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Linear vs. threshold cointegration approaches to price discovery: The case of the Warsaw Stock Exchange

1. Introduction

Financial markets are continuously exposed to new information, which exerts a significant impact on the price formation process. According to the informational efficiency hypothesis proposed by Fama (Fama 1970), in a fully efficient market, asset prices should instantaneously reflect all available information. The complete absorption of new information into the price of a given instrument is referred to as *price discovery*. In practice, however, the vast majority of markets fail to meet the assumptions of strong-form informational efficiency, and the adjustment of prices to new information arriving in the market is a gradual process that unfolds over time.

Due to the specific nature of derivative instruments, their prices are closely linked to the prices of the corresponding underlying assets traded in the spot market. Incoming market information usually triggers similar investor reactions in both markets (spot and futures). The strong connection between these two markets ensures that any temporary mispricing is exploited by arbitrageurs, whose activity restores equilibrium and guarantees price consistency.

Nevertheless, price changes in the two markets are not perfectly synchronized. In practice, new information may be absorbed more quickly by either the spot or the futures market. When the price of the underlying asset or derivative adjusts first, it is assumed that the other price will follow in a short period of time. Such relationships are associated with the existence of Granger causality between spot and futures prices – past price changes in one market may improve

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the forecasting accuracy of price changes in the other, compared to forecasts made without that information.

In the literature, it is generally assumed that the market that first incorporates new price-relevant information plays the leading role in the price discovery process. It should be noted that fulfilling the price discovery function is regarded as one of the key functions of the futures market. A mature and liquid futures market is expected to actively participate in price discovery and contribute to the efficiency of the spot market. For this reason, the analysis of the price discovery process constitutes an important element in assessing the maturity of a derivatives market as well as its impact on the efficiency and functioning of the spot market.

One of the most commonly employed methods of analyzing the price discovery process is the use of a bivariate vector error correction model (VECM). This model enables the identification of both long-run and short-run causal relationships between the prices of futures contracts and their corresponding spot market instruments. This approach has been applied, among others, by Bohl and colleagues (Bohl et al. 2011), Mutlu and Arik (Mutlu, Arik 2015), Marcinkiewicz and Kompa (Marcinkiewicz, Kompa 2013), Chen and colleagues (Chen et al. 2019, 2021), and Suliga (Suliga 2025). However, the classical VECM relies on assumptions that may limit its usefulness in studies of price discovery, particularly in the context of the complex and dynamic nature of financial markets.

First, the model assumes linear and time-invariant relationships between price changes in the two markets. This implies that the response of one variable to the other remains unchanged throughout the entire sample period. In practice, especially when the estimation horizon spans several years, this assumption may be unrealistic. Markets are subject to structural shocks such as financial crises, regulatory changes, or macroeconomic turbulence, which may permanently alter the relationships between derivatives and their underlying assets. The VECM framework does not account for such changes.

Second, the model presumes that the error correction mechanism is independent of both the direction and the magnitude of deviations from the long-run equilibrium. In reality, however, adjustment may occur only after a certain threshold is exceeded; for instance, when the deviation becomes economically significant from the perspective of market participants. Moreover, asymmetric responses are possible; for example, the futures market may react more rapidly to overpricing than to underpricing of the derivative. Such nonlinear and state-dependent reactions cannot be captured by the classical VECM.

Given these limitations, it appears reasonable to apply a nonlinear threshold vector error correction model (TVECM) in the study of price discovery. This model allows the adjustment mechanism to vary depending on the magnitude and direction of deviations from equilibrium, making it possible to identify situations in which the error correction mechanism actually becomes active. As a result, the

TVECM framework may more accurately reflect real market conditions such as transaction costs, limited liquidity, or delayed arbitrage responses. Thus, it offers a more flexible and precise tool for analyzing the price discovery process.

The objective of this study is to compare the usefulness of the classical bivariate VECM and its nonlinear threshold version (TVECM) in analyzing the price discovery process on the Warsaw Stock Exchange. The analysis is conducted for the most liquid derivative instruments traded on the exchange, namely WIG20 index futures and their corresponding underlying index.

The study employs daily data from January 2018 to December 2024. During this period, two major events severely disrupted the domestic financial market and led to a sharp increase in investment risk: the outbreak of the COVID-19 pandemic and the onset of the war in Ukraine. Both during the initial phase of the pandemic and in the first weeks of the war, the WIG20 index and its associated futures contracts experienced steep declines. It can be assumed that the structural shocks triggered by these events also influenced the relationship between the spot and the futures markets. In such a context, the results of a classical linear VECM estimated on data from 2018 to 2024 would likely be distorted and would fail to reliably capture the causal relationships between the analyzed markets.

The comparison of VECM and TVECM outcomes aims to determine whether a nonlinear framework – accounting for market reactions that vary with the magnitude and direction of deviations from equilibrium – can more accurately capture the price discovery mechanism under conditions of heightened volatility and structural disturbances. In particular, the questions are whether the threshold model better reflects actual market behavior during crisis periods, and whether it provides a more effective analytical tool than the classical linear specification.

The remainder of this paper is structured as follows. Section 2 presents a review of the literature on the price discovery process, with particular emphasis on the application of the VECM and TVECM models as tools for investigating causal linkages between derivative markets and their underlying instruments. Section 3 describes the data and outlines the methodological framework. Section 4 presents and discusses the empirical results. Section 5 concludes with a summary, final remarks and a discussion of the study’s limitations.

2. Literature review

2.1. Previous research findings from various markets worldwide

A mature futures market, characterized by high liquidity, should play an important role in the process of price discovery. The dominant role of the derivatives market in this process has been confirmed by studies conducted with respect to the United States market (e.g., Hasbrouck 2003; Chou, Chung 2006), the United Kingdom (Brooks et al. 2001; Gwilym, Buckle 2001), Germany (Booth et al. 1999; Gaul, Theissen 2008), and France (Alphonse 2000; Buckle et al. 2019).

The features that enable a mature futures market to reflect new incoming information faster than the spot market include, among others, lower transaction costs, the leverage inherent in derivative instruments, the absence of short-sale restrictions, and the significant share of institutional investors (cf. Theissen 2002; Bohl et al. 2011; Mutlu, Arık 2015; Fassas, Siriopoulos 2019).

In the case of emerging markets, previous research on the contribution of futures contracts to the price discovery process differs considerably across markets. For example, with regard to the Korean market, Min and Najand found that index futures began to dominate the price discovery process almost immediately after their introduction (Min, Najand 1999). By contrast, as demonstrated by Guo and colleagues, on the Chinese market, along with the development of the futures segment, futures contracts gradually took over the leading role in price discovery from the spot market (Guo et al. 2013). Similarly, in the case of the Greek market, early research on futures contracts indicated inefficiency, with futures prices appearing to be biased as forecasts of spot prices (cf. Kenourgios 2005; Andreou, Pierides 2008). However, studies conducted in later periods demonstrated the dominant role of futures in the price discovery process on that market (cf. Kavussanos et al. 2008; Kavussanos, Visvikis 2011).

Research on the price discovery process on the Warsaw Stock Exchange (WSE) is relatively scarce. An analysis concerning the early years of the WSE's derivatives market (1998–2009) was conducted by Bohl and colleagues (Bohl et al. 2011). Using daily closing prices of the WIG20 index and the corresponding futures contracts, and applying a bivariate VECM, they showed that the spot market played the leading role in price discovery during the analyzed period. By splitting the sample into two subperiods, they also demonstrated that after investment funds were allowed to participate in the futures market in 2004 – which significantly increased the share of institutional investors in derivatives trading – the role of WIG20 futures in price discovery rose. While the average contribution of the futures market to price discovery was estimated at around 16% during 1998–2004, it was assessed at around 35% in 2005–2009.

Causal relationships between the WIG20 index and its futures contracts were also investigated by Marcinkiewicz and Kompa using VAR and VECM models (Marcinkiewicz, Kompa 2013). The authors employed both daily data and high-frequency data (5-, 15-, and 30-minute intervals) from 2008 to 2011. In a summary table, they presented the direction and significance of the identified causal linkages. Their finding that the error correction coefficient was significant in each of the estimated VECM models only in the equation describing futures price changes was, however, followed by an incorrect conclusion: "this means that in the long run causality runs from futures to spot." The conclusion should be the opposite, since the significance they reported indicates that the WIG20 index is the Granger cause of futures prices. In light of this interpretation and the absence of full estimation results, the evidence provided by Marcinkiewicz and Kompa may not be sufficient to draw firm conclusions about the contribution of index futures to price discovery on the Warsaw Stock Exchange in 2008–2011 (Marcinkiewicz, Kompa 2013). Writing in the conclusion of their work that the futures market reacted faster than the spot market to incoming information and that the influence of the index market on futures was weaker than the reverse causal relationship, they likely overstated the role of the futures market in price discovery during the analyzed period.

Studies on the price discovery process on the Warsaw Stock Exchange were also conducted by Mutlu and Arik (Mutlu, Arik 2015) and Suliga (Suliga 2025). However, these studies focused on causal relationships between the prices of single-stock futures (SSFs) and the prices of their underlying shares. Both studies employed a bivariate VECM, and both indicated that it was the stock market, rather than the futures market, that played the leading role in price discovery.

Mutlu and Arik, in addition to the Polish market, examined the Russian, Korean, and Indian markets (Mutlu, Arik 2015). With respect to Poland, for twenty pairs of instruments (SSF contracts and their underlying stocks), they used daily data covering the period from the introduction of each series of contracts up to August 15, 2014, as well as 60-minute data from April 1 to August 15, 2015. The authors did not include the full estimation results of the VECM models in their paper. They provided only the average contributions of the spot and futures markets to price discovery, estimated on the basis of those models. For the Polish SSF market, this contribution was reported to be around 40% when daily data were used and around 38% when intraday data were applied.

Suliga carried out an analysis of the price discovery process for single-stock futures with the highest contract multiplier (1000) and their underlying shares (Suliga 2025). The study covered the years 2020–2023 and was conducted using daily data. The results demonstrated that in most cases, stock prices were the Granger cause of futures prices, both in the context of short-run and long-run relationships. Causality running from stock prices to futures prices was found in the long run for only three companies and in the short run for only one out of the eight analyzed firms. In particular, Suliga's study showed that a high SSF contract multiplier (and thus higher leverage) does not guarantee the dominance of these contracts in the price discovery process, especially when futures trading volumes are relatively low (Suliga 2025).

2.2. VECM and TVECM models as tools for examining the price discovery process

The classical bivariate vector error correction model (VECM) has been employed in numerous studies conducted so far relating to the price discovery process. This model has been applied to the analysis of causal relationships between the spot and futures markets in relation to the Chinese (Hou, Li 2013; Liu, Qiao 2017; Xu 2018), Greek (Fassas, Siriopoulos 2019), Indian (Kumar, Tse 2009; Mutlu, Arik 2015; Curran et al. 2020), Korean (Mutlu, Arık 2015; Kang et al. 2016), Polish (Bohl et al. 2011; Marcinkiewicz, Kompa 2013; Mutlu, Arik 2015; Suliga 2025), Russian (Mutlu, Arik 2015), and Taiwanese (Chen et al. 2019; Chen et al. 2021) markets.

In the classical VECM, it is assumed that the structure of the cointegrating relationship remains stable throughout the adopted research period and that the causal relationships between the analyzed time series are linear. In practice, these assumptions are not always met, and the economic realism of the model is simplified. For instance, during a long research period, the nature of the cointegrating relationship may change as a result of a crisis or significant regulatory shifts that alter the structure of participants in the futures market. In some studies on price discovery, the research period is therefore divided into subperiods, and the VECM is estimated separately for each (e.g., Bohl et al. 2011; Marcinkiewicz, Kompa 2013; Xu 2018; Chen et al., 2021). This division usually serves the purpose of testing whether an important event significantly influenced the strength and direction of causal relationships between the spot and futures markets.

The linear VECM also assumes that the process of returning to equilibrium is symmetric and linear, regardless of whether the deviation from the long-run equilibrium is positive or negative. In reality, however, it is possible that the response to disequilibria between markets is asymmetric; for example, prices may adjust more rapidly in cases of undervaluation than in cases of overvaluation (or vice versa). Applying the VECM with a division of the research period into subperiods does not solve this problem and does not allow for the detection of potential asymmetries in market mechanisms. Modeling such asymmetries is possible through the application of the threshold vector error correction model (TVECM), originally introduced by Balke and Fomby (Balke, Fomby 1997) and further developed by Hansen and Seo (Hansen, Seo 2002), which extends the classical VECM. In this model, the concept of a threshold (or thresholds) is introduced, assuming that once the threshold is exceeded, the error correction mechanism changes.

In research on price discovery and causal relationships between derivative prices and the prices of their underlying assets, the TVECM has been employed by, among others, Martens and colleagues (Martens et al. 1998), Kim and colleagues (Kim et al. 2010), Mamatzakis and Remoundos (Mamatzakis, Remoundos 2010), Ters and Urban (Ters, Urban 2020), and Xi and colleagues (Xi et al. 2023).

Mamatzakis and Remoundos (Mamatzakis, Remoundos 2010) applied a threshold VECM with a single threshold to analyze cointegration between spot and futures prices of Brent crude oil, using data from 1990 to 2009. Their results confirmed the existence of cointegration, with two distinct regimes emerging. Applying a threshold model instead of the classical linear VECM allowed them to more precisely capture the relationships between the examined markets.

Xi and colleagues (Xi et al. 2023) employed both the classical VECM and a threshold VECM with a single threshold to study the efficiency of the CO_2 futures market in the second phase of the European Union Emissions Trading System (2008–2012). Their findings suggest that the analyzed derivatives played an important role in the price discovery process during the research period and that the use of past futures prices could improve spot price forecasts. By employing the threshold model, Xi and colleagues (Xi et al. 2023) found that, in the short run, the futures market under study was not fully efficient. The error correction mechanism was activated only when the basis fell below −0.07. Futures prices exhibited very slow adjustments to long-run equilibrium in the presence of deviations. The authors attributed this phenomenon to the global financial crisis of 2008 and its aftermath.

Ters and Urban presented a methodology for estimating a three-regime TVECM with an unknown cointegrating vector, based on a novel dynamic grid evaluation (Ters, Urban 2020). The authors highlighted that applying a three-regime threshold model made it possible to estimate the range in which arbitrageurs have no incentive to trade, meaning that the error correction mechanism is not activated. It becomes active only when the basis exceeds a critical threshold, making potential profits from arbitrage greater than the necessary transaction costs. Only then can one expect arbitrageurs to enter the market and execute transactions. Such a mechanism supports the presence of nonlinear adjustment dynamics and the existence of three regimes, with the middle regime corresponding to the no-arbitrage band. These regimes may differ significantly in terms of the characteristics of the relationships between cointegrated spot and futures markets. In the final part of their study, Ters and Urban also presented two short applications of the TVECM with two thresholds, relating to the palladium market and to S&P 500 index futures (Ters, Urban 2020).

A three-regime TVECM was also applied by Martens and colleagues and Kim and colleagues in their studies of causal relationships between the S&P 500 index and its futures contracts (Martens et al. 1998; Kim et al. 2010). Their choice of model was justified similarly to that of Ters and Urban (Ters, Urban 2020), arguing that arbitrageurs enter the market only when the mispricing between the spot and futures markets is large enough to compensate for the transaction costs they incur and the associated risks of interest rate and dividend changes. If the error correction term ec_t deviates only slightly from zero, arbitrage is not profitable. For this reason, the authors considered the use of a three-regime threshold model to be appropriate for studying the price discovery process, with the middle regime corresponding to $|ec_t| \approx 0$.

The results obtained by Martens and colleagues (Martens et al. 1998) showed that in their research period (S&P 500 futures contracts expiring in June and December 1993), the impact of the futures market on the spot market was greater when the error correction term was negative, and that the influence of deviations from long-run equilibrium on current returns increased with the degree to which futures prices diverged from their theoretical values.

An analysis of the nonlinear dynamics of causal relationships between the S&P 500 index and its futures contracts was also conducted by Kim and colleagues using a three-regime TVECM (Kim et al. 2010). The authors identified two thresholds defining the no-arbitrage band (small deviations of prices from the long-run equilibrium relationship), consistent with the cost-of-carry model Kim et al. 2010).

Kim and colleagues found that when prices move outside this band, i.e., when the error correction term is sufficiently large in absolute value, spot and futures price series become stationary mean-reverting processes (Kim et al. 2010). Prices within the no-arbitrage band, however, were found to be non-stationary. Furthermore, Kim and colleagues (Kim et al. 2010) confirmed the earlier findings of Martens and colleagues (Martens et al. 1998), namely that futures prices dominate in the price discovery process (the index adjusts to futures prices, not the other way around).

Taking into account the arguments of Ters and Urban (Ters, Urban 2020) as well as the results obtained by Martens and colleagues (Martens et al. 1998) and Kim and colleagues (Kim et al. 2010), the threshold VECM with two thresholds (three regimes) appears to be a more suitable tool for analyzing the price discovery process in the spot and futures markets than the model with two regimes (a single threshold). In this study, therefore, the classical VECM will be compared in terms of its usefulness for analyzing price discovery on the Warsaw Stock Exchange with the three-regime threshold model. The information criterion (AIC) further confirmed that the TVECM with two thresholds provides a better fit to the analyzed data than the model with a single threshold.

3. Data and research methodology

This section presents the data and methodology used in the empirical analysis. It begins with a description of the dataset, including the WIG20 index and its futures contracts, followed by the specification of the econometric models applied to examine the price discovery process.

3.1. Data used

The study employed daily closing prices of the WIG20 index and its futures contracts for the period from January 2, 2018, to December 30, 2024. Futures contracts on the WIG20 index are the most liquid instruments on the Warsaw Stock Exchange's derivatives market. In the years 2018–2024, the average annual turnover value of these derivatives amounted to PLN 262,695 million, which represented on average 90.92% of the annual turnover of all futures contracts traded on the Warsaw Stock Exchange. The multiplier of WIG20 futures contracts has a value of 20, which means that a change of one index point in the underlying index results in a change of PLN 20 in the contract value. Each quarter, a new series of WIG20 futures contracts is introduced into trading, which remains listed on the exchange for one year. The expiration days of the derivatives are the third Fridays of March, June, September, and December. Thus, at any given time, four consecutive contract series are simultaneously traded. The series of WIG20 contracts with the nearest expiration date usually exhibits the highest liquidity, while on the expiration day of that series, the subsequent series takes over the dominant position in terms of liquidity.

In order to conduct the analysis of the price discovery process using the VECM and TVECM models, it was necessary to construct a uniform time series covering daily closing prices of futures contracts from January 2018 to December 2024. This series was built on the basis of closing prices of the contracts with the nearest delivery date, whereby the transition to the subsequent series was set at the expiration day of the current one. This method of constructing the series makes it possible to obtain the most reliable representation of prices, taking into account trading volume and the number of open positions in the futures market (see Fassas, Siriopoulos 2018).

Figure 1 presents the time series used in the study. The blue line (WIG20) represents the series of WIG20 index closing prices, while the red line (FW20) represents the series of daily closing prices of the futures contracts on this index, constructed according to the principles described above. Figure 1 illustrates the close relationship between spot and futures price levels, which is reflected in the nearly parallel trajectories of their time series. In many instances, these series overlap, and the differences visible at certain points in time remain relatively small.

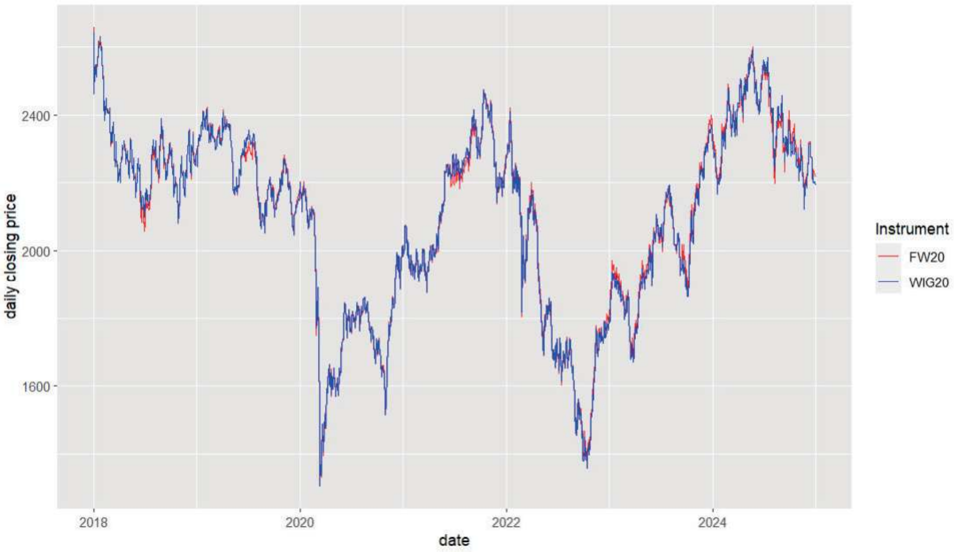


Figure 1. Daily closing prices of the WIG20 index and WIG20 futures (January 2018 – December 2024).

The conducted study used logarithms of closing prices of the WIG20 index (S_t) and its futures contracts (Ft), as well as daily log returns. The following notations of log prices were adopted:

$$s_t = ln\left(S_t\right), f_t = ln\left(F_t\right) \tag{1}$$

and of log returns:

$$\Delta s_t = s_t - s_{t-1},\ \Delta f_t = f_t - f_{t-1} \tag{2}$$

Table A1 in the Appendix reports the basic descriptive statistics for the log returns of the WIG20 index and its futures contracts over the sample period. The interpretation of these results is provided directly below the table. As indicated there, the values of the individual descriptive statistics are very similar across both markets.

3.2. Research methodology

According to the cost-of-carry model, which is the classical economic model used for the valuation of derivative instruments and which assumes the absence of arbitrage opportunities, the fair price F_t of a futures contract at time t is expressed by the formula:

$$F_t = S_t e^{(r-d)(T-t)} \tag{3}$$

In this equation, S_t denotes the price of the underlying instrument, r is the risk-free interest rate, d is the dividend yield, while $T - t$ represents the time remaining until the expiration of the derivative.

Logarithmizing equation (3) on both sides allows us to observe that the cost-of-carry model assumes the existence of a linear relationship between the log prices of spot and futures:

$$f_t = s_t + (r-d)(T-t) \tag{4}$$

In econometric models, the variability of returns, dividends, and the time remaining until contract expiration is often omitted. As a result, relation (3) is simplified by assuming that the time series s_t and f_t are cointegrated of order (1,1). This assumption means that both series are integrated of the first order (that is, they are non-stationary with a stochastic trend, while their first differences, i.e., the log returns $\Delta s_{t,}$ Δf_t are stationary), and that there exists a linear combination of them, ec_t, which is stationary. Without loss of generality, one may assume that in this combination, one of the coefficients equals 1, and therefore, for example, that the futures price is a linear function of the spot price:

$$f_t = \beta_1 s_t + ec_t \tag{5}$$

From equation (5), it follows that the stationary linear combination ec_t, called the error correction term, can be expressed as:

$$ec_t = f_t - \beta_1 s_t \tag{6}$$

The vector of coefficients of this combination, $[1, -\beta_1]$, is called the cointegrating vector.

Classical VECM Model

The classical two-dimensional Vector Error Correction Model (VECM), used to describe the relationships between cointegrated spot and futures prices, takes the following form:

$$\Delta f_t = \mu_f + \alpha_f ec_{t-1} + \sum_{j=1}^{p} \gamma_{fs,j} \Delta s_{t-j} + \sum_{j=1}^{p} \gamma_{ff,j} \Delta f_{t-j} + \varepsilon_{f,t} \tag{7}$$

$$\Delta f_t = \mu_f + \alpha_f ec_{t-1} + \sum_{j=1}^{p} \gamma_{fs,j} \Delta s_{t-j} + \sum_{j=1}^{p} \gamma_{ff,j} \Delta f_{t-j} + \varepsilon_{f,t} \tag{8}$$

In the general case, the *n*-dimensional VECM model takes the following matrix form:

$$\Delta Y_t = \mu + \Pi Y_{t-1} + \sum_{j=1}^{p} \Gamma_j \, \Delta Y_{t-j} + \varepsilon_t \tag{9}$$

where $Y_t = \left[y_{t1}, y_{t2}, \ldots, y_{tn} \right]$ is a vector of non-stationary time series.

In order to examine the existence of cointegration between the non-stationary time series s_t and f_t, and thus confirm the validity of employing the VECM model expressed by equations (7) and (8), one may apply the Johansen test (Johansen 1991, 1992), which is based on the estimation results of the VECM model and requires the computation of two test statistics:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{n} \ln\left(1 - \lambda_i\right) \tag{10}$$

$$\lambda_{max}(r) = -T \ln\left(1 - \lambda_{r+1}\right) \tag{11}$$

The λ_i values appearing in equations (10) and (11) are the eigenvalues of the matrix Π from the general form (9) of the *n*-dimensional VECM model, ordered in ascending sequence. By means of the trace statistic $\lambda_{trace}(r)$, one tests the null hypothesis that the number of distinct cointegrating vectors is not greater than *r* against the alternative that there are more. The maximum eigenvalue statistic $\lambda_{max}(r)$, on the other hand, is used to verify the null hypothesis of the existence of exactly *r* cointegrating vectors, against the alternative that there are $r + 1$. Naturally, in the case of the two-dimensional model described by equations (7) and (8), at most one cointegrating vector may exist. Hence, performing the Johansen test will serve to confirm that such a vector exists, and thus that spot and futures price series are cointegrated of order (1,1).

In the VECM model defined by equations (7) and (8), the coefficients α_s and α_f accompanying the error correction term ec_{t-1} measure the speed of price adjustment in each market following a disturbance in the long-run equilibrium relation defined by equation (5). For example, a value of α_s significantly different from zero would indicate that, in the event of a disruption of this relation, a significant price adjustment takes place in the spot market. This would signal the existence of a significant long-run causal relationship running from the futures market to the spot market, pointing to a substantial contribution of the futures market to the price discovery process. Conversely, a significant value of the coefficient α_f would indicate an analogous causality running in the opposite direction, and thus a significant contribution of the spot market to price discovery, considered in the context of long-run causal relationships.

The examination, based on the VECM model, of the existence of short-run causal relationships between the prices of the derivative and the prices of its corresponding underlying instrument will be based on determining the significance of the parameters $\gamma_{sf,j}$ ($\gamma_{fs,j}$) appearing in equations (7) and (8) with the lagged values of returns. Confirmation of the joint significance of the parameters $\gamma_{sf,j}$ in equation (7) will mean that futures prices are the Granger cause of spot prices. Conversely, the existence of short-run causality running in the opposite direction will be indicated by the significance of the parameters $\gamma_{fs,j}$ in equation (8).

TVECM with three regimes

As mentioned earlier, the TVECM model was introduced by Balke and Fomby (Balke, Fomby 1997) and later extended by Hansen and Seo (Hansen, Seo 2002). The model employs a regime-based approach that allows the adjustment process to vary depending on the size of the deviations from the long-run equilibrium. Estimation of the three-regime TVECM proceeds as follows. In the first step, the cointegrating relation (5) is estimated and, based on the coefficients of the cointegrating vector obtained, the values of the error-correction term ec_t are computed. Subsequently, depending on the value of ec_{t-1}, the values of the time series s_t and f_t are partitioned into three regimes (R_{-1}, R_0, R_1) according to the rule:

$$\begin{cases} ec_{t-1} < \tau_1 \Rightarrow s_t, f_t \in R_{-1} \\ \tau_1 \le ec_{t-1} \le \tau_2 \Rightarrow s_t, f_t \in R_0 \\ ec_{t-1} > \tau_2 \Rightarrow s_t, f_t \in R_1 \end{cases} \tag{12}$$

where $\tau_1 < 0$, $\tau_2 > 0$ are threshold values defined in such a way that:

$$0.05 \le P\left(ec_{t-1} \in R_i\right) \le 0.95 \text{ for } i \in \{-1, 0, 1\}, \tag{13}$$

where P denotes probability. The selection of the optimal thresholds τ_1, τ_2 is based on searching a preliminary grid of values. For each combination of potential thresholds, the parameters of the TVECM model are estimated, and the most optimal of those are chosen, which minimize the sum of squared residuals of the model.

The lower regime R_{-1} will include those observations s_t and f_t, for which, on the previous day ($t - 1$), the futures price was sufficiently undervalued relative to the spot price, whereas the upper regime R_1 will consist of those values s_t, f_t, for which, on the preceding day, the derivative price was correspondingly too high relative to the price of the underlying instrument. The middle regime, in turn, comprises those values of the time series s_t and f_t, for which the error correction term ec_{t-1} falls between the thresholds defining the lower and upper regimes, meaning that deviations from the long-run equilibrium described by equation (5) are moderate.

After partitioning the data into three regimes, the VECM model is estimated in each of them separately with the same lag order p. The TVECM model can therefore be described by the following equations:

$$\Delta s_t = \mu_{s,i} + \alpha_{s,i} ec_{t-1} + \sum_{j=1}^{p} \gamma_{ss,j,i}\, \Delta s_{t-j} + \sum_{j=1}^{p} \gamma_{sf,j,i}\, \Delta f_{t-j} + \varepsilon_{s,t,i} \tag{14}$$

$$\Delta f_t = \mu_{f,i} + \alpha_{f,i} ec_{t-1} + \sum_{j=1}^{p} \gamma_{fs,j,i}\, \Delta s_{t-j} + \sum_{j=1}^{p} \gamma_{ff,j,i}\, \Delta f_{t-j} + \varepsilon_{f,t,i} \tag{15}$$

where $i \in \{-1,0,1\}$,

To prevent overparameterization, the optimal lag length p in both the VECM and TVECM models is selected based on the Bayesian Information Criterion (BIC), with adjustments made if autocorrelation of the residuals is observed. According to the results obtained by Martens and colleagues (Martens et al. 1998), it may be expected that in the TVECM model, the impact of the error correction term on the values of index and futures returns will be considerably smaller in the middle regime, corresponding to the no-arbitrage band, than in the lower and upper regimes. One may therefore expect the following inequalities: $|\alpha_{s,-1}| > |\alpha_{s,0}|$ and $|\alpha_{s,1}| > |\alpha_{s,0}|$ in the equation describing the changes in spot prices as well as $|\alpha_{f,-1}| > |\alpha_{f,0}|$ and $|\alpha_{f,1}| > |\alpha_{f,0}|$ in the equation relating to futures price movements.

In the final step of the analysis, after estimating the VECM and TVECM, the measures of the average contribution of the spot and futures markets to the price discovery process, the common factor weights, were calculated. These measures were first defined by Schwarz and Szakmary (Schwarz, Szakmary 1994) and have subsequently been employed in other studies addressing the issue under consideration (see, e.g., Bohl et al. 2011; Mutlu, Arik 2015; Fassas, Siriopoulos 2019; Suliga 2025):

$$\theta_s = \frac{|\alpha_f|}{|\alpha_s| + |\alpha_f|} \tag{16}$$

$$\theta_f = 1 - \theta_s = \frac{|\alpha_s|}{|\alpha_s| + |\alpha_f|}. \tag{17}$$

As follows from equations (16) and (17), the common factor weights are based solely on the adjustment coefficients α_s and $\alpha_{f,}$ and therefore reflect the average contribution of a given market to the price discovery process only in the context of long-run (error-correction) causal linkages. How should these values be interpreted? The larger the ratio of α_f to α_s in absolute terms, the stronger the response of the futures market (relative to the spot market) to a disturbance from the long-run equilibrium relationship between prices in the two markets. The market that reacts more strongly does not dominate price discovery; rather, it tends to follow the other market, adjusting to changes that have occurred there earlier. A larger absolute ratio α_f / α_s thus translates into a higher value of θ_s, which indicates the spot market's contribution to price discovery. In the abovementioned research, the measures θ_s and θ_f have been computed based on estimates from the classical bivariate VECM. In the present study, they will also be computed within each of the regimes of the threshold VECM (TVECM).

4. Empirical results

The empirical investigation, the results of which are presented in this chapter, followed the structure outlined below. At the outset, whether the analyzed time series satisfy the fundamental assumptions regarding stationarity and cointegration was verified, which are necessary for the application of error-correction models. The main part of the study, conducted after validating these assumptions, was divided into two stages. In the first stage, a classical bivariate vector error correction model was fitted to the data, on the basis of which preliminary conclusions were drawn concerning the contribution of the spot and futures markets to the price discovery process over the period under study. In the second stage, a threshold VECM with three regimes was estimated using the same dataset. The conclusions regarding the roles of the spot and futures markets, derived from the threshold model, were then compared with the results of the first stage of the study in order to assess their consistency.

The results of the Augmented Dickey–Fuller (ADF) test, reported in Table A2 in the Appendix, indicate that the log price series s_t and f_t are nonstationary in levels but stationary in first differences, confirming that they are integrated of order one, I(1).

Table 1 presents the results of the cointegration tests for the series s_t and f_t using Johansen's procedure. The values of both test statistics, λ_{trace} and λ_{max}, clearly indicate that the null hypothesis of no cointegrating vector should be rejected in favor of the alternative hypothesis that the series s_t and f_t are cointegrated. The final column of the table reports the estimated coordinates of the cointegrating vector.

Table 1

Cointegration test results

Johansen test statistics values				Cointegrating vector
λ_{trace}		λ_{max}		
$H_0: r = 0$	$H_0: r \leq 1$	$H_0: r = 0$	$H_0: r = 1$	
40.90^{***}	5.64	35.27^{***}	5.64	[1; −0.987]

Note: The symbol *** denotes statistical significance at the 0.01 level

The results presented in Table A2 and Table 1 confirm that the data satisfy the fundamental assumptions of the VECM framework. In the next step of the analysis, the parameters of the classical VECM specified by equations (7) and (8) were therefore estimated. The Bayesian Information Criterion (BIC) indicated an optimal lag order of 2; however, due to the presence of autocorrelation in the residuals, the lag order was increased to $p = 3$, which resolved this issue. Residuals were examined for heteroskedasticity, which was detected; therefore, parameter significance was assessed using robust Newey–West standard errors. The estimation results of the model are reported in Table 2.

With respect to the coefficients on the error-correction term, only the parameter α_f differs significantly from zero (at the 0.05 significance level). In the context of long-run causal relationships between spot and futures prices, this result indicates the existence of significant unidirectional causality running from the WIG20 index to its futures contracts. This suggests that during the analyzed sample period, in situations where the long-run equilibrium relationship between spot and futures prices was disrupted, significant price adjustment occurred exclusively in the futures market. Hence, it was the spot market that initiated the disequilibrium, while the derivatives market followed the price changes of the index.

Table 2

Estimation results of the VECM model

Equation	Coefficient values								C.f.w.
Δs_t	μ_s	α_s	$\gamma_{ss,1}$	$\gamma_{ss,2}$	$\gamma_{ss,3}$	$\gamma_{sf,1}$	$\gamma_{sf,2}$	$\gamma_{sf,3}$	θ_s [%]
	0.007	−0.065	0.190	0.038	0.031	−0.202	−0.044	0.022	68.33
Δf_t	μ_f	α_f	$\gamma_{ff,1}$	$\gamma_{ff,2}$	$\gamma_{ff,3}$	$\gamma_{fs,1}$	$\gamma_{fs,2}$	$\gamma_{fs,3}$	θ_f (%)
	0.014^{**}	-0.127^{**}	-0.634^{***}	−0.248	−0.034	0.621^{***}	0.256	0.085	31.67

Note: The symbols *** and ** indicate statistical significance at significance levels of 0.01 and 0.05, respectively, based on Newey–West robust standard errors. The last column (C.f.w.) contains the values of the common factor weights

The coefficient $\gamma_{fs,1,}$ which appears in the equation describing changes in futures prices, is statistically significant (at the 0.01 significance level). This indicates that during the analyzed period, the daily return on WIG20 futures contracts depended significantly on the previous day's return of the WIG20 index, thereby confirming the existence of short-run causality running from the spot market to the futures market. The daily futures returns also depended on their own lagged value, as indicated by the significant (at the 0.01 level) coefficient $\gamma_{ff,1}$. It should be noted that in the equation describing Δs_t, none of the coefficients are significantly different from zero. In particular, none of the coefficients indicating potential causality from the futures market to the spot market (α_s, $\gamma_{sf,1}$, $\gamma_{sf,2}$, $\gamma_{sf,3}$) are significant.

The final column of Table 2 presents the values of the common factor weights, computed according to equations (16) and (17). These results suggest that over the period 2018–2024, the average contribution of the spot market to the price discovery process amounted to approximately 68.33%, compared with a contribution from the futures market that was less than half of this.

In summary, the results presented in Table 2 provide no basis for concluding that WIG20 index futures played a significant role in the price discovery process on the Warsaw Stock Exchange during the period under investigation. Instead, they suggest that this process occurred primarily in the spot market, with the derivatives market following it and performing only a secondary function. Thus, the findings are consistent with earlier results reported by Bohl and colleagues (Bohl et al. 2011) for the WIG20 index and its futures contracts. Taking into account prior studies on the role of index futures in price discovery in other emerging markets (Min, Najand 1999; Kenourgios 2005; Andreou, Pierides 2008; Kavussanos et al. 2008; Kavussanos, Visvikis 2011; Guo et al. 2013), the results reported in Table 2 suggest that the Polish market behaves in an atypical way. While in the cases of the Korean, Chinese, and Greek markets, it has been shown that the derivatives market – if not from the outset, then at least with further development – gradually took over the price discovery function from the spot market and expanded its contribution over time, the findings discussed above for the Polish futures market indicate that its role in the price discovery process remains limited despite more than two decades of existence.

However, as noted in the previous section, the classical VECM suffers from important analytical limitations. Above all, as a linear model, it assumes stability of the cointegrating relationship and homogeneity of adjustment mechanisms throughout the entire sample period. In reality, this assumption may be overly restrictive, particularly in the context of dynamically changing market conditions. The sample period – 2018 to 2024 – includes episodes of exceptionally strong economic disturbances, most notably the global COVID-19 pandemic and the outbreak of the war in Ukraine. These events may have significantly influenced market participants' behavior and the structure of the linkages between the spot and futures markets. In particular, the years 2020–2022 were characterized by heightened volatility, uncertainty, and structural instability in financial markets.

Under such conditions, the assumption of a homogeneous, linear adjustment process may lead to distorted inferences. The return to equilibrium between spot and futures markets may have taken different forms during stable and unstable periods, and it may also have been asymmetric – i.e., stronger in response to negative shocks than to positive ones. The classical VECM is incapable of capturing such nonlinearities and regime shifts.

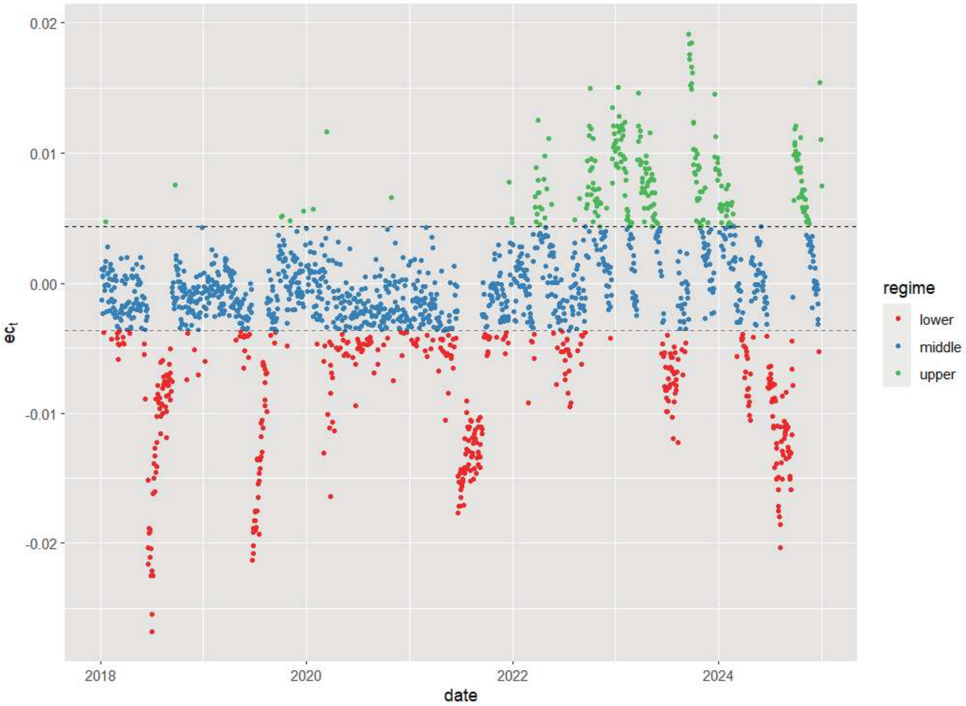
For this reason, the TVECM model was employed in the subsequent stage of the analysis. This framework makes it possible to account for asymmetries and varying adjustment dynamics depending on the extent of the deviation from equilibrium. By allowing for different error-correction mechanisms across regimes, the TVECM may provide a more adequate tool for analyzing periods characterized by instability and structural shocks.

The appropriate specification of the three-regime TVECM was selected on the basis of the Bayesian Information Criterion (BIC). This criterion indicated a TVECM with a lag order of $p = 1$. However, the BIC for the TVECM ($BIC_{TVECM} = -35{,}234.70$) is higher than that for the classical VECM ($BIC_{VECM} = -35{,}339.94$) employed in the previous stage, suggesting that the VECM provides a better fit according to this criterion. The higher BIC value for the TVECM may reflect its greater complexity, as BIC imposes a stronger penalty for additional parameters than AIC.

In contrast, the AIC for the TVECM is lower than for the VECM ($AIC_{TVECM} = -35{,}376.80$, while $AIC_{VECM} = -35{,}247.04$), indicating some support for the TVECM specification. Overall, the mixed evidence from the information criteria suggests that both models provide a comparable fit to the empirical data.

The estimated threshold values appearing in equation (12) amounted to (rounded to three decimal places): $\tau_1 = -0.004$ and $\tau_2 = 0.004$. A total of 26.1% of the observations fell into the lower regime. This means that on approximately one quarter of the trading days under consideration, the error-correction term given by equation (6) was negative and lower than τ_1, implying that the daily closing price of WIG20 futures contracts was undervalued relative to the underlying index. The middle regime comprised 59.5% of the observations, indicating that on most of the trading days, the spot and futures prices did not deviate largely from their long-run equilibrium relationship. Finally, 14.4% of the observations fell into the upper regime, in which the error-correction term ec_t was positive and greater than τ_2, signifying that the WIG20 futures contracts were overpriced relative to the index.

The division of observations across regimes is illustrated in Figure 2. The vast majority of observations are located in the middle regime (blue), corresponding to minor deviations from equilibrium, where adjustment mechanisms are likely to remain inactive. The lower regime (red) is more numerous than the upper regime (green), suggesting that instances of futures undervaluation occurred more frequently than cases in which futures were overvalued relative to the spot market during the sample period. This may point to asymmetric arbitrage costs or the existence of other market frictions. Horizontal dashed lines have been added to the figure to clearly separate the three regimes.



Note: The lower regime corresponds to the values $ec_t < -0.004$, while the upper regime includes the values $ec_t > 0.004$. The values $ec_t \in [-0.004; 0.004]$ determine the middle regime

Figure 2. Division of correction error term values ec_t across the three regimes

The results of the TVECM estimation are presented in Table 3. The residuals were found to be free of autocorrelation but exhibited heteroskedasticity; consequently, the statistical significance of the estimated parameters was evaluated using robust Newey–West standard errors. In the upper part of Table 3 (Panel A), the estimated parameter values of the model corresponding to the lower regime are reported. Values of both coefficients related to the error correction term (α_s and α_f are not significantly different from zero. This suggests that when futures prices are undervalued relative to the WIG20 index, neither the spot nor the futures market responds in a systematic way to restore equilibrium. The error correction mechanism appears to be locally inactive.

This finding may be explained by the presence of some market frictions that limit arbitrage activity. As a result, observed mispricing may not be perceived as sufficiently profitable relative to the associated risk. Furthermore, the absence of a significant error correction mechanism may be linked to the turbulent market conditions observed during the sample period, including the COVID-19 pandemic and the outbreak of the war in Ukraine. Periods of heightened uncertainty and volatility are typically associated with reduced liquidity and increased risk aversion, which can limit arbitrage activity and lead to a temporary breakdown of the equilibrium adjustment process.

Moreover, Figure 2 reveals clustering of observations within the lower regime. This indicates that deviations from equilibrium are persistent rather than transitory, as the system tends to remain in this regime for several consecutive periods once the threshold is crossed. Such regime persistence is consistent with the insignificance of the error correction coefficients and suggests that the adjustment process is not only weak but also delayed.

The statistically significant $\gamma_{fs,1}$ and $\gamma_{ff,1}$ (at the 0.1 and 0.01 levels, respectively), which appear in the lower regime in the equation related to changes in futures prices, indicate that the daily return on WIG20 index futures depended on both its own lagged value and on the previous day's return of the index. The value of the coefficient $\gamma_{fs,1}$ in particular indicates the potential presence of unidirectional short-run causality: if futures prices are too low relative to the closing value of the WIG20 index on a given day, only the futures market reacts to the information contained in spot prices the following day. Such a unilateral transmission of information between the markets was also detected in the first part of the study, as the classical VECM model indicated short-run causality running only from the WIG20 index prices to the futures prices.

Panel B, constituting the middle part of Table 3, presents the results of the TVECM parameter estimation obtained for the middle regime. As in the lower regime, the values of both coefficients related to the error correction term (α_s and α_f are statistically insignificant. This result is consistent with expectations, since the middle regime corresponds to observations for which the value of the error correction term was close to zero, and thus no significant deviation from the long-run equilibrium between the spot and futures markets was present. Under such conditions, the adjustment mechanism toward equilibrium was not activated. According to the suggestions of Ters and Urban (Ters, Urban 2020), this regime defines the no-arbitrage band, within which the basis of the futures contract is too small (in absolute terms) for arbitrage transactions to be financially viable.

Table 3

Estimation results of the TVECM model

Equation	Coefficient values				C.f.w.
Panel A: lower regime					
Δs_t	μ_s	α_s	$\gamma_{ss,1}$	$\gamma_{sf,1}$	θ_s [%]
	0.002	0.106	0.153	−0.350	58.09
Δf_t	μ_f	α_f	$\gamma_{ff,1}$	$\gamma_{fs,1}$	θ_f [%]
	0.003^{**}	0.147	-0.714^{***}	0.521^{*}	41.91
Panel B: middle regime					
Δs_t	μ_s	α_s	$\gamma_{ss,1}$	$\gamma_{sf,1}$	θ_s [%]
	-0.001^{*}	−0.033	0.254	−0.149	88.86
Δf_t	μ_f	α_f	$\gamma_{ff,1}$	$\gamma_{fs,1}$	θ_f [%]
	-0.001^{**}	−0.266	-0.451^{**}	0.549^{***}	11.14
Panel C: upper regime					
Δs_t	μ_s	α_s	$\gamma_{ss,1}$	$\gamma_{sf,1}$	θ_s [%]
	0.006^{**}	-0.501^{**}	−0.199	0.056	51.28
Δf_t	μ_f	α_f	$\gamma_{ff,1}$	$\gamma_{fs,1}$	θ_f [%]
	0.005^{**}	-0.528^{**}	−0.278	0.151	48.72

Note: The symbols ***, ** and * indicate statistical significance at significance levels of 0.01, 0.05 and 0.1, respectively, based on Newey-West robust standard errors. The last column (c.f.w.= common factor weights) reports the values of the common factor weights defined in equations (16) and (17)

It is worth noting that the middle regime accounts for about 60% of all observations. This means that on the majority of days covered by the analysis, no significant disturbance of the equilibrium relation between the markets occurred, and the correction mechanism remained inactive. This fact casts doubt on the results obtained in the first stage of the study using the classical VECM model, in which it was assumed that the adjustment mechanism operates regardless of the scale of the deviation from the long-run equilibrium between spot and futures prices.

With respect to the coefficients $\gamma_{ss,1}$ and $\gamma_{ff,1,}$ it is noteworthy that in the middle regime – similarly to the previous regime – only $\gamma_{ff,1}$ is statistically significant (at the 0.01 level). This suggests that, during periods of relative stability between the markets, futures returns depend on their own lagged values, whereas this is not the case for WIG20 index returns.

For the short-term relationships between the markets, only the coefficient $\gamma_{fs,1}$ is statistically significant (at the 0.01 level). This indicates, once again, the presence of unidirectional short-run causality running from the spot market to the futures market: futures prices respond to past values of the underlying index, while no reverse relationship is observed. These findings are also consistent with the results obtained from the classical VECM model, which also indicated short-run causality running from the spot market to the futures market.

The lower part of Table 3 (Panel C) presents the estimation results of the TVECM parameters corresponding to the upper regime. In this case, both error correction coefficients, α_s and $\alpha_{f,}$ are significantly different from zero at the 0.05 level. This regime corresponds to a positive term ec_t, meaning that $f_t > \beta_1 s_t$. The negative estimate of ($\alpha_f = -0.528$) is justified. It means that futures prices decrease to get back to the equilibrium state. However, the estimate α_s was also obtained as negative ($\alpha_s = -0.501$). It means that in the situation of a large positive error correction term, the spot prices also decrease, which is not conducive to restoring an equilibrium state. Bohl and colleagues (Bohl et al. 2011) suggest that it is possible that the error correction coefficients of the spot and futures markets have the same sign. However, to restore the equilibrium in the long run, the inequality $|\alpha_s| < |\alpha_f|$ should hold. This holds in our case in absolute values, but the test of the statistical difference between α_s and α_f does not provide grounds to reject the null hypothesis of equality of the coefficients. The findings thus are contrary to the standard expectations of the error correction mechanism and suggest that both the spot and futures markets respond in a similar direction to deviations from equilibrium. Instead of correcting the deviation, both markets appear to move in parallel, which may prevent the spread from narrowing in a systematic way. This suggests a breakdown of the classical arbitrage-based error correction mechanism in the upper regime.

One possible explanation for this behavior is the presence of common informational shocks affecting both markets simultaneously. In periods of heightened volatility, such as during financial or geopolitical crises, arbitrage activity may become constrained due to increased risk or market frictions. As a result, prices on both markets may adjust concurrently to new information rather than through a mechanism aimed at restoring equilibrium. As can be seen in Figure 2, the upper regime consists predominantly of observations from the period 2022–2024, corresponding to the time of the war in Ukraine. This confirms that the regime may be associated with increased uncertainty. Moreover, similarly to the lower regime, a clear clustering of observations can be observed. This indicates that once the system enters this regime, it tends to remain there for several consecutive periods, which suggests that the adjustment process in the regime is delayed or weakened rather than immediate. This is consistent with the obtained α_s and α_f values. The results therefore point to the dominance of common factors driving both markets, rather than the operation of an effective arbitrage mechanism linking them.

With respect to short-run dynamics, none of the estimated coefficients are statistically significant. This indicates that daily returns of both the WIG20 index and its futures contracts do not depend on either their own lagged values or the lagged returns observed in the other market.

This lack of short-run interdependence further supports the interpretation that price dynamics are driven primarily by contemporaneous informational shocks in the upper regime rather than by systematic interactions between the two markets. In other words, standard short-run transmission mechanisms appear to be dominated by exogenous factors affecting both markets simultaneously.

The last column of Table 3 contains the values of the common factor weights, calculated according to formulas (16) and (17), separately for each regime. In both the upper and lower regimes, the two coefficients – θ_s and θ_f – are close to 50%, although a slightly higher contribution to price discovery is attributed to the spot market in both cases. The values of θ_s and θ_f computed in the middle regime amounted to 80.19% for the spot market and only 19.81% for the futures market. These results differ substantially from those based on the classical VECM specification (see Table 2), which suggested a dominant role of the spot market in the price discovery process throughout the entire sample period. This discrepancy highlights an important limitation of the linear VECM framework, which assumes a constant adjustment mechanism and stable relationships between markets over time.

In contrast, the TVECM model reveals that the error correction mechanism is either weak, statistically insignificant, or operates in a non-standard way in different regimes. As a result, the process of restoring long-run equilibrium cannot be regarded as uniform across market conditions. Common factor weights rely on the existence of a well-functioning error correction mechanism. However, in the regimes where such a mechanism is absent or ineffective, the estimated weights cannot be considered reliable indicators of the relative contributions of the spot and futures markets to price discovery. The results suggest that the linear VECM model, although it provides a convenient summary measure, oversimplifies the underlying dynamics between the stock and futures markets by averaging across fundamentally different market states.

When comparing the values of the coefficients related to the error correction term across regimes, the following inequalities can be observed: $|\alpha_s^1| > |\alpha_s^{-1}| > |\alpha_s^0|$ and: $|\alpha_f^1| > |\alpha_f^0| > |\alpha_f^{-1}|$. These inequalities between α_f values are not consistent with earlier expectations that the absolute values of the error correction coefficients in the lower and upper regimes should be higher than in the middle regime. Once again, these results highlight the non-standard behavior of the error correction coefficients across regimes, deviating from the conventional expectations.

When comparing the TVECM estimation results with the findings of similar studies conducted for the U.S. market by Martens and colleagues (Martens et al. 1998) and Kim and colleagues (Kim et al. 2010), partial consistency can be observed with respect to the existence of a no-arbitrage band. The results of the present study suggest that such a band may also be present in the Polish market; however, the adjustment mechanism is not uniformly activated once the threshold is exceeded as deviations from equilibrium are not always effectively eliminated in either the upper or the lower regime.

Furthermore, the results obtained for the WIG20 index and its futures differ from those reported for the S&P 500 index and its derivatives with regard to the price discovery process. While studies for the U.S. market point to a dominant role of the futures market, the evidence for the Warsaw Stock Exchange suggests that price discovery is not stable over time and varies across regimes. In particular, the spot market appears to play a leading role in some regimes, while the relationship between the markets weakens or is dominated by common informational shocks in others. Overall, the findings indicate that, unlike in more mature markets, the price discovery process in the Polish market is less efficient and more sensitive to changing market conditions.

5. Summary and conclusions

The aim of the study was to compare the usefulness of the classical Vector Error Correction Model (VECM) and its nonlinear threshold version (TVECM) in analyzing the price discovery process on the Warsaw Stock Exchange. The study covered daily data on the closing prices of the WIG20 index and its futures contracts from 2018 to 2024, thus encompassing both periods of relative market stability and periods of pronounced market turbulence associated with the outbreak of the COVID-19 pandemic and the war in Ukraine. Such a broad time horizon made it possible to assess to what extent the two models are capable of capturing the mechanisms of price discovery under conditions of volatility, asymmetry, and potential structural changes.

The estimation results of the classical VECM model indicated unequivocally the dominant role of the spot market in the price discovery process. Both the analysis of the error correction coefficients and the investigation of short-run causality suggested that in the years 2018–2024, it was the WIG20 index that initiated changes, while the futures contracts followed. The measures of the contribution of spot and futures markets to the price discovery process (common factor weights) suggested that the average contribution of the spot market to price discovery was more than twice as high as that of the futures market. These findings confirmed earlier results in this respect (Bohl et al. 2011) and implied a secondary role for WIG20 futures contracts in the price discovery process.

However, when analyzing the estimation results of the classical VECM model, attention was drawn to its limitations stemming from its linear nature and the assumption of constant adjustment mechanisms over the entire sample period. It was therefore hypothesized that the conclusions derived from the VECM model might be oversimplified, particularly in light of the crises that occurred in the market during the analyzed period. To verify this hypothesis, the threshold TVECM model was employed in the subsequent stage of the study, which made it possible to incorporate nonlinearity and differentiated market reactions depending on the magnitude and direction of deviations from equilibrium.

The estimation results of the TVECM model revealed that the error correction mechanism behaves differently and, at least in part, unexpectedly. In line with theoretical expectations, no significant adjustment toward equilibrium was observed in the middle regime, which can be interpreted as a no-arbitrage band. However, no significant adjustment toward the long-run equilibrium was observed in the lower regime, while both markets reacted in a similar direction to deviations in the upper regime, which did not lead to an effective reduction of disequilibrium. These findings indicate that deviations from equilibrium may persist over time and are not necessarily eliminated through the classical arbitrage mechanism.

Moreover, the regime-based analysis showed that short-run relationships between the spot and futures markets are unstable. In the lower and middle regimes, short-run causality from the spot market to the futures market was observed, while no significant interactions were detected in the upper regime, indicating that price transmission mechanisms cannot be adequately described by a single, stable structure.

A comparison of the two models highlights that the VECM tends to oversimplify dynamics by imposing a constant adjustment mechanism, whereas the TVECM captures regime-dependent variations in both the error correction process and short-run interactions. Nevertheless, even the TVECM may not fully account for price discovery during highly turbulent periods, reflecting the inherent limitations of both models.

Importantly, the results indicate that the contribution of the spot and futures markets to price discovery is not stable over time. While the VECM suggests a dominant role of the spot market, the TVECM indicates that this dominance is conditional and may vary across regimes, with extreme market turbulence potentially driven by common informational shocks rather than classical arbitrage mechanisms.

Finally, it should be noted that the study's results are subject to important limitations stemming from the use of daily data, which were employed due to the lack of access to high-frequency intraday data. While daily data are suitable for analyzing long-term relationships, in modern markets, the informational leadership of a given market in the price discovery process can manifest within minutes, seconds, or even milliseconds. For example, Hasbrouck showed for the U.S. market that informational contributions to price discovery, which remain unclear at a one-second frequency, may only be revealed at the sub-millisecond level (Hasbrouck 2021).

In recent years, most studies have relied on high-frequency intraday data to analyze price discovery and the relative informational contributions of spot and futures markets (e.g., Aggarwal, Thomas 2019; Baur, Dimpfl 2019; Zhou et al. 2021; Han et al. 2025). Daily data cannot capture the information flows with all the details or the true speed of price adjustments across markets. This limitation is particularly relevant during periods of heightened market volatility, such as the COVID-19 pandemic and the outbreak of the war in Ukraine, when intraday information transmission was rapid and markets were highly reactive. Therefore, the findings of this study should be interpreted with caution with respect to the high-frequency dynamics of the price discovery process. Future research will aim to extend this analysis using high-frequency intraday data, which would allow for a more precise assessment of the speed and microstructural dynamics of price discovery between the spot and futures markets.

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Appendix

Table A1 presents the basic descriptive statistics for the log returns of the WIG20 index and its futures contracts. The results indicate that the individual descriptive statistics are very similar across both markets. Both return series exhibit negative skewness, while their high kurtosis values suggest a strong concentration of observations around a mean close to zero. Moreover, the significantly positive Jarque–Bera test statistics at the 0.01 significance level indicate that neither distribution can be considered normal.

Table A1

Descriptive statistics for the log returns of the WIG20 index and its futures contracts in the years 2018–2024

Statistics	Time series	
	Δs_t	Δf_t
mean	−0.0001	−0.0001
median	−0.0001	−0.0004

Table A1 cont.

minimum	−0.1425	−0.1275
maximum	0.0810	0.0900
standard deviation	0.0148	0.0152
skewness	−0.7512	−0.5862
kurtosis	11.6772	10.3662
J-B test statistic	5648.3***	4052.1***
N	1748	1748

Note: The symbol *** denotes statistical significance at the 0.01 level

Table A2 reports the results of the stationarity tests for the log prices of the WIG20 index and its corresponding futures contracts, as well as for the log returns. The Augmented Dickey–Fuller (ADF) test statistic obtained for the price series s_t and f_t is not statistically significant for all variants of the ADF test. This implies that there is no basis for rejecting the null hypothesis that the series of log spot and futures prices are nonstationary and exhibit a stochastic trend. By contrast, the ADF test applied to the first differences confirms the stationarity of the log returns Δs_t and Δf_t, as test statistics reported on the right-hand side of the table are statistically significant (at the 0.01 or 0.05 significance levels, respectively). The results presented in Table A2 therefore confirm that the series s_t and f_t are integrated of order one, I(1).

Table A2

Results of stationarity tests for the time series of log prices and log returns of the WIG20 index and its futures contracts

Alternative hypothesis H_1	ADF test statistic			
	s_t	f_t	Δs_t	Δf_t
no drift no trend	−0.349	−0.331	−42.9***	−44***
drift	−2.57	−2.66	−42.9**	−44***
drift and trend	−2.51	−2.61	−42.9***	−44***

Note: The symbols *** and ** indicate statistical significance at significance levels of 0.01 and 0.05, respectively

Summary

This study evaluates and compares the usefulness of the classical Vector Error Correction Model (VECM) and the Threshold Vector Error Correction Model (TVECM) in analyzing the price discovery process on the Warsaw Stock Exchange. The empirical analysis uses daily data on the WIG20 index and its futures contracts from 2018 to 2024. The VECM results indicate unidirectional long-run and short-run causality from the spot market to the futures market, with the latter primarily adjusting to deviations from equilibrium. The estimated common factor weights suggest that the spot market accounts for about two-thirds of the overall price discovery. Based on these findings alone, one might conclude that the dominance of the spot market is stable and persistent. However, the TVECM reveals substantial nonlinearities and regime-dependent dynamics that challenge this conclusion. It identifies three regimes, corresponding to undervaluation, near-equilibrium, and overvaluation of futures, within which the adjustment mechanisms differ notably. In both the lower and middle regimes, the error correction mechanism is weak or statistically insignificant, indicating that deviations from equilibrium are not systematically eliminated. In particular, the middle regime, which accounts for the majority of observations, can be interpreted as a no-arbitrage band in which mispricing is too small to trigger arbitrage activity. In the upper regime, although both markets respond to deviations, their adjustments occur in the same direction, preventing the restoration of equilibrium and suggesting a breakdown of the classical arbitrage mechanism. This behavior may reflect the presence of common informational shocks and heightened market uncertainty rather than a stable lead–lag relationship between the markets. The comparison demonstrates that while the VECM provides a convenient summary of average relationships, it oversimplifies the underlying dynamics by assuming a constant adjustment process. The TVECM offers a more informative framework by capturing regime-specific behavior and revealing that the price discovery process is unstable, asymmetric, and sensitive to market conditions. These findings highlight the importance of nonlinear approaches in analyzing financial market dynamics, particularly in periods of increased volatility.

JEL codes: C32, C58

Keywords: *price discovery, Granger causality, futures on WIG20 index, VECM model, TVECM model*