnormal returns SAX'_{it} are obtained:

$$SAX'_{it} = \begin{cases} SAX_{it} & t = -50,\dots,-6 \\ SAX_{it} / S(SAX_t) & t = t_0 \end{cases} \tag{10}$$

where $t_0 \in \{-5, \dots, 2\}$ and $S(SAX_t)$ is a cross-sectional standard deviation of variables SAX_{it} on day t, given by the formula:

$$S(SAX_t) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(SAX_{it} - \overline{SAX_t}\right)^2} \tag{11}$$

and N is the numer of events. If the null hypothesis of no expiration effect on day t_0 is true, SAX'_{it_0} are zero mean and unit variance random variables. It should be mentioned that the decision on which days t_0 in the event window variables SAX'_{it} should be adjusted, is made after the anasysis of appropriate plots of standard deviation (e.g. in case of volume effect the correction of volatility is made only for expiration day).

After calculation of SAX'_{it} the test is carried out for each day ($t_0 \in \{-5, \dots, 2\}$ separately, with the use of standardized ranks defined as:

$$U_{it} = \frac{rank(SAX'_{it})}{47} - \frac{1}{2} \tag{12}$$

where $t_0 \in \Omega\{-50, \dots, -6, t_0\}$, $i = 1, \dots, N$ and $rank(SAX'_{it})$ is a rank of SAX'_{it} within the vector consisting of SAX'_{it_0} and adjusted standardized abnormal returns from the pre-event window. The null hypothesis of no event effect on day t_0 means that $E(U_{it_0} = 0)$. The generalized rank test constructed by Kolari and Pynnönen (2011) to test this hypothesis has test statistic τ_{grank} defined as:

$$\tau_{grank} = Z\sqrt{\frac{T-2}{T-1-Z^2}} \tag{13}$$

where:

$$Z = \frac{\bar{U}_{t_0}}{S_{\bar{U}}}, \quad \bar{U}_{t_0} = \frac{1}{N}\sum_{i=1}^{N} U_{it_0}, \quad S_{\bar{U}} = \sqrt{\frac{1}{46}\sum_{t\in\Omega}\bar{U}_t^2} \tag{14}$$

If the null hypothesis is true, the distribution of the τ_{grank} test statistic converges to t-student distribution with 44 degrees of freedom, when the sample size N increases to infinity.

3.3. Research hypotheses

Based on the results obtained in previous research on expiration day effects regarding various foreign markets as well as the Polish market, the main research hypotheses were formulated.

First of all, the studies show that the anomalies are highly probable when the final settlement price of the derivative is equal to a single price of an underlying asset (see e.g. Park, Lim, 2004; Bhaumik, Bose, 2007; Chuang et al., 2008; Narang, Vij, 2013). Considering the method of calculating the final settlement price of stock futures on the WSE, we suspect that despite the development of the market, expiration day effects occurred on expiration days during the whole period under study. Thus, we formulate the first hypothesis:

Conjecture 1: In the period 2018–2020 expiration of stock and index derivatives was accompanied by price and volume anomalies on the stock market.

We also phrase more detailed conjectures relating to each effect separately. Essentially, abnormally high trading volume occurring on the spot market on expiration days was detected on every market for which research was carried out. Suliga (2020, 2021) checked that in the period 2012–2017 this anomaly occurred on the WSE mostly during the time when the final settlement prices of stock and index futures were being calculated. We suppose that this has not changed in recent years, so the next two hypotheses are:

Conjecture 2: On expiration days, trading volume of underlying stocks increases significantly above expectations.

Conjecture 3: An unusual increase in trading volume on expiration days occurs during the last hour of continuous trading as well as at the close and during overtime.

As in the case of the effect of increased trading volume, research conducted on the basis of data from the period 2012–2017 confirmed the occurrence of the effect of increased volatility on the spot market of the WSE during the time of the formation of the final settlement prices of futures (see Gurgul and Suliga 2020, Suliga 2020). We suppose that the effect has not disappeared in the following years and formulate the following hypothesis:

Conjecture 4: Intraday volatility of stock prices is abnormally high on expiration days. The increase in volatility takes place during the last hour of continuous trading and at the close.

In reference to the price reversal effect, Suliga (2017) showed that this anomaly occurred on the WSE after futures expirations and that was stronger than after Fridays on which derivatives did not expire. However, it should be noted that the study conducted by Suliga (2017) covered a very long period (2001–2016) and all expiration days from this time were studied together. In another research (Suliga, 2020) she analyzed price reversal for each of the stock under study separately and found that on average, this anomaly was only unusually strong after futures expirations in case of 3 out of 14 companies. Research relating to other markets also indicates that price reversal is a phenomenon that occurs on the market not only after the expiration days of futures. Moreover, even if it occurs after expiration, it is not higher than after other days (see e.g. Alkebäck, Hagelin 2004; Hsieh, Ma, 2009; Debasish, 2010). Thus, we predict the following:

Conjecture 5: Price reversal is not a phenomenon that constantly accompanies the expiration of stock and index futures on the WSE and even if it occurs after the expiration, it is not unusually high.

As price shock is an effect analyzed in few studies (Vipul, 2005; Xu, 2014; Suliga, 2020), which do not give a clear answer to the question about its occurrence, it is not easy to formulate an unambiguous hypothesis regarding the occurrence of this anomaly on the WSE. In reference to the Polish market, the study on the price shock effect was carried out, to our knowledge, only by Suliga (2020). Using the measure PS_t^1, given by the equation (3b), Suliga (2020) detected this anomaly only in case of six out of 14 companies she considered. However, the examination carried out by Suliga (2020) for all companies together confirmed the occurrence of price shock effect on the WSE during the period under study (January 2012 – March 2017). In this research, in addition to the measure PS_t^1 we employ the measure PS_t^2 defined in equation (4b) to check if it is possible that price shock is a very short-lived effect and is already visible at the opening of the first trading session after futures expiration. We formulated the following preliminary hypothesis:

Conjecture 6: The price shock effect is visible in stock prices after stock and index futures expirations. On average, differences between the return from the last hour of the trading session on expiration day and the return from the beginning of the next trading session are higher than normal.

The final hypothesis concerns the impact of the outbreak of the COVID-19 pandemic on the occurrence and strength of expiration day effects on the WSE. The negative impact of this event on the Polish stock market was analyzed, among other markets, and confirmed in several studies (e.g. Ashraf, 2020; Czech et al., 2020a). In detail, functioning of the spot and futures markets of the WSE in the initial phase of the pandemic (March – May 2020) was studied by Czech et al. 2020b).[1]

Since futures contracts are used as tools to hedge open positions on a spot market and the pandemic has led to an increase in the risk of investing in shares, it could be suspected that the futures market attracted more hedgers in the initial phase of the pandemic, or at least encouraged existing hedgers to increase their positions on the market (cf. Corbet et al., 2020). The latter assumption seems to be consistent with the results obtained by Czech et al. (2020b) who checked that the futures market did not attract many new investors in March 2020 but was characterized by an increase in investor activity measured by the size of transactions. Thus, it can be assumed that the importance of the actions taken on expiration days by arbitrageurs and speculators during the COVID-19 pandemic, whose activity on spot market is perceived as a main source of anomalies, weakened considerably. What is more, the underlying instruments of futures only include the largest and most liquid companies from the main market. Meanwhile, Czech et al. (2020b) detected a retreat of investors in the initial phase of the pandemic from the main market to the New Connect market, i.e. to investment in shares of smaller companies. In May 2020, capitalization of the main market of GPW was 26.5% lower than in the same month of the previous year, while the capitalization of the New Connect market increased by 70.9% during the same period (see Czech et al., 2020b). This also suggests potential outflow from the main market of investors responsible for the occurrence of the anomalies. The above deliberations allow us to formulate the last hypothesis:

Conjecture 7: The outbreak of the pandemic COVID-19 has contributed to a significant weakening of expiration day effects on the WSE.

4. Empirical results

As described in detail in the previous section, the research on expiration day effects was conducted with the use of event study analysis. In this section, the results obtained for individual effects and for the three considered research periods are presented.

[1] We do not present a detailed analysis of the Polish spot and futures markets in the initial phase of the COVID-19 pandemic in this paper, so as not to unnecessarily lengthen an already long article. We only briefly present those aspects of the functioning of the market that, in our opinion, may shed light on the problem we are interested in: what impact the outbreak of the pandemic had on the strength of the expiration day effects.

4.1. Event study analysis for the period 2018–2020

At the beginning, the occurrence of the effects was checked for the period January 2018 – December 2020. Table 2 presents the results of the event study of the abnormal trading volume effect. As a measure of the activity of investors, in each of the tree specified subperiods of a trading session, the natural logarithm of turnover value was used. Values of the measure in the event window (starting 5 days before the expiration day and ending two days after it) were compared with their "normal" values, defined as an average value from the pre-event window (consisting of 45 days directly before the event window). In Table 2, the mean abnormal values of the log-turnover for each day in the event window are presented with corresponding values of the Kolari–Pynnönen test statistic τ_{grank}.

Table 2

The effect of abnormal trading volume

t	From opening to the last hour of continuous trading		In the last hour of continuous trading		At the close and in overtime	
	$\overline{A\ln(V_{t,1})}$	τ_{grank}	$\overline{A\ln(V_{t,2})}$	τ_{grank}	$\overline{A\ln(V_{t,3})}$	τ_{grank}
−5	0.150	1.251	0.036	0.295	0.051	0.553
−4	0.022	0.157	0.077	0.722	0.015	0.222
−3	0.142	1.138	0.084	0.536	0.195	1.205
−2	0.136	1.203	0.181*	1.982	0.140	0.789
−1	0.077	0.481	0.039	0.201	0.114	0.791
0	**0.106**	**0.921**	**1.123*****	**7.999**	**2.440*****	**7.500**
1	0.165	0.431	−0.071	−1.162	−0.066	−0.545
2	0.013	−0.265	−0.128	−1.642	−0.069	−0.656

This table presents the results of the event study analysis employed to detect abnormal trading volume effect. Log-volume of stocks, which are underlying assets of stock futures, was used to check the occurrence of the effect during three subperiods of a trading session: from opening to the last hour of continuous trading, during the last hour of continuous trading, at the close and during overtime. For each day in the event window (starting from the 5th day before expiration and ending on the 2nd day after it) and for each of the three subperiods, average abnormal trading volumes are presented with corresponding statistic values.
* and *** denote significance at the 10% and 1% levels, respectively.

The left-most part of the table contains average abnormal values of log-volume from before the last hour of continuous trading. They are all positive, but test statistic values suggest that none of the corresponding expected values of abnormal variable $A\ln(V_{t,1})$ differs significantly from zero. It means that during this part of a trading session activity of investors is not abnormally high on any of the days in the event window.

Results obtained in reference to the last hour of continuous trading are presented in the middle part of the table while the right part contains results for the closing phase and overtime. In these two parts of the table, averages obtained for $t = 0$ are positive. The test confirms the occurrence of a strong volume effect on the expiration day during the triple witching hour, indicating that expected values of the variable $A\ln(V_{0,2})$ are significantly positive (at a 1% level). Trading in shares which are underlying assets of stock futures, and which pose parts of the underlying indices of index futures, increases unusually during the time when final settlement prices of expiring derivatives are being shaped. This observation is fully consistent with the analogous results obtained by Gurgul and Suliga (2020) as well as by Suliga (2020) for the period 2012–2017. However, Gurgul and Suliga (2020) (as well as Suliga and Wójtowicz (2019) who performed a study in reference to a similar period but using daily data), detected an unusual increase in trading volume of stocks also on day $t = -1$ what indicated abnormal activity occurring on the stock market of the WSE already one day before expiration. In our research, only one significantly positive expected value of abnormal measure of the volume effect was detected before the expiration day – for day $t = -2$ in the last hour of continuous trading. However, it might be considered significantly different from zero only if a 10% significance level is assumed. All other averages presented in Table 2 which are related to days before expiration, are also positive but corresponding expected values of abnormal variables are found as insignificantly different from zero. This suggests that during the period under study, there could be still some increase in activity of investors on the spot market before expiration days connected with futures expiration, but it was certainly weaker than in previous years. Such an increase in trading volume on days directly before expiration can be a symptom of the early unwinding of arbitrage positions (Vipul, 2005; Debasish, 2010; Agarwalla, Pandey, 2013). Our results suggest, however, that in the process of time and with the development of the market, the occurrence of the volume effect was shortened and narrowed down mostly to the time interval in which trading on stocks has a direct impact on the final settlement prices of expiring futures. This observation fully confirms Conjectures 2 and 3.

In Table 3, the results of the analysis of the volatility effect are presented. Measures of this anomaly related to the three subperiods of a trading session were defined by equations (1) and (2). Test statistic values suggest that all the expected values of abnormal variables related to the days before the expiration ($t < 0$) are insignificantly different from zero. On expiration day ($t = 0$) significantly positive (at a 1% level) expected values were detected for abnormal measures of volatility in the last hour of continuous trading ($Ad_{t,2}$) as well as at the close ($A|R_t^d|$). Thus, similarly to the volume effect, the effect of abnormally high intraday volatility of stock prices does not occur before the expiration, while on expiration day it only appears during the time of the formation of final settlement prices of futures. This observation is a confirmation of Conjecture 4.

Test statistic value related to the average abnormal mean deviation of 5-min returns obtained one day after the expiration (t = 1) before the last hour of continuous trading indicates that expected value of corresponding variable $Ad_{1,1}$ is positive and significantly different from zero (at a 5% level). It can be a preliminary signal that prices temporarily distorted on expiration day return to their normal levels at the beginning of the first trading session after expiration, translating into an increase in volatility. As such a return can occur relatively quickly (Suliga, 2020) thus, except for this one value $\left(\overline{Ad_{1,1}}\right)$, in Table 3 there are no other significantly different form zero values related to the days after the expiration. This interpretation will be verified by the results of price reversal and price shock effects.

Table 3

The effect of abnormal volatility of 5-min returns

t	From opening to the last hour of continuous trading		In the last hour of continuous trading		At the close	
	$\overline{Ad_{t,1}}$ [%]	τ_{grank}	$\overline{Ad_{t,2}}$ [%]	τ_{grank}	$\overline{A\|R_t^d\|}$	τ_{grank}
−5	0.043	1.223	0.014	0.784	0.051	1.210
−4	0.034	0.446	0.031	0.359	−0.021	−1.241
−3	0.028	0.443	0.010	−0.759	−0.001	−0.292
−2	0.031	0.356	0.012	−0.080	−0.022	−1.099
−1	0.017	0.221	0.003	−0.296	−0.030	−0.796
0	**0.018**	**0.736**	**0.051*****	**3.636**	**0.384*****	**8.518**
1	0.038**	2.434	0.008	0.408	−0.005	0.051
2	0.011	0.683	0.008	0.841	0.008	0.055

This table presents the results of the event study analysis employed to detect abnormal volatility effect. Mean deviation of 5-min returns from before the last hour of continuous trading, mean deviation of 5-min returns during the last hour of continuous trading, as well as absolute values of returns at the close were used to check the occurrence of the effect during three subperiods of trading session. For each day in the event window (starting from the 5th day before expiration and ending on the 2nd day after it) average abnormal values of above-mentioned measures are presented with corresponding statistic values.

** and *** denote significance at the 5%, and 1% levels, respectively.

Table 4 presents the results of the event study analysis employed to detect price reversal and the price shock effect. This time event window contains only two days (t = –1,0). Due to the nature of the anomalies, the expected values of variables obtained for $t = 0$ are of particular interest. In the left part of Table 4 abnormal absolute values of the measures REV_t^1 and REV_t^2 of price reversal effect are presented with corresponding τ_{grank} test statistic values. On expiration day, both averages are very close to zero and test statistic values indicate that the expected values of considered variables are not significantly different from zero. Thus, the obtained results do not confirm the occurrence of a price reversal effect on the WSE during the period under study. However, the measures were defined with the use of intraday returns, namely: return from the last hour of trading session on the expiration day and return from the first hour or from the opening of the next trading session respectively (see eq. (3) and (4)). Thus, they allow to check if the anomalies only occur at the beginning of the first trading session after futures expiration.

Table 4

The effects of price reversal and price shock

	Price reversal measures				Price shock measures			
t	$\overline{AREV_t^1}$ [%]	τ_{grank}	$\overline{AREV_t^2}$ [%]	τ_{grank}	$\overline{APS_t^1}$ [%]	τ_{grank}	$\overline{APS_t^2}$ [%]	τ_{grank}
−1	−0.239	−0.296	−0.205	−0.899	−0.035	−0.174	−0.024	−0.655
0	0.075	0.228	−0.005	0.982	0.584***	4.083	0.502***	4.567

This table presents the results of the event study analysis employed to detect price reversal as well as price shock effect. For expiration day (t = 0) and for the previous day (t = –1)average abnormal values of measures given by equations (3a), (3b), (4a) and (4b) are presented with corresponding statistic values. *** denote significance at the 1% level.

As already mentioned, the measure REV_t^1 was employed by Suliga (2020) who checked the occurrence of price reversal for individual stocks. However, Suliga (2020) used other methods in her research. She compared the distribution of the values of the measure between expiration and control days (defined as the third Friday of the months in which derivatives do not expire). Only in the case of 3 out of 14 companies under study were significant differences between the distributions found, suggesting that on average, share prices of these three companies reverse after derivative expirations and the reversal is significantly stronger than after other days. An analogous comparison made by Suliga (2020) for all the stocks together also indicated that in general there are grounds for assuming that after futures expiration slight price reversal on average take place, while after control days the direction of price change at the beginning of the next session is the same as at the end of the previous session. In Table 4, $\overline{AREV_t^1}$ calculated for $t = 0$ is slightly positive which also means that the value of the measure on expiration day is on average slightly higher than normally. However, the test statistic values indicate that this difference is insignificant. In conclusion, it should be stated that results obtained for price reversal measures support Conjecture 5.

Results presented in the right part of Table 4 show the weakness highlighted by Vipul (2005) of the measures of price reversal as tools for detecting anomalies in prices. While these measures do not confirm the occurrence of any unusual price changes, averages received on day $t = 0$ for both measures of price shock are positive and test statistic indicates that corresponding expected values are significantly higher than zero (at a 1% level). This confirms the occurrence of price shock effect on the WSE during the period under study as it was assumed in Conjecture 6: changes in prices between the triple witching hour and the beginning of the first trading session after derivatives expiration are abnormally high. This observation is probably the result of unusually large price changes occurring not only at the end of the trading session on the expiration day but also at the beginning of the next session. Indeed, at an earlier stage of this study (see Table 3) a significant increase in volatility was detected on day $t = 1$ in the initial hours of a trading session.[2]

4.2. Results of the analysis conducted in subperiods: before and in the initial phase of COVID-19 pandemic

To check if the outbreak of the COVID-19 pandemic has had an impact on the occurrence and strength of expiration day effects on the WSE, an event study analysis was conducted in two subperiods separately: January 2018 – December 2019 (Subperiod 1, before the pandemic, 254 events) and January 2020 – December 2020 (Subperiod 2, 143 events). As the first expiration day in 2020 was on the third Friday of March, the latter subperiod can be also described as March 2020 – December 2020 and identified with the initial phase of the pandemic.

[2] Price shock measures were calculated with the use of stock returns realized between expiration on the Friday and the subsequent Monday. Thus, one might surmise that the detected price shock is a sign of the weekday effect rather than expiration day effect. The use of eight-day event window (as was employed to the two previously considered effects), containing day $t = -5$, which is also a Friday, may dispel such a doubt, if the expected values of the measures assigned to this day would be insignificantly different from zero. We checked this using eight-day event window in calculations and anomalies were not detected five days before expiration. We decided, however, to present two-days event window in the article to shorten the notation and focus only on the days when price effects are expected to occur.

The results for abnormal trading volume effect are reported in Table 5. A comparison between Panel A (containing results for subperiod 1) and Panel B (results for subperiod 2) confirms the existence of the anomaly both before and at the beginning of the pandemic.

Table 5

The effect of abnormal trading volume in subperiods

		From opening to the last hour of continuous trading		In the last hour of continuous trading		At the close and in overtime	
	t	$\overline{A\ln(V_{t,1})}$	τ_{grank}	$\overline{A\ln(V_{t,2})}$	τ_{grank}	$\overline{A\ln(V_{t,3})}$	τ_{grank}
Panel A: Subperiod 1	−5	0.024	0.042	−0.052	−0.587	−0.065	−0.017
	−4	−0.066	−0.696	0.054	0.408	−0.002	0.124
	−3	0.076	0.575	0.039	−0.006	0.207	1.243
	−2	0.099	0.982	0.220**	2.078	0.177	1.035
	−1	0.038	0.089	−0.040	−0.807	0.131	0.715
	0	**0.102**	**1.225**	**1.266*****	**8.338**	**2.494*****	**6.575**
	1	0.154	−0.141	−0.106	−1.602	−0.045	−0.637
	2	−0.005	−0.412	−0.124	−1.438	0.059	0.024
Panel B: Subperiod 2	−5	0.375	1.888	0.192	0.981	0.258	1.038
	−4	0.177	0.884	0.119	0.641	0.045	0.244
	−3	0.258	1.220	0.164	0.769	0.173	0.569
	−2	0.202	0.947	0.112	0.846	0.075	0.081
	−1	0.148	0.657	0.180	1.054	0.083	0.505
	0	**0.112**	**0.330**	**0.870*****	**3.683**	**2.344*****	**5.179**
	1	0.184	0.790	−0.008	−0.129	−0.104	−0.160
	2	0.045	−0.030	−0.134	−0.969	−0.295	−1.237

This table presents results of event study analysis employed to detect abnormal trading volume effect in two subperiods: January 2, 2018, through December 30, 2019 (Subperiod 1, results presented in Panel A, 254 events) and January 2, 2020, through December 30, 2020 (Subperiod 2, results presented in Panel B, 143 events). For both subperiods the analysis was conducted separately in each of the three session intervals: before the last hour of continuous trading phase, in the last hour of continuous trading, at the close and during overtime. For each day in the event window (starting from the 5th day before expiration and ending on the 2nd day after it), average abnormal log-turnover values are presented with corresponding statistic values.

** and *** denote significance at 5% and 1% levels, respectively.

Test statistic values related to averages calculated for expiration days ($t = 0$) indicate that the activity of investors only increase unusually in the last hour of continuous trading as well as at the close and during overtime. The averages are positive and values of τ_{grank} statistic suggest that expected values of abnormal measures are significantly different from zero (at a 1% level). This observation is true for both subperiods and confirms the direct relationship between the time of the occurrence of the anomaly and the procedure for calculating the final settlement prices of futures: trading on underlying stocks is abnormally high during the time when final settlement prices of corresponding derivatives are shaped.

In Panel A, the average abnormal value obtained in reference to the last hour of continuous trading on day $t = -2$ suggest that before the pandemic, trading volume increased abnormally two days before expiration, in the final part of the continuous trading phase. An analogous result was reported in Table 2, when the existence of the volume effect was checked for the whole period under study. We interpreted this as a potential sign of the early increase of investor activity, connected with expiration of futures. However, comparing this observation with results obtained by Gurgul and Suliga (2020) as well as with Suliga and Wójtowicz (2019) we concluded that in the years covered by their research (2012–2017), the volume effect occurred over a longer period (they also discovered clear signals of this anomaly one day before expiration) while in the following years, the time of its occurrence has been shortened. The results in Table 5 additionally support the thesis. In 2020, the volume effect only appeared in the triple witching hour. Has the strength of the anomaly decreased after the outbreak of the pandemic? Comparison of the appropriate abnormal average values ($\overline{A\ln\left(V_{t,1}\right)}$ and $\overline{A\ln\left(V_{t,2}\right)}$ for $t = 0$) between Panel A and B may suggest this, as the values related to the latter subperiod are smaller. However, the final answer to this question requires to conduct an appropriate test, what will be done at a later stage of the research.

The results of event study analysis performed for volatility measures in subperiods are presented in Table 6. On the grounds of the data from Panel A, an abnormal volatility effect can be confirmed in the first subperiod only on day $t = 0$ and during the time when final derivatives' prices are being calculated. Averages of the measures obtained on expiration day in the last hour of continuous trading as well as at the close are positive and the corresponding values of τ_{grank} test statistic indicate that expected values of the abnormal variables are significantly positive (at a 1% level). Beyond this short extract of a trading session on expiration day, the volatility effect was not detected in any other part of the event window. In particular, there is no basis for concluding that an unusual increase in the intraday volatility of stock prices occurred at the beginning of the first trading session after the expiration before the pandemic. Such an anomaly was reported on the grounds of the data from Table 3 which contains results of the event study analysis performed for the entire period 2018–2020. However, the outcomes presented in Panel B of Table 6 indicate that this anomaly occurred during the initial phase of the COVID-19 pandemic. In this panel, average abnormal value of the measure $d_{t,1}$ is positive on $t = 1$ and the Kolari–Pynnönen test suggests that expected value of the abnormal variable is significantly positive (at a 5% level). This means that the conclusions which were drawn with regard to the analogous observation from Table 3 only remain valid for the latter of the two considered subperiods. Interestingly, an abnormal volatility effect was only detected on expiration days in 2020 at the close, but its existence cannot be confirmed in the last hour of continuous trading. Mean abnormal value of the measure $|R_0^d|$, although smaller than in the first subperiod, is still significantly positive (at a 1% level) whereas expected value of $Ad_{0,2}$ does not differ significantly from zero. How can such a reduction in anomaly time be explained?

The main source of the expiration day effects is the activity of arbitrageurs and speculators who have open position in expiring futures. Such an activity is undertaken during the time when their transactions on a spot market influence the final prices of expiring derivatives. Thus, anomalies occurring on the WSE in the last hour of continuous trading on expiration day should mainly be identified with the activity of investors who have open positions in index derivatives while effects at the close of the trading session – with the activity of holders of stock futures. Efforts of speculators who try to control final settlement prices of futures, or even deliberately manipulate their prices, may be successful inasmuch as they constitute a significant group operating on the spot market at a given time. However, if the market is dominated by investors using contracts in hedging strategies, the occurrence of significant price anomalies is much less likely. Their activity compensates for the activity of speculators and limits the possibility of rapid changes in share prices (and ipso facto in prices of expiring derivatives).

As highlighted in subsection 3.1, market risk increased significantly with the outbreak of the pandemic and the prices of the stocks of many companies included in the most important indices listed on the WSE declined sharply. At the same time, there was a clear increase in the turnover of the futures market, which can be read as a signal that the crisis caused by the pandemic prompted investors to look for tools to hedge against the increasing risk of investing in shares. In the period March–December 2020, the trading volume of index futures was approximately 65% higher than in the corresponding period in 2019 (see Czech et al., 2020b).

This sharp increase in turnover should be identified (if not in its entirety, then at least in a major part) with an increase in hedging positions on the futures market, and thus with a decrease in the importance (impact on prices) of actions undertaken by arbitrageurs and speculators.

How to explain the occurrence of abnormal volatility effect at the close on expiration days in 2020 in this context? A comparison of relevant averages from Panel A and B in Table 6 suggests that the effect was probably weaker during the pandemic than before it (it will be tested later in the study), but it was still visible. The difference in the procedure for calculating final settlement prices of stock and index futures seems to be crucial. Such a single value of stock is much easier to influence than the average value of the index. Thus, thoughtful transactions made by speculators on the shares of a particular company at the very end of the trading session on expiration day (for example, placing appropriate closing orders) may cause an unusually large change in this share's closing price, reflected in increased volatility at the close. This is possible even during a crisis, when many investors use derivatives as hedging tools.

The results presented in Panel B of Table 6 suggest, however, that such disturbances in the level of stock prices are only temporary – the increase in volatility detected at the beginning of the first trading session after expiration might be a sign of a fast return of prices to a "normal" level, what will be further investigated by the measures of the price reversal and price shock effects.

Table 6

The effect of abnormal volatility of 5-min returns in subperiods

		From opening to the last hour of continuous trading		In the last hour of continuous trading		At the close	
	t	$\overline{Ad_{t,1}}$ [%]	τ_{grank}	$\overline{Ad_{t,2}}$ [%]	τ_{grank}	$\overline{A\,\lvert R_t^d \rvert}$	τ_{grank}
Panel A: Subperiod 1	−5	0.006	0.484	0.000	0.234	0.027	0.716
	−4	−0.001	−0.316	−0.007	−0.420	−0.038	−1.438
	−3	−0.003	−0.027	−0.012	−1.213	−0.050	−1.459
	−2	0.001	0.064	0.000	−0.299	−0.007	−0.685
	−1	−0.006	−0.774	−0.014	−1.115	−0.033	−0.695
	0	**−0.001**	**0.034**	**0.059*****	**3.586**	**0.437*****	**7.546**
	1	0.013	1.356	−0.007	−0.329	−0.037	−0.694
	2	0.003	0.331	0.011	0.751	0.030	0.621

Table 6 cont.

		From opening to the last hour of continuous trading		In the last hour of continuous trading		At the close	
	t	$\overline{Ad_{t,1}}$ [%]	τ_{grank}	$\overline{Ad_{t,2}}$ [%]	τ_{grank}	$\overline{A\mid R_t^d \mid}$	τ_{grank}
Panel B: Subperiod 2	−5	0.108	1.481	0.039	0.960	0.095	1.192
	−4	0.097	1.195	0.098	1.094	0.009	−0.234
	−3	0.083	0.805	0.048	0.277	0.085	1.526
	−2	0.083	0.532	0.033	0.240	−0.048	−1.036
	−1	0.059	1.406	0.034	0.897	−0.025	−0.467
	0	**0.051**	**1.045**	**0.036**	**1.304**	**0.288***	**4.617**
	1	0.080**	2.432	0.033	1.053	0.052	1.071
	2	0.024	0.748	0.004	0.424	−0.031	−0.772

This table presents results of event study analysis employed to detect abnormal volatility effect in two subperiods: January 2, 2018, through December 30, 2019 (Subperiod 1, results presented in Panel A, 254 events) and January 2, 2020, through December 30, 2020 (Subperiod 2, results presented in Panel B, 143 events). In both subperiods the measures of intraday volatility of stock prices are: mean deviation of 5-min returns from before the last hour of continuous trading, mean deviation of 5-min returns during the last hour of continuous trading, as well as absolute values of returns at the close. For each day in the event window (starting from the 5th day before expiration and ending on the 2nd day after it) average abnormal values of these measures are presented with corresponding statistic values. ** and *** denote significance at the 5% and 1% levels, respectively.

Table 7 presents the results of the analysis of the price reversal and price shock effects in subperiods. In Panel A, average abnormal values of both measures of price reversal are positive on expiration days. This means that, on average and before the COVID-19 pandemic, after futures expiration the direction of the changes in stock prices was opposite to the direction at the end of expiration day. However, the Kolari–Pynnönen test statistic indicates that the expected value of the abnormal variable $AREV_t^1$ does not differ significantly from zero while the expected value of the second variable $AREV_t^2$ can be considered as significantly positive on day $t = 0$ only if 0,1 significance level is assumed. These results do not give a clear confirmation of the occurrence of price reversal effect in the first subperiod. However, the results presented in the right part of panel A indicate the occurrence of a significant price shock effect. The averages of both price shock measures are positive on day $t = 0$ and the test confirms that expected values of abnormal measures APS_t^1 and APS_t^2 are positive (at a 1% level). It means that differences between the return from the last hour of trading session on expiration day and appropriate return from the beginning of the next trading session (the return at the opening or the return from the first hour of continuous trading) are unusually high compared to the average of analogous differences observed in pre-event widow.

The results in Panel B, relating to the initial phase of the pandemic, do not differ widely from those discussed above. This time, however, there are no indications of the price reversal effect. The averages of abnormal measures are both negative – not only on expiration day but also on day $t = -1$. It means that, on average, on these days stock prices at the beginning of trading session tend to continue the trend from the last hour of a previous session. As in the Kolari–Pynnönen test, a one-sided alternative hypothesis was employed (see eq. (6b)) the test obviously suggests that none of the expected values of abnormal variables of price reversal are significantly higher than zero. As in the first subperiod, price shock effect is detected on expiration day by both measures. Based on the test statistic values it can be concluded that expected values of abnormal variables APS_t^1 and APS_t^2 are significantly positive on day $t = 0$ while they are not higher than zero one day before the expiration. Average values of the measures are higher than in Panel A which may suggest the intensification of price shocks in the COVID-19 pandemic. However, it relates to the increase in volatility of prices which is a typical phenomenon during a crisis. Thus, to be done properly, the comparison of the strength of the anomaly before and during the pandemic should be carried out with the use of standardized, abnormal values of the measures, given by equation (7).

Comparing the results described above with the results from Table 4, related to the study performed for the entire period 2018–2020, it can be concluded that they are consistent. In particular, the data from Table 7 confirm that price shock effect, detected at an earlier stage of the research, occurred both before and in the initial phase of the pandemic. The division of the research period into two parts did not provide valuable, additional information on the occurrence of price reversal effect, while giving some indications of this anomaly before the outbreak of the pandemic. It is possible that it only occurred in the prices of some of the stocks under study (as Suliga, 2020) showed in the study related to the period 2012–2017) and that research conducted to each of the companies separately would give more information on this point.

To sum up the analysis of the expiration day effects conducted in the subperiods, it was confirmed that expiration day effects occurred on the WSE both before and in the initial phase of COVID-19 pandemic. On expiration days, during the last hour of continuous trading as well as at the close and during overtime activity of investors on a spot market intensified, as reflected in the increased trading volume of underlying stocks. This statement is true for both researched subperiods.

Moreover, the results suggest that before the pandemic trading volume increased abnormally two days before expiration (in the last hour of continuous trading) but in 2020 the volume effect only occurred during the triple witching hour.

Table 7

The effects of price reversal and price shock in subperiods

	Price reversal measures				Price shock measures			
t	$\overline{AREV_t^1}$ [%]	τ_{grank}	$\overline{AREV_t^2}$ [%]	τ_{grank}	$\overline{APS_t^1}$ [%]	τ_{grank}	$\overline{APS_t^2}$ [%]	τ_{grank}
Panel A: Subperiod 1								
−1	−0.075	−0.230	0.026	0.220	−0.167	−0.857	−0.147	−1.392
0	0.205	1.182	0.108*	1.592	0.466***	3.621	0.394***	3.649
Panel B: Subperiod 2								
−1	−0.529	−0.207	−0.614	−1.606	0.199	0.536	0.195	0.484
0	−0.155	−0.905	−0.205	−0.742	0.793**	2.390	0.693***	2.616

This table presents results of event study analysis employed to detect price reversal and price shock effect in two subperiods: January 2, 2018, through December 30, 2019 (Subperiod 1, results presented in Panel A, 254 events) and January 2, 2020, through December 30, 2020 (Subperiod 2, results presented in Panel B, 143 events). For both subperiods average abnormal values of measures given by equations (3a), (3b), (4a) and (4b) are presented with corresponding statistic values for expiration day ($t = 0$) and for the previous day ($t = -1$).

*, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

The result suggests that the time of the occurrence of the volatility effect on expiration day shortened after the outbreak of COVID-19: in 2018–2019 it was detected, as with the volume effect, during the time when stock and index futures final settlement prices are being calculated. In 2020 abnormal growth in intraday volatility of stock prices only occurred on expiration days at the close but, additionally, it also appeared at the beginning of the next trading session.

As expected, the research confirmed that price reversal is not a phenomenon that constantly accompanies the expiration of stock and index futures on the WSE. Only in the first subperiod and in the case of one of the two measures of the anomaly some indication of the existence of abnormally strong price reversal was obtained. In the initial period of the pandemic this anomaly did not occur on the WSE. However, the analysis of price shock confirmed Vipul's (2020) statement that price reversal measures are insufficient to express all potential perturbations in the price formation process after futures expiration. The existence of a significant price shock was detected in both subperiods by both of the measures employed.

4.3. Comparison of the strength of the expiration day effects between the subperiods

The results obtained for the two subperiods described in the previous subsection are insufficient to verify the validity of the hypothesis 7, which states that futures expiration effects weakened after the outbreak of the pandemic. Admittedly, they indicate a shortening of the duration of the volume (no anomaly before the expiration day) and volatility effect (no effect in the last hour of continuous trading) in the initial phase of the pandemic compared to the period before its outbreak. They also give some indication of price reversal in the first subperiod but give any basis for concluding that this effect occurred after expiration days in 2020. However, in case of the anomalies whose occurrence was confirmed in a given time interval before both the pandemic and in its initial phase, foregoing results do not provide sufficient grounds to conclude whether the strength of these anomalies differs significantly between the compared subperiods. To verify this, a Mann–Whitney *U* test was conducted (see Mann, Whitney, 1947). This non-parametric test allows to check if two independent data samples come from the same distribution. The test was applied to the abnormal, standardized values of only these measures, which confirmed the occurrence of a given expiration day effect in both of the two analyzed subperiods.

Table 8 presents the results of the conducted Mann–Whitney *U* test. It contains means and medians of the standardized, abnormal values of the measures calculated in two subperiods separately (2018–2019 and 2020). In the rightmost part of the table *p*-values of the *U* test, comparing the distributions of the measures, are presented.

The volume effect was confirmed on expiration days in both subperiods during the last hour of continuous trading as well as at the close. Thus, the *U* test was employed to abnormal, standardized values of measures $\ln(V_{0,2})$ and $\ln(V_{0,3})$. *P*-values of the test, which are close to zero, suggest that compared distributions differ significantly (at a 1% level). Means and medians are greater in the first sample, related to the period before the pandemic, indicating that this anomaly really weakened after the outbreak of COVID-19. Similar results were obtained in reference to the volatility effect at the close on expiration day. Mean and median of the standardized, abnormal measure $SA|R_0^d|$ are lower in the second subperiod and the test confirms a significant difference between the distributions of the two compared samples.

In the case of the effect of price shock, the conducted Mann–Whitney *U* test does not give grounds to conclude that the strength of this effect changed significantly after the outbreak of the pandemic. The means and medians of both employed measures are slightly greater in the second sample, related to the data from 2020, but *p*-values of the test indicate that there are no differences between the compared distributions. Thus, it should be concluded that the strength of the price shock effect, occurring at the beginning of the session following expiration days, was comparable during the pandemic to its strength in previous years.

Table 8

Comparison of the strength of the expiration day effects between the period before the outbreak of the COVID-19 pandemic (2018–2019) and the initial phase of the pandemic (2020). The results of Mann–Whitney ***U*** test applied to abnormal, standardized measures of the effects

Measure	Subperiod 1		Subperiod 2		p-value
	mean	median	mean	median	
$SA\ln(V_{0,2})$	2.046	2.058	1.339	1.272	**0.000**
$SA\ln(V_{0,3})$	2.943	2.794	2.498	2.494	**0.005**
$\mathrm{SA}\lvert R_0^d\rvert$	1.529	1.118	0.774	0.338	**0.000**
$SAPS_0^1$ [%]	0.463	0.056	0.554	0.172	0.305
$SAPS_0^2$ [%]	0.478	0.209	0.617	0.225	0.626

This table presents values of means and medians of abnormal, standardized measures of expiration day effects in two subperiods: January 2, 2018, through December 30, 2019 (Subperiod 1, results presented in the left part of the table) and January 2, 2020, through December 30, 2020 (Subperiod 2, results presented in the right part of the table). Only these measures which confirmed the occurrence of a given anomaly in both of the two analyzed subperiods were included in the table. Distributions of these measures' values were compared between the two subperiods with the use of nonparametric Mann–Whitney *U* test. *P*-values of the test are presented in the most-right column of the table. *P*-values smaller than 0.01 are in bold indicating that the compared distributions differ significantly at 1% level.

Concluding the results from the previous subsection and those from Table 8, hypothesis 7 should be regarded as partially true. It can only be unambiguously confirmed that the effects had weakened for the volume and volatility effects occurring on expiration day after the outbreak of the COVID-19 pandemic.

5. Summary and conclusions

In this study, the existence of expiration day effects of stock and index futures on the spot market of the Warsaw Stock Exchange was examined. Four types of anomalies, namely abnormal growth of trading volume of underlying stocks, significant increase in volatility of their prices, price reversal and price shock, were analyzed. To detect the effects and precisely determine the time of their occurrence, event study analysis was employed to high-frequency data.

According to the authors' knowledge, this study is the first to check the occurrence of expiration day effects on the Polish market in the period 2018–2020, as earlier studies in this area covered periods ending before or at most in 2017. As they confirmed the occurrence of the anomalies on the WSE, one of the important goals of this study was to check if the effects have been visible in recent years. Since the Polish stock market was classified by the FTSE Russell agency as a developed one in 2018, anomalies, which hit mostly developing markets, should disappear or at least weaken. As the research period covered the initial phase of the COVID-19 pandemic, the authors also checked if its outbreak in March 2020 had significantly influenced the strength of the detected expiration day effects. With this end in view, the research period was divided into two parts (January 2018 – December 2019, January 2020 – December 2020) and the Mann–Whitney *U* test was employed to compare distributions of the measures of anomalies between the two subperiods.

The results confirmed most of the hypotheses formulated in section 3.3. In the period 2018–2020 expiration day effects of futures still occurred on the spot market of the WSE. On expiration days, the trading volume of the underlying stocks increased unusually. This increase only occurred, however, during the time when final settlement prices of expiring derivatives were being formed, that is in the last hour of continuous trading as well as at the close and during overtime. Before the outbreak of the COVID-19 pandemic, an abnormal growth of trading volume was also detected two days before the expiration. This can be a sign of early unwinding arbitrage positions. Analogous results, indicating the existence of the volume effect on days directly before expiration were obtained by Suliga and Wójtowicz (2019) as well as by Gurgul and Suliga (2020) in reference to the period 2012–2017. However, in 2020, the volume effect was only detected on expiration days in the final part of trading session. The results suggest that with the development of the market, the time of the occurrence of this anomaly was shortened and narrowed to the time of the formation of derivatives' final prices.

The research also confirmed that the intraday volatility of stock prices is abnormally high on expiration days. The division of the research period into two parts allowed us to discover that before 2020 this anomaly, just as trading volume effect, appeared on expiration days during the last hours of continuous trading and at the close. However, in the initial period of the COVID-19 pandemic, the intraday volatility of underlying stocks was significantly higher than normally on expiration days only at the close and, additionally, at the beginning of the next trading session. Considering the difference in the procedure of calculating final prices of stock and index derivatives on WSE, it can be assumed that the increase in price volatility in 2020 resulting from the expiration of stock futures continued to occur on expiration days, while the effect of the increase in volatility associated with the expiration of index futures disappeared. This may stem from both the development of the WSE futures market and from the outbreak of the pandemic. In March 2020, when the risk of investing in leading stocks skyrocketed, futures market turnover (in particular, turnover of WIG20 future, the most popular derivative instrument on the WSE) increased significantly. This growth of hedging positions on the derivative market certainly reduced the importance of actions undertaken on expiration days by arbitrageurs and speculators with open positions in expiring index derivatives.

The research of price reversal confirmed that this effect is not a phenomenon which appears constantly after futures expiration. One of the two applied measures suggest the existence of this anomaly in the period 2018–2019 but there were no grounds to conclude this effect still occurred in the initial phase of the pandemic. However, the existence of price shock, understood as an abnormal difference between stock returns from expiration day and from the next day, was confirmed for the whole period under study, indicating that on expiration days stock prices are distorted but this distortion is very short-lived and they come back to their normal levels, reflecting intrinsic values, just at the beginning of the next trading session.

Comparison of the strength of the expiration day effects between the subperiods revealed that volume effect in the triple witching hour, as well as the volatility effect at the close on expiration days, weakened after the outbreak of the pandemic. However, they were still visible in 2020. Moreover, the Mann–Whitney *U* test conducted suggests that the strength of the price shock effect had not changed after the outbreak of COVID-19 pandemic.

To sum up, the study confirmed the occurrence of expiration day effects on the WSE. Although over the years and with the development of the market, some anomalies have weakened and their duration have been shortened, significant price distortion at the close of the trading session on expiration day have been still visible. Even during COVID-19 pandemic, when the importance of the activity hedgers in the futures market certainly increased, expiration of stock futures was associated with abnormal increase in volatility of underlying assets' prices. Many studies related to various foreign markets confirmed that if the final price of a derivative depends on a single value of underlying stock, expiration day effects are highly probable (see e.g. Hsieh, Ma, 2009; Xu, 2014). However, if this price is calculated as an average of many values, the anomalies are the weaker the longer is the time of price formation (see e.g. Kan, 2001; Chow et al., 2003; Fung, Yung, 2009). Would such a change in the procedure of calculating final settlement prices of stock futures on the WSE be desirable? This is a question that only the Polish

Financial Supervision Authority can answer. However, this research, as well as previous research on the expiration day effects on the Polish market, allows us to suppose that as long as the method of calculating final settlement prices of stock futures does not change, expiration effects will occur.

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Summary

This paper examines the existence of expiration day effects of stock and index derivatives on the Warsaw Stock Exchange. Event study analysis is employed to high-frequency data to check the occurrence of four types of anomalies: abnormal increase in trading volume and in intraday volatility of underlying stocks, price reversal and price shock. The study confirms that on expiration days trading volume of underlying stocks increase unusually during the time when final settlement prices of expiring futures are being calculated. Intraday volatility of stock prices is also abnormally high on expiration days. However, before 2020 this price effect occurred on expiration days during triple withing hour, while in the initial phase of COVID-19 pandemic it has been visible on expiration days only at the close and additionally at the beginning of the next trading session. The analysis of price reversal and price shock effects revealed that only the second anomaly is a phenomenon which constantly appears after futures expiration, indicating the distortion of stock prices on expiration days and their return to normal levels at the beginning of the next trading session. Division of the research period (2018-2020) into two parts allow to find out that after the outbreak of the pandemic, when the importance of hedgers' activity on the futures market have increased, some of the analyzed anomalies have weakened and their duration have been shortened. However, distortions of underlying stock prices have been still visible at the close of the trading session on expiration days. This suggests that as long as the final settlement prices of stock future are equal to closing prices of underlying stocks, expiration day effects will occur on the WSE.

JEL codes: C14, C32

Keywords: *COVID-19, event study, expiration day effects, futures market, high-frequency data, stock market, Warsaw Stock Exchange*

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The decision making criteria of a rational investor on the example of an investment portfolio analysis of listed companies and a basket of currencies

1. Introduction

Investing is the activity of engaging in something in the hope of obtaining future benefits. The Accounting Act defines investments, in accordance with general regulations, as assets acquired in order to achieve economic benefits resulting from the increase in the value of these assets, obtaining income from them in the form of interest, dividends or other benefits, including also from a commercial transaction, in particular financial assets and those real estate and intangible assets that are not used by the entity, but were acquired in order to obtain benefits (Ustawa z 29 września 1994 r. o rachunkowości).

The increase in access to information has contributed to the emergence of a growing range of opportunities in the field of multiplying wealth. Currently, of course, depending on the volume of capital held, one can invest in shares, cryptocurrencies, gold, real estate, among others. All investing should focus on selecting diversified assets while maintaining appropriate asset values in the portfolio.

All investment decisions on stock exchanges are conditioned by a wide range of factors. Among them, psychological determinants are of great importance, constituting the area of behavioral finance research. One of the premises of the classical theory of finance is based on the idea of rational choices made by economic agents. A rational investor generally acts in such a way as to maximize her profits, and at the same time does not succumb to emotions or pressure from the environment. In her decisions, such an investor is only guided by information obtained from solid financial analysis. All her decisions should be reduced

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to minimizing the risk with the assumed, expected rate of return or with the simultaneous assumption of maximizing the expected rate of return (Wasilewski, Juszczyk, 2016). A rational investor should strive to maximize the expected rate of return on investment (Markowitz, 1952).

In 1952, Harry Markowitz, later a Nobel laureate, proposed the portfolio theory, which is widely used today. This is one of the most useful discoveries in the field of modern finance. Among the conditions for the correct interpretation and application of the portfolio theory is the treatment of the investment portfolio as an integral whole. The correlation between portfolio elements is important (Stepaniuk, 2015). Consistent with the above theory, the law of average covariance illustrates how the reduction in dispersion that is achieved through diversification is constrained by this tendency (Markowitz, 1976). Markowitz's theory says that higher rates of return on an investment portfolio involve taking more risk. Before selecting a specific investment portfolio, the level of risk for a specific entity intending to make investments must first be determined. In the case of a private investor, the choice of risk is directly related to their age, financial and family situation, and therefore she selects the risk level that is comfortable for her. Cash and bonds have the lowest level of risk, and the highest is related to stocks and derivatives. It is safer to keep funds in a bank deposit than to invest them in the stock market. However, in the situation of the SARS-CoV-2 pandemic, in which low interest rates occurred, with the simultaneous increase in inflation, it is worth rethinking broadly understood investing. Investors are rational when they are risk averse, i.e. they strive to obtain the highest possible rates of return with the lowest possible risk (Kontek, 2011).

The next independent theory, which was developed by William Sharpe and Jan Mossin, as another extremely important theory of capital markets, was an outgrowth of earlier work by Markowitz and Tobin – The Capital Asset Pricing Model – CAMP (Megginson, 1996). The CAMP model was the next step in the development of the equilibrium theory of capital markets. Investors have found its use in the valuation of securities as a function of statistical risk. Sharp won the Nobel Prize in Economics for his groundbreaking contribution to its development (Myles, 2013). The above presentation of theories and their authors motivates the choice of methodology used in this article.

2. Purpose of the work

The aim of the work is to present a portfolio of listed companies, in which entities from the property development, mining and metallurgical industries, as well as the energy industry, were selected in a collision with a portfolio consisting of purchased currencies. The first observation is December 30, 2019, and the last one is December 30, 2020. Of course, the observation period for the analyzed data, both in terms of companies and currency purchases, coincides in order to maintain the ability to compare rates of return. The period that should be examined using the methods cited depends on the individual preferences of the investor, as well as the availability of historical data and forecasts. For the purposes of this article, a period of one year was examined. However, in most cases, examining historical data over many years can provide better information on asset behavior and risk ratios.

Studying a portfolio of companies from various industries and a portfolio of currencies based on the theories of Markowitz and CAPM is valuable as it aims to understand portfolio diversity, manage risk, and assess expected returns.

The portfolio was built using tools such as: standard deviation, variance, covariance, correlation, rate of return, coefficient of volatility, beta coefficient and rate of return base. The selection of the research sample, in the form of companies from the development industry, mining and metallurgical industries, as well as the energy industry, in relation to the basket of currencies, was used to diversify assets in the investment portfolio.

For companies, daily closing rates for individual days of the year were used, due to the occurrence of holidays and holidays, the number of observations was 252. Data were downloaded from Stooq.com. For the second set of data, referring to the exchange rates of purchased currencies, tables for the euro and the Australian dollar were used from the website of the National Bank of Poland. The conducted analysis is an attempt to estimate the rate of return on invested capital for the described investment portfolios.

3. Characteristics of the analyzed data

The selection of companies in the portfolio was not accidental. Due to the growing interest in the real estate market, entities dealing with development services were selected, i.e. Archicom, Dom Development, Inpro, JW. Construction Holding, Atal, Global Trade Centre. In addition, quotation values for Jastrzębska Spółka Węglowa as the largest producer of high-quality hard coking coal and a leading producer of coke used for steel smelting were adopted for the analysis (JSW, 2021). ArcelorMittal, the largest steel producer, for which we are observing a significant increase in value in 2020, was selected as a company from a related industry. In addition, the portfolio includes companies from the energy sector, i.e. Enea and Energa.

The second portfolio included currencies: the Australian dollar and the euro, which recorded an increase in exchange rates in the analyzed period. Due to different rates for currency exchange in bureaux de change or via a bank account, the cost of currency purchase/sale was not included in the analysis. It is also worth paying attention to the fact that the commission appears in banks from a high balance at the end of 2020 (for most 5 million PLN and 100,000 in a foreign currency). Currently, limits only apply to corporate accounts, while private investors also need to be aware of such fees in the future and should constantly monitor the tables of fees and commissions for the bank accounts used.

4. Analysis

According to the assumptions of the Markowitz model, investors are rational when they are risk averse, i.e. they strive to obtain the highest possible rates of return with the lowest possible risk. The analysis uses the formula for the simple rate of return of the portfolio, which is presented below.

Pattern 1

$$R = \frac{FV - PV}{PV}$$

Source: (Pera et al., 2014)

In the formula, PV is the purchase price of the stock, while FV is the final value of the investment. The simple rate of return is the base rate and is the least accurate measure of investment return. It is the ratio of the obtained or expected income to the outlays that have been or will be incurred. In addition, it should be noted that the simple rate of return is devoid of additivity. For further analysis, the logarithmic rate of return was used, which is the basic method for calculating the relative return on investment for continuous capitalization. The analytical form of the equation for the logarithmic rate of return is such a transformation of the future value formula for the aforementioned continuous capitalization so that the interest rate is on its left side. Hence, the relationship presented below is true (Pera et al., 2014).

Pattern 2

$$r_{ln} = \ln(FV) - \ln(PV) = \ln\left(\frac{FV}{PV}\right)$$

Source: (Pera et al., 2014)

Therefore, it can be said that each n-period logarithmic rate of return is the sum of single-period logarithmic rates of return, i.e. it is additive (Pera et al., 2014). In further analysis for the indices of the companies mentioned above, the results for the logarithmic rate of return for each day were used to calculate the rate of return and standard deviation.

Standard deviation as a measure of risk was used to estimate the level of volatility in the quotations of the audited entities, as well as to further present the formation of the volatility coefficient in the entities under consideration. The results of the calculations are presented in the Table 1.

Table 1

Expected rate of return, standard deviation and coefficient of variation for portfolio I companies

	Archicom S.A. (ARH)	Dom Development S.A. (DOM)	Inpro S.A. (INP)	JW. Construction Holding S.A. (JWC)	Atal S.A. (1AT)	Global Trade Centre S.A. (GTC)	Jastrzębska Spółka Węglowa S.A. (JSW)	ArcelorMittal S.A. (MT.US)	Energa S.A. (ENG)	Enea S.A. (ENA)
Expected rate of return [%]	0.17	0.11	0.00	–0.01	–0.06	–0.12	0.08	0.12	0.04	–0.08
The standard deviation of the rate [%]	3.38	2. 39	3.18	1.83	2.67	3.02	5.33	4.71	2.10	3.45
Coefficient of variation	20.1968	22.1250	1423.7562	–231.8276	–43.0746	–25.7205	69.3756	40.5265	49.5495	–45.3147

Source: own calculations based on quotations available on https://stooq.pl (access date: 1.08.2021)

The above data show that despite the popularity of the indicated industries, the rates of return vary. It should be remembered that due to the pandemic, there has been reluctance in society to invest in real estate. The fear of losing one's job and a steady source of income caused a decline in investments in this area. It is only in 2021 that we notice an increase in interest in real estate purchases. It is also directly related to rising inflation, as well as low interest rates in banks. To estimate the level of volatility, the basic measure of volatility was used, which is standard deviation. The larger the observed standard deviation, the further the recorded values are away from the average. In the presented period, the lowest standard deviation from the rate of return, compared to historical data, was recorded by JW. Construction Holding. ARE. The calculated rate of return and standard deviation were used to determine the volatility coefficient, which is a relative measure of risk assessment. It informs about the amount of risk per unit of rate of return. It is a measure referring to the characteristics of a rational investor mentioned in the introduction who invests his funds in such a way as to maximize his profits while minimizing the risk taken. According to this theory, the coefficient of variation should be as small as possible (Pera et al., 2014).

Cell fields referring to the coefficient of variation were filled in gray deliberately. The results have been obscured because considering the coefficient in question only makes sense when it takes values greater than zero.

Similarly, the analysis was performed for a basket of currencies. In the same period, i.e. on December 30, 2019, an Australian dollar was purchased, marked later in the essay with the code AUD and euro – EUR. The determined rate of return for the currencies in question is as follows in the Table 2.

Table 2

Expected rate of return, standard deviation and coefficient of volatility for currencies from portfolio II

	Australian dollar [AUD]	Euro [EUR]
Expected rate of return [%]	0.235	0.0243
The standard deviation of the rate [%]	0.66	0.43
Coefficient of variation	27.9182	17.6679

Source: own calculations based on https://www.nbp.pl/home.aspx?f=/kursy/arch_a.html (access date: 20.08.2021)

The next stage of the study was the creation of the Spearman correlation matrix, after calculating the weights for individual companies. This coefficient applies to any monotonic relationship. The calculation results are presented in the Table 3 (Pera et al., 2014).

Table 3

Spearman's correlation coefficient for companies from portfolio I

	Spearman correlation matrix									
	ARH	DOM	INP	JWC	1AT	GTC	JSW	MT.US	ENG	ENA
ARH	**1.0000**	0.2464	0.0293	0.1247	0.1852	0.1628	0.2450	–0.0019	0.1438	0.2669
DOM	0.2464	**1.0000**	0.0352	0.1687	0.2786	0.2179	0.2769	–0.0499	0.2282	0.2032
INP	0.0293	0.0352	**1.0000**	–0.1062	0.1276	0.0391	0.0528	0.0938	–0.0098	0.1258
JWC	0.1247	0.1673	–0.1062	**1.0000**	0.0437	0.0494	0.2311	0.0293	0.1431	0.1204
1AT	0.1852	0.2796	0.1276	0.0437	**1.0000**	0.1140	0.2036	0.0644	0.0519	0.1637
GTC	0.1628	0.2179	0.0391	0.0494	0.1140	**1.0000**	0.3018	0.1218	0.0528	0.2104
JSW	0.2450	0.2769	0.0528	0.2311	0.2036	0.3018	**1.0000**	0.1228	0.1385	0.4108
MT.US	–0.0019	–0.0499	0.0938	0.0293	0.0644	0.1218	0.1228	**1.0000**	–0.0780	0.0394
ENG	0.1438	0.2282	–0.0098	0.1431	0.0519	0.0528	0.1385	–0.0780	**1.0000**	0.2892
ENA	0.2669	0.2032	0.1258	0.1204	0.1637	0.2104	0.4108	0.0394	0.2892	**1.0000**

Source: own calculations based on https://stooq.pl (access date: 1.08.2021)

The highest correlation, at the level of 0.41, is recorded by Jastrzębska Spółka Węglowa and Enea. The above value fits into the correlation at a moderate level. On the other hand, the remaining observations in the matrix of companies show a weak or even no relationship (Statystyka, 2021).

On the other hand, the Spearman correlation matrix for the values of exchange rates is characterized by a weak relationship (Table 4).

Table 4

Spearman's correlation coefficient for portfolio currencies II

	Spearman correlation matrix	
	AUD	EUR
AUD	**1.00**	0.2642
EUR	0.2642	**1.00**

Source: own calculations based on
https://www.nbp.pl/home.aspx?f=/kursy/arch_a.html (access date: 20.08.2021)

In the vast majority of both portfolios, the ratio shows a weak or no relationship, thanks to which the risk of a sudden decrease in the value of portfolios is low.

Additionally, covariance and correlation matrices were created for all indices, and then the beta coefficient was calculated, the calculation results are shown in the tables below (Tables 5–10).

Table 5

Covariance matrix for companies from portfolio I

	Covariance matrix									
	ARH	DOM	INP	JWC	1AT	GTC	JSW	MT.US	ENG	ENA
ARH	0.0011	0.0003	0.0001	0.0001	0.0003	0.0001	0.0005	–0.0001	0.0001	0.0004
DOM	0.0003	0.0006	0.0001	0.0001	0.0002	0.0002	0.0004	0.0000	0.0001	0.0002
INP	0.0001	0.0001	0.0010	0.0000	0.0001	0.0000	0.0002	0.0001	0.0000	0.0002
JWC	0.0001	0.0001	0.0000	0.0003	0.0000	0.0000	0.0002	0.0000	0.0000	0.0001
1AT	0.0003	0.0002	0.0001	0.0000	0.0007	0.0001	0.0004	0.0000	0.0001	0.0002
GTC	0.0001	0.0002	0.0000	0.0000	0.0001	0.0009	0.0005	0.0002	0.0001	0.0002
JSW	0.0005	0.0004	0.0002	0.0002	0.0004	0.0005	0.0028	0.0000	0.0001	0.0008
MT.US	–0.0001	0.0000	0.0001	0.0000	0.0000	0.0002	0.0000	0.0022	–0.0001	0.0000
ENG	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	–0.0001	0.0004	0.0002
ENA	0.0004	0.0002	0.0002	0.0001	0.0002	0.0002	0.0008	0.0000	0.0002	0.0012

Source: own calculations based on quotations available on https://stooq.pl (access date: 1.08.2021)

Table 6

Covariance matrix for currencies from portfolio II

	Covariance matrix	
	AUD	EUR
AUD	0.00004	0.00001
EUR	0.00001	0.00002

Source: own calculations based on quotations available on https://www.nbp.pl/home.aspx?f=/kursy/arch_a.html (access date: 20.08.2021)

The covariance illustrates the relationship between the rates of return and indicates the direction of changes. However, it does not measure the strength of the relationship between the rates of return (Pera et al., 2014).

Table 7

Correlation matrix for companies from portfolio I

	Correlation matrix									
	ARH	DOM	INP	JWC	1AT	GTC	JSW	MT.US	ENG	ENA
ARH	1.0000	0.3181	0.0802	0.1110	0.3173	0.1088	0.2706	–0.0558	0.1463	0.3742
DOM	0.3181	1.0000	0.1160	0.1573	0.2709	0.2908	0.2854	–0.0273	0.1732	0.2797
INP	0.0802	0.1160	1.0000	–0.0337	0.1190	0.0201	0.0960	0.0469	–0.0661	0.2002
JWC	0.1110	0.1573	–0.0337	1.0000	0.0666	–0.0162	0.2054	–0.0274	0.1266	0.1086
1AT	0.3173	0.3822	0.0896	0.0666	1.0000	0.1621	0.2755	–0.0087	0.0913	0.2179
GTC	0.1088	0.2908	0.0201	–0.0162	0.1621	1.0000	0.2896	0.1117	0.0918	0.2147
JSW	0.2706	0.2854	0.0960	0.2054	0.2755	0.2896	1.0000	–0.0167	0.0716	0.4442
MT.US	–0.0558	–0.0273	0.0469	–0.0274	–0.0087	0.1117	–0.0167	1.0000	–0.0723	–0.0306
ENG	0.1463	0.1732	–0.0661	0.1266	0.0913	0.0918	0.0716	–0.0723	1.0000	0.2616
ENA	0.3742	0.2797	0.2002	0.1086	0.2179	0.2147	0.4442	–0.0306	0.2616	1.0000

Source: own calculations based on quotations available on https://stooq.pl (access date: 1.08.2021)

Table 8

Correlation matrix for currencies from portfolio II

	Correlation matrix	
	AUD	EUR
AUD	1.4831	–0.0169
EUR	–0.0169	0.6743

Source: own calculations based on quotations available on https://www.nbp.pl/home.aspx?f=/kursy/arch_a.html (access date: 20.08.2021)

The positive correlations observed in the vast majority of cases indicate that an increase in one feature is accompanied by an increase in the average values of the other feature.

Table 9

Beta coefficient for companies from portfolio I

	Betafactor [%]									
	ARH	DOM	INP	JWC	1AT	GTC	JSW	MT.US	ENG	ENA
ARH	–	44.89	8.53	20.51	40.11	12.17	17.14	–4.00	23.49	36.70
DOM	22.55	–	8.74	20.60	34.25	23.05	12.81	–1.39	19.70	19.45
INP	7.55	15.40	–	–5.86	10.66	2.11	5.72	3.17	–9.98	18.47
JWC	6.00	12.00	–1.94	–	4.56	–0.98	7.04	–1.06	10.99	5.76
1AT	25.09	42.66	7.54	9.75	–	14.34	13.81	–0.49	11.60	16.90
GTC	9.72	36.68	1.91	–2.68	18.32	–	16.40	7.17	13.17	18.82
JSW	42.70	63.55	16.11	59.94	54.97	51.12	–	–1.89	18.15	68.76
MT.US	–7.77	–5.37	6.95	–7.06	–1.53	17.42	–1.48	–	–16.17	–4.18
ENG	9.11	15.22	–4.38	14.58	7.19	6.39	2.83	–3.23	–	15.98
ENA	38.15	40.24	21.70	20.46	28.08	24.48	28.70	–2.24	42.81	–

Source: own calculations based on quotations available on https://stooq.pl (access date: 1.08.2021)

The above calculations show, for example, that for the pair of Archicom and Dom Development, an increase in the rate of return by one unit for the first company generates an increase in the rate of return for Dom Development by 0.22 percentage point.

Table 10

Beta coefficient for currencies from portfolio II

	Betafactor [%]	
	AUD	EUR
AUD	–	33.88
EUR	14.48	–

Source: own calculations based on quotations available on
https://www.nbp.pl/home.aspx?f=/kursy/arch_a.html (access date: 20.08.2021)

For currency basket calculations, it can be deduced that an increase in the return of the Australian dollar by one unit generates an increase in the return of the euro by 0.14 percentage points.

5. Conclusions

Summing up the analyses carried out within the portfolio of shares and currencies, it can be stated that investing in companies listed on the stock exchange was a better solution. The average rate of return for the portfolio of companies, being the arithmetic mean of simple rates of return in the analyzed period, was 9.16%, and for the basket of currencies 6.20% (assuming an equal share of securities in portfolio I). Among the analyzed companies, the highest simple rate of return was recorded by Archicom S.A. – 52%, while in logarithmic terms – 42%. The loss was generated by the Global Trade Center, with a simple rate of return of −26% and a logarithmic rate of return of −30%. For the basket of currencies, the average rate of return was 6.20%, and the logarithmic rate was 6.01%. As already mentioned, a rational investor strives to minimize the risk with the assumed expected rate of return or with the simultaneous assumption of maximizing the expected rate of return. Therefore, drawing conclusions from historical data, it can be assumed that he would invest in a portfolio of companies.

This article is both about the importance of the topic of investing, and also emphasizes the importance of diversifying the investment basket. A wide spectrum of investment opportunities should be examined and analyzed, especially during periods of rising inflation, in order to secure the assets held. The presented method of analyzing investment activities can not only be used in academia, but also in economic life in the real world. The above work is a call for further research in the field of investing and presents the possibilities of their measurement. A good investor is not only characterized by a rational approach, but also by making responsible investment decisions, striving to achieve their financial goals in a sustainable and effective manner. The above article is also an incentive to undertake financial education in order to be aware of various financial instruments, as well as the possibilities of their use. It is worth mentioning that there are no perfect investors but one should strive to develop and build those features that can help to build an effective and balanced investment portfolio.

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Summary

In today's globalized world, investing is a key financial management strategy related to the hope of obtaining future economic benefits. Access to more and more information has created new opportunities for wealth creation, including investments in stocks, cryptocurrencies, gold, real estate and many other assets. The work emphasizes the importance of investing and diversifying the investment portfolio, especially in periods of inflation, using the example of a portfolio of listed companies compared to a currency portfolio. The study covers the period from December 2019 to December 2020, and the analysis aims to estimate the rate of return on investment based on the Markowitz and CAPM theories. The presented analysis method is applicable both in the academic environment and in real economic scenarios, encouraging further research on investments.

JEL codes: G17, G24, G32, G41

Keywords: *economics, finance, investment portfolio, Markowitz, CAMP*

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3. The journal uses double blind peer review policy, i.e. neither the author nor the referee knows the identity of the other. In addition, each referee signs a declaration of no conflict of interests, whereby the conflict of interest is defined as a direct personal relationship (kinship to the second degree, legal relationships, marriage) or a professional scientific cooperation between the referee and the author which took place at least once in the past two years preceding the year of accepting the invitation to review.

4. Only written referee reports are considered (journal does not accept face-to-face or phone-call- based reports). Each report should clearly express the referee's final recommendation (i.e. whether the article should be accepted for publication, revised or rejected). The referees are kindly requested to fill the review form which can be found in "For reviewers" section. In general, the referees are asked to:

– assess:

- the scientific importance of the submission's topic,
- the quality of research;

– verity whether:

  - the Abstract is concise and informative,
  - the facts and interpretations are satisfactorily separated in the text,
  - the interpretations and conclusions follow from the data,
  - the length and structure of the paper is appropriate,
  - the paper can be shortened without loss of quality,

- all the tables and figures are necessary,
- the diagrams and photographs are of good quality,
- there are all essential figures that should be prepared,
- all the references are exact,
- the manuscript requires proof reading by native speaker,
- there is sufficient attention given to previous research.

5. The names of the referees of particular articles are classified.

6. Once a year the journal publishes the complete list of referees.