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Somdeb Lahiri*

A statistical interpretation of a market demand curve for a commodity obeying the law of demand

1. Introduction

It is well known that in demand theory "budget constrained utility maximization" implies the weak axiom of revealed preference. Kihlstrom, Mas-Colell and Sonnenschein (1976) investigate demand theory based on the assumption that the demand function satisfies the weak axiom of revealed preference. Kihlstrom, Mas-Colell and Sonnenschein show that satisfaction of the weak axiom of revealed preference implies the "law of compensated demand", i.e., quantity demanded of a commodity, changes in a direction that is opposite to the direction of a price change, provided the initial consumption bundle is exactly affordable (i.e. budget-balance for the initial consumption bundle) at the new price. This follows, since if the original bundle is affordable but not chosen at new prices, then by the weak axiom of revealed preference, at original prices the new consumption bundle is required to be unavailable. We provide an independent derivation of this result in Appendix A of this paper. The converse of this result holds provided the Marshallian demand function satisfies "budget balance". For a normal good, the "law of compensated demand" applied to the Slutsky equation for own prices, implies the "law of demand", i.e., quantity demanded of a commodity, changes in a direction that is opposite to the direction of a price change. Thus, the weak axiom of revealed preference implies the "law of demand" for normal goods. A very lucid account of the related demand theory is available in chapter 2 of Mas-Colell, Whinston and Green (1995). A comprehensive discussion of demand theory, using money as a numeraire, and the related welfare economic theory of consumer surplus when the demand function satisfies the "law of demand" is

* PDEU, India, e-mail: somdeb.lahiri@gmail.com

rigorously presented in Lahiri (2022a, 2022b), the genesis of which is available in Lahiri (2020).

In this note we provide a statistical interpretation of the Marshallian market demand curve of a commodity which obeys the "law of demand" and whose unit of consumption is fixed (e.g., single units of refrigerator, air conditioner or full tank capacity of fuel oil of a Maruti 800) and which has a finite and positive level of satiation. Alternatively, we could consider the demand function to be a "compensated market demand function", except in section 5, where we discuss the Marshallian demand curve generated by "budget constrained linear utility maximization". The "finite and positive level of satiation" means that allowing for free disposability of the good, there is a finite positive amount of the good after which the consumers are not willing to pay anything for incremental units of the good. We refer to this positive level of satiation as "market size". The market size may depend on the tastes and preferences and income distribution of the consumers as well as the prevailing prices of other goods and services. Further, suppose that the unit of consumption is infinitesimally small compared to the market size. This latter assumption allows the good to be viewed as "homogeneous and infinitely divisible", as far as the demand analysis of the good is concerned. An interesting consequence of the statistical approach that we adopt in this paper is that in the context of two goods, we are able to obtain demand functions which are very similar to those obtained by "budget-constrained Cobb-Douglas utility maximization", but now as a result of a "budget-constrained linear utility maximization" exercise, although our budget constraint is "slightly different" from the one that would be used for the former optimization problem. This is discussed in section 5 of this paper.

A standard reference for what follows is Peterson and Lewis (1999). For alternative and more advanced demand theory one may refer to Katzner (1970, 2008), Lahiri (2022a, 2022b).

2. The model

Suppose $\bar{q} > 0$ is the "market size". In what follows, we can assume without loss of generality that $\bar{q} = 1$.

A "reservation price" for a particular unit of the good in the market is the maximum amount that some consumer (in the market) would be willing to pay for that particular unit of the good. We allow for the same buyer to have different reservation prices for successive units of the good. Let $F:[0, +\infty)\rightarrow[0, 1]$ be a function, with at most a finite number of points of discontinuity, denoting the "distribution function of the reservation prices" for the good in the market, where F may depend on the tastes, preferences and income distribution of the consumers as well as the prevailing prices of other goods and services. Thus for $p \in [0, +\infty)$, the reservation prices for first $1 - F(p)$ units of the good – and not more – are greater than or equal to 'p'. Thus at price $p \in [0, +\infty)$, consumers would be willing to buy at most $q(p) = 1 - F(p)$ units of the good.

In what follows, we assume the following, which is known as the "uncompensated" Law of Demand: F is strictly increasing on the set $\{p \in [0, +\infty) \mid 0 < F(p) < 1\}$.

Thus the function $1 - F(\cdot)$ is strictly decreasing on the set $\{p \in [0, +\infty) \mid 0 < F(p) < 1\}$.

Clearly the function F is invertible with the inverse function $F^{-1}:[0, 1) \rightarrow [0, +\infty)$ being strictly increasing on (0, 1) and satisfying $F^{-1}(0) = 0$.

Consider the function $P:(0, 1] \rightarrow [0, +\infty)$ defined by $P(q) = F^{-1}(1 - q)$.

The function P (which may depend on prices of other goods and services as well as the distribution of income of the consumers) is the "(Marshallian) statistical demand curve" for the good. In what follows we will refer to P as the "demand curve" for the good.

For $q \in (0, 1]$, P(q) is the maximum price the consumers are willing to pay for q units of the good. This is possible if and only if P(q) is the "lowest" reservation price when q units – and no more – of the good is bought in the market. Units of the good in excess of the first q units have reservation prices less than P(q) and hence, are not bought, at price P(q).

Let $W: [0, 1] \rightarrow [0, +\infty)$ with $W(0) = 0$ be a function such that for all $q \in (0, 1]$, W(q) is the total amount of the good that consumers are willing to pay for q units of the good. W is the "**willingness to pay function**".

Clearly W may depend on tastes, preferences and the distribution of income of the consumers as well as the prices of other goods and services.

At price $p \geq 0$ and quantity of the good $q \in (0, 1]$ the "consumers' surplus" is given by $W(q) - pq$.

The consumers are said to be "surplus maximizers" if for all $p > 0$, such that $0 < F(p) < 1$, $q(p) = [1 - F(p)]$ solves

Maximize $W(q) - pq$

s.t. $0 < q < 1$.

We know that at price p, only those units of the good will be bought whose reservation prices are greater than or equal to 'p' – no more and no less.

In the following sections we compute the willingness to pay functions for linear and piece wise linear demand curves for surplus maximizing consumers.

3. Linear Demand Curves

Recall that if the consumers are surplus maximizers, then for all $p > 0$, such that $0 < F(p) < 1$, $q(p) = 1 - F(p)$ solves

Maximize $W(q) - pq$

s.t. $0 < q < \bar{q}$.

For a real number $a > 0$, let $F(p) = \frac{p}{a}$ for all $p \in [0, a]$ and $F(p) = 1$ for all $p > a$.

Then $q(p) = 1 - \frac{p}{a}$ for all $p \in [0, a]$, and $q(p) = 0$ for all $p > a$.

Hence, $P(q) = a - aq$ for $q \in (0, 1]$.

Let q belong to the open interval (0, 1).

The market's total willingness to pay for q units denoted by $W(q) = aq - \frac{a}{2}q^2$, for $q \in (0, 1]$.

A derivation of this result without using calculus is provided in Appendix B of this paper.

It is easy to see that for $q \in (0, 1]$, $W(q)$ is the area under the straight-line $P(q') = a - aq'$ from '0' to 'q'.

Let $p > 0$ be the price of the good. We know that for $p \geq a$, the quantity demanded is zero. Hence suppose $p < a$.

The surplus obtained from consuming q units of the good, where $q \in (0, 1)$ is given by $(a - p)q - \frac{a}{2}q^2 = -\frac{a}{2}(q^2 - 2(1 - \frac{p}{a})q) = -\frac{1}{2}[(q - (1 - \frac{p}{a}))^2 - (1 - \text{—})^2]$.

The surplus is maximized for the value of q that minimizes $(q - (1 - \frac{p}{a}))^2 - (1 - \frac{p}{a})^2$ subject to $q \in (0, 1)$. Thus, the surplus is maximized at $q = 1 - \frac{p}{a}$, i.e., the point $q \in (0, 1)$ satisfying $P(q) = p$.

The interesting thing to note is that for $q \in (0, 1]$, $W(q) = aq - \frac{a}{2}q^2 = aq - aq^2 + + \frac{a}{2}q^2 = \frac{(a + (a - aq))q}{2}$ = area of trapezium below the demand curve from 0 to q.

Note: It is easy to see that for the linear demand curve $W(q) = \int_0^q (a - aq')dq' = = aq - \frac{a}{2}q^2$ for $q \in (0, 1]$.

4. Piece-wise Linear Demand Curves

For a positive integer $n \geq 2$, let $a_0 > a_1 > \ldots > a_n = 0$ (the entire array of prices possibly depending on the income distribution of the consumers as well as the prevailing prices of other goods and services) and let $0 = q_0 < q_1 < \ldots < q_n = 1$ (the entire array possibly depending on the tastes, preferences and income distribution of the consumers as well as the prevailing prices of other goods and services) be such that for all $j \in \{0, \ldots, n\}$, q_j is the quantity demanded at price a_j. Let $G(p) = 0$ for all $p \geq a_0$, $G(p) = (a_0 - p)q_1$ for all $p \in (a_1, a_0]$, and for all $j \in \{1, \ldots, n - 1\}$

let $G(p) = G(a_j) + \frac{(a_j - p)}{(a_j - a_{j+1})}(q_{j+1} - q_{(j)})$ for all $p \in (a_j, a_{j+1}]$. Let $F(p) = 1 - G(p)$ for all $p \in (0, a_0]$.

Thus, for $q \in (0, 1]$, the reservation prices for units corresponding to the quantity demanded q are greater than or equal to $P(q) = a_j - \frac{(q - q_j)}{(q_{j+1} - q_j)}(a_j - a_{j+1})$ if $q \in (q_j, q_{j+1}]$.

By an argument similar to the one used for linear demand curves, we get that the market's willingness to pay for 'q' units of the good is measured by "the area of the polygon below the demand curve from 0 to q".

Thus, $W(q) = \frac{1}{2}q(2a_0 - \frac{a_0 - a_1}{q_1}q)$ if $q \in (q_0, q_1]$

and for $q \in (q_j, q_{j+1}]$ with $j \in \{1, \ldots, n-1\}$, $W(q) = \sum_{(k=1)}^{j-0} \frac{(q_{k+1} - q_{(k)})(a_{k+1} + a_k)}{2}$ +

$+ \frac{1}{2}(q - q_j)(2a_j - \frac{(q - q_j)}{(q_{j+1} - q_j)}(a_j - a_{j+1}))$.

5. Linear utility maximization

Of considerable interest is the demand function obtained by budget constrained "linear" utility maximization subject to a satiation constraint. In order to present the result in its full generality, in this section we will relax the assumption the market size, $\bar{q}$ is 'one' and allow it to be any positive real number. Let $\mu > 0$ be the money available to the consumers who are willing to pay a maximum price $u > 0$ for the commodity. The commodity has the feature of a "quasi-essential" good, so that up to the market size $\bar{q}$, a strictly positive share $\alpha < 1$ of the entire amount of money that is available, is spent on the good, after which if there is any money left, that is used for the consumption of other goods and services. We assume that $u > \frac{\alpha\mu}{\bar{q}}$ and the distribution of reservation prices has a discontinuity at u. Thus, $F:[0, +\infty) \rightarrow [0, 1]$ is defined as follows:

For $p \in (u, +\infty)$, $F(p) = 1$; for $p \in \left[\frac{\alpha\mu}{\bar{q}}, u\right]$, $F(p) = 1 - \frac{\alpha\mu}{p\bar{q}}$; and for $p \in \left[0, \frac{\alpha\mu}{\bar{q}}\right)$, $F(p) = 0$.

The associated demand function, which is very similar to the one generated by the Cobb-Douglas utility function within the price range $\left[u, \frac{\alpha\mu}{\bar{q}}\right]$, is obtained as an optimal solution to the following maximization problem:

Choose 'q' to

Maximize $u(\min\{q, \bar{q}\}) + y$

s.t. $y + pq \leq \alpha\mu$,

$q \geq 0$, $y \geq 0$.

The interval on which the demand functions generated by the above maximization exercise coincides with those generated by "budget-constrained Cobb-Douglas utility maximization" expands as u and/or $\bar{q}$ increases.

6. Surplus maximization using calculus

In this section we use (Newtonian) calculus to obtain the relationship between the demand curve and the willingness to pay function. Clearly, the results in this section are not applicable for piecewise linear demand curves.

Assumption 1: $\int_0^q P(q')dq'$ exists and $0 < \int_0^q P(q')dq' < +\infty$ for all $q \in (0, \bar{q}]$.

Assumption 2: There exists a differentiable function $W: (0, 1] \rightarrow [0, +\infty)$ with $W':(0, 1) \rightarrow [0, +\infty)$ (where $W'(q) = \frac{dW(q)}{dq}$ for all $q \in (0, 1)$) positive valued and strictly decreasing, that gives for each $q \in (0,1]$ the buyers' willingness to pay for q units of the commodity, the latter possibly depending on the tastes, preferences, income distribution of the buyers as well as prices of other goods.

Theorem 1: The consumers are surplus maximizers if and only if for all $q \in (0, q)$, $W'(q) = \frac{dW(q)}{dq} = P(q)$.

Proof: It is easy to see that if consumers are surplus maximizers, then for all $q \in (0, 1)$, $W'(q) = \frac{dW(q)}{dq} = P(q)$.

Hence suppose that for all $q \in (0, 1)$, $W'(q) = \frac{dW(q)}{dq} = P(q)$.

Let $p > 0$, such that $0 < F(p) < 1$, and suppose $q^0 \in (0, 1)$ solves

Maximize $W(q) - pq$

s.t. $0 < q < 1$.

Then clearly, $W'(q^0) = \frac{dW(q^0)}{dq} = p$.

However, by assumption $W'(q^0) = \dfrac{dW(q^0)}{dq} = P(q^0)$.

Thus, $P(q^0) = p$.

Since P is strictly decreasing (follows from $W'(q) = \dfrac{dW(q)}{dq} = P(q)$ for all $q \in (0, 1)$ and W' is strictly decreasing) $p = P(q^0) = F^{-1}(1 - q^0)$, i.e. $q^0 = 1 - F(p)$.

This proves the theorem. Q.E.D.

An immediate corollary of the above is the following.

Corollary of theorem 1: The consumers are surplus maximizers if and only if for all $q \in (0, 1)$, $W(q) = \int_0^q P(q')dq'$.

Applying a change of variable theorem argument to the above, we get $W(q(p)) = \int_p^{+\infty} q(p')dp' + pq(p) = \bar{q}\int_p^{+\infty} 1 - F(p')]dp' + pq(p)$

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Appendix A

For $L \geq 2$, let D: $(\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++} \to \mathbb{R}^{L}_{+}$ be the demand function for L goods of which the first L – 1 goods are non-monetary goods and the L^{th} good is monetary savings for the future or monetary savings for non-monetary goods other than those considered in one of the first L – 1 goods. The price of money is fixed at '1'. At price vector $p \in (\mathbb{R}^{L-1}_{++} \times \{1\})$ and monetary value of wealth $\mu \in \mathbb{R}_{++}$, $D(p, \mu)$ is the vector denoting the quantities that are demanded for each of the L goods – monetary as well as non-monetary.

We assume that D satisfies the **Weak Axiom of Revealed Preference (WARP)**, i.e., for all (p^0, μ^0), $(p^1, \mu^1) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++}$: $[D(p^0, \mu^0) \neq D(p^1, \mu^1)$ and $p^{0T}D(p^1, \mu^1) \leq \mu^0]$ implies $[p^{1T}D(p^0, \mu^0) > \mu^1]$.

Let H: $(\mathbb{R}^{L-1}_{++} \times \{1\}) \times (\mathbb{R}^{L}_{+} \setminus \{0\}) \to \mathbb{R}^{L}_{+}$ be the function defined by

$H(p, x) = D(p, p^Tx)$ for all $(p, x) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times (\mathbb{R}^{L}_{+} \setminus \{0\})$.

H is said to be the **compensated demand function.**

Lemma 1: For all $(p, \mu) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++}$, $H(p, D(p, \mu)) = D(p, \mu)$.

Proof: Towards a contradiction, suppose they are not equal at some price income-pair. Note that, for all $(p, \mu) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++}$ we have $H(p, D(p, \mu)) =$
$= D(p, p^TD(p, \mu))$.

Hence at some $(p, \mu) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++}$ we have

$H(p, D(p, \mu)) = D(p, p^TD(p, \mu)) \neq D(p, \mu)$.

Thus, $p^TD(p, p^TD(p, \mu)) \leq p^TD(p, \mu) \leq \mu$, and $p^TD(p, \mu) \leq p^TD(p, \mu)$.

Thus, $D(p, p^TD(p, \mu))$ is available and not chosen when $D(p, \mu)$ is chosen and $D(p, \mu)$ is available and not chosen when $D(p, p^TD(p, \mu))$ is chosen.

This contradicts the Weak Axiom of Revealed Preference (WARP) and proves the lemma. Q.E.D.

Proposition 1: Suppose D satisfies WARP. Then, for all $p, p^0 \in (\mathbb{R}^{L-1}_{++} \times \{1\})$ and $\mu^0 \in \mathbb{R}_{++}$ it must be the case that $(p - p^0)^T(H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))) \leq 0$, with strict inequality if $H(p, D(p^0, \mu^0)) \neq H(p^0, D(p^0, \mu^0))$.

Proof: By definition of H, for all $(p, \mu) \in (\mathbb{R}^{L-1}_{++} \times \{1\}) \times \mathbb{R}_{++}$, $H(p, D(p^0, \mu^0)) = D(p, p^T D(p^0, \mu^0))$ and by Lemma 1, $H(p, D(p, \mu)) = D(p, \mu)$.

Hence,

$p^T[H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))] = p^T[D(p, p^T D(p^0, \mu^0)) - D(p^0, \mu^0)] \le 0.$

If $H(p, D(p^0, \mu^0)) = H(p^0, D(p^0, \mu^0))$, then Proposition 1 is obviously correct. Hence suppose $H(p, D(p^0, \mu^0)) \neq H(p^0, D(p^0, \mu^0)) = D(p^0, \mu^0)$

$p^T D(p^0, \mu^0) = p^T D(p^0, \mu^0)$ means $D(p^0, \mu^0)$ is available but not chosen when $D(p, p^T D(p^0, \mu^0))$ is chosen.

Thus, by WARP $p^{0T} D(p, p^T D(p^0, \mu^0)) > I^0 \ge p^{0T} D(p^0, \mu^0)$.

Thus, $p^{0T}[D(p, p^T D(p^0, \mu^0)) - D(p^0, \mu^0)] > 0$.

Hence, $-p^{0T}[D(p, p^T D(p^0, \mu^0)) - D(p^0, \mu^0)] > 0$, i.e.,

$-p^{0T}[[H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))] < 0$.

Combined with $p^T[H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))] \le 0$, we get

$(p - p^0)^T(H(p, D(p^0, \mu^0)) - H(p^0, D(p^0, \mu^0))) < 0$.

This proves the lemma. Q.E.D.

Appendix B

Let $P(q) = a - aq$ for $q \in (0, 1]$.

Let q^0, q satisfying $0 \le q^0 < q$, belong to the open interval (0, 1).

Suppose that consumers are already consuming an amount q^0 and we want to find their willingness to pay for an additional amount of $q - q^0$.

For $m \in \mathbb{N}$, let us subdivide the interval (q^0, q) into 'm' equal and non-overlapping intervals of length $\frac{q-q^0}{m}$ each.

For the first $\frac{q-q^0}{m}$ units, the market's average willingness to pay for an incremental unit is less than equal to $a - aq^0$ and greater than or equal to $a - a\left(q^0 + \frac{q-q^0}{m}\right)$. Hence, for the first $\frac{q-q^0}{m}$ units, the market's total willingness to pay is greater than or equal to $\left[a - a\left(q^0 + \frac{q-q^0}{m}\right)\right]\frac{q-q^0}{m}$ and less than or equal to $[a - aq^0]\frac{q-q^0}{m}$.

For the j^{th} $\frac{q-q^0}{m}$ unit, with $j\in\{2, \ldots, m\}$, the market's total willingness to pay is greater than or equal to $\left[a-a\left(q^0+j\frac{q-q^0}{m}\right)\right]\frac{q-q^0}{m}$ and less than or equal to $\left[a-a\left(q^0+(j-1)\frac{q-q^0}{m}\right)\right]\frac{q-q^0}{m}$.

Hence the market's willingness to pay for the extra '$q - q^0$' units is greater than or equal to $\frac{1}{m}\sum_{j=1}^{m}\left[a-a\left(q^0+j\frac{q-q^0}{m}\right)\right]\frac{q-q^0}{m}$ and less than or equal to $\frac{1}{m}\sum_{j=1}^{m}\left[a-a\left(q^0+(j-1)\frac{q-q^0}{m}\right)\right]q-q^0$.

$$\frac{1}{m}\sum_{j=1}^{m}\left[a-a\left(q^0+j\frac{q-q^0}{m}\right)\right](q-q^0)=a(q-q^0)-aq^0(q-q^0)-a\left(\frac{q-q^0}{m}\right)^2\sum_{j=1}^{m}j=$$
$$=a(q-q^0)-aq^0(q-q^0)-a\left(\frac{q-q^0}{m}\right)^2\frac{m(m+1)}{2}=a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m+1}{2m}.$$

Similarly,

$$\frac{1}{m}\sum_{j=1}^{m}\left[a-a\left(q^0+(j-1)\frac{q-q^0}{m}\right)\right](q-q^0)=a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m-1}{2m}.$$

Hence, the market's willingness to pay for $q - q^0$ additional units is greater than or equal to $a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m+1}{2m}$ and less than or equal to $a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m-1}{2m}$ for all $m\in\mathbb{N}$.

Now,

$$\lim_{m\to\infty}\left[a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\frac{m+1}{2m}\right]=a(q-q^0)-aq^0(q-q^0)-$$
$$-\frac{a}{2}\left(q-q^0\right)^2=a[q-\frac{1}{2}q^2]-a[q^0-\frac{1}{2}q^{02}].$$

Similarly,

$$\lim_{m\to\infty}\left[a(q-q^0)-aq^0(q-q^0)-a\left(q-q^0\right)^2\right]=a[q-\frac{1}{2}q^2]-a[q^0-\frac{1}{2}q^{02}].$$

Hence, the market's total willingness to pay for the additional $q - q^0$ units $W(q) - W(q^0) = a[q - \frac{1}{2}q^2] - a[q^0 - \frac{1}{2}q^{02}]$, where $W(q) = aq - \frac{a}{2}q^2$ for $q \in (0, 1]$, obtained by letting $q^0 = 0$, is the market's willingness to pay for the first q^{th} units of the good.

Summary

In this note we provide a statistical interpretation of the Marshallian market demand curve of a commodity that obeys the law of demand and which has a finite and positive level of satiation. A consequence of our approach is that in the context of two goods, we are able to obtain demand functions which are very similar to those obtained by "budget-constrained Cobb–Douglas utility maximization", but now as a result of a "budget-constrained linear utility maximization" exercise, although our budget constraint is "slightly different" from the one that would be used for the former optimization problem.

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