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## **CAPM applications for appropriate stock pricing – impact of speculation companies**

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### **1. Introduction**

The phenomena of incompatible pricing with the classic CAPM may be, for example, the size effect of Banz (1981), the January effect, the reversal of long-term returns documented by DeBondt and Thaler (1985), or the continuation of short term returns found by Jegadeesh and Titman (1993). The effect of DeBondt and Thaler is captured by the Fama-French three-factor model. The above anomalies deny pricing in light of the CAPM.

Research on stock pricing on the Polish market has been presented by Adamczak (2000), Jajuga (2000), Bołt and Miłobędzki (2002), Osińska and Stempińska (2003), Byrka-Kita and Rozkrut (2004), Zarzecki et al. (2004–2005), Fiszeder (2006), Czapkiewicz and Skalna (2010), and Gurgul and Wójtowicz (2014). The vast majority of the results deny pricing in light of the CAPM.

Czapkiewicz and Wójtowicz (2014) simulated returns of Polish stocks by a four-factor pricing model. Urbański (2012) tested the Fama and French (1993) model and proposed its modification based on the Fama and French (1995) work. In his further work, Urbański (2015) investigated whether stocks pricing simulated by the classic and modified Fama and French models is consistent in light of the ICAPM. The author also explored the impact of speculative stocks on the ICAPM stock pricing results. He stated that, if speculative stocks are eliminated, pricing errors generated by the model decrease, and the model generates multifactor-efficient portfolios.

Is commonly known that ICAPM applications and the classic CAPM can be employed to estimate the cost of company capital. Used for this purpose, ICAPM applications provide a lot of difficulties, and the procedures built on the clas-

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sic CAPM are usually applied. However, if the pricing is not consistent with the pricing that could be observed with the CAPM rules (which is confirmed by the above-shown work), the estimated capital cost may be incorrect.

To our knowledge, there are no studies to explain the reasons for the incorrect stock pricing in light of the classic CAPM. Thus, leaning again on Urbański's (2015) work and the need for a simple-to-use and correct capital cost estimation, it seems justified to also examine the influence of speculation stocks on pricing in light of the classic CAPM.

In this paper, we explain the reasons of inconsistent stock pricing. The basis of the correct test of the CAPM application is the appropriate formation of portfolios. Therefore, we use the injunctions by Cochrane (2001). He said: "Finally, I think much of the attachment to portfolios comes from a desire to more closely mimic what actual investors would do rather than simply form a test." (see Cochrane, 2001, p. 445). "If your portfolios have no spread in average returns – if you just choose 25 random portfolios – then there will be nothing for the asset pricing model to test." (see Cochrane, 2001, p. 453).

The present work is a continuation of the preliminary study in this field presented by Urbański et al. (2014) and Urbański (2015). They note that many speculative stocks (described by bad financial indicators and penny prices) are characterized by extremely high returns (in the USA, a penny stock is defined by the Securities and Exchange Commission [SEC] as a security whose price is below 5 \$ per share, while in the UK, this threshold is 1 £. Penny stocks in the USA are often traded on over-the-counter markets. The SEC has defined specific rules for the sale of penny stocks.)

Therefore, we expect that the following conjectures are true:

### **Conjecture 1**

Speculative stocks are the cause of inconsistent stock pricing in light of the CAPM.

### **Conjecture 2**

Improper procedures for the construction of test portfolios are an additional factor leading to incompatible stock pricing.

Section 2 presents the data and procedures of portfolio construction. Section 3 widely analyzes the results of pricing in light of the classic CAPM for each procedure presented in Section 2. Section 4 tests the influence of additional boundary conditions on the pricing correctness in light of the classic CAPM. Section 5 presents our conclusions.

## 2. Data and procedures of portfolio construction

The study is based on stocks quoted on the WSE during the period of 1995–2012. The full-sample observations are divided into two separate sub-periods: 1995–2005 (the years preceding Poland's accession to the EU) and 2005–2012 (the years of Poland's membership in the EU). Necessary data characterizing the inspected companies (such as fundamental indicators and stock quotes) were provided by Notoria Serwis Company and the Warsaw Stock Exchange.

Procedures of portfolio forming are defined in two variants. According to Cochrane's guidelines, Variant 1 provides practical investment strategies. In this variant, the model of portfolio management described in the work of Urbański (2012) and briefly presented in the appendix is used. In Variant 2, portfolios are built according to the methodology of Fama and French (1993).

In each variant, three modes of samples are analyzed. Mode M1 considers all WSE stocks except for companies characterized by a negative book value. In Mode M2, we eliminate speculative stocks that meet one of the following boundary conditions: a)  $MV/BV > 100$ ; b)  $ROE < 0$  and  $BV > 0$  and  $r_{it} > 0$ ; c)  $MV/BV > 30$  and  $r_{it} > 0$ , where  $r_{it}$  is return of stock  $i$  in period  $t$ . In Mode M3, we eliminate speculative stocks meeting an additional condition d)  $MV/E < 0$ . The speculative stocks, defined in Mode M2 appear from Q1 of 2005. The speculative stocks defined in Mode M3 appear throughout the whole analyzed period. The number of analyzed companies decreased from 14% in 2005 to 21% in 2012 after the exclusion of the speculative stocks defined in Variant 3 (see Tab. 1) (230 companies were listed on WSE in 2004, while 426 were listed at the end of 2011).

The analyzed securities are sorted into quintile portfolios built on the basis of fundamental functional FUN as well as the NUM and DEN functions presented in the appendix – in Variant 1 (five portfolios are formed on FUN, five on NUM, and five on DEN) as well as on BV/MV and CAP (five portfolios are formed on BV/MV and five on CAP) in Variant 2. FUN, NUM, DEN, BV/MV, and CAP are calculated for all analyzed securities at the beginning of each investment period in which the return is to be calculated. FUN, NUM, DEN, BV/MV, and CAP for the portfolios constitute the average arithmetical values of these functions of various portfolio securities. Returns on the given portfolios are average stock returns weighted by market capitalizations. Function NUM represents an investor forming a portfolio that consists of the best fundamental companies, whereas DEN represents an investor portfolio that consists of the undervalued stocks. Functional FUN represents an investor building a portfolio that consists of the best fundamental and simultaneously undervalued stocks (the argumentation relates to long investments).

Table 1 presents the number of listed companies classified into quintile portfolios during the chosen periods. Table 2 shows the return spreads of the

portfolios formed on the maximal (Quintile 1) and minimal (Quintile 5) values of FUN, NUM, DEN, BV/MV, and CAP.

**Table 1**  
Number of companies in quintile portfolios

Portfolio	IQ1996			IQ2005			IQ2012		
	M1	M2	M3	M1	M2	M3	M1	M2	M3
1	11	11	11	33	30	27	63	61	50
2	11	11	11	33	30	27	63	61	50
3	11	11	11	33	30	27	63	61	50
4	11	11	11	33	30	27	63	61	50
5	13	13	10	34	28	29	62	60	49

In M1 negative-BV stocks are excluded from portfolios. Mode M2 eliminates speculative stocks meeting one of the following boundary conditions: 1)  $MV/BV > 100$ ; 2)  $ROE < 0$  and  $BV > 0$  and  $r_{it} > 0$ ; 3)  $MV/BV > 30$  and  $r_{it} > 0$ , where  $MV$  is stock market value,  $ROE$  is return on book value (BV),  $r_{it}$  is return of portfolio  $i$  during period  $t$ . Mode M3 eliminates speculative stocks meeting additional condition 4)  $MV/E < 0$ , where  $E$  is average earning for last four quarters.

Source: own research

The maximal return spreads are for portfolios formed on FUN, NUM, and DEN in M2 and M3 (p-values < 0.001%). The spreads for the portfolios formed on BV/MV are lower and are insignificantly different from zero in the first sub-period. The spreads for the portfolios formed on CAP are insignificantly different from zero in all of the tested periods (p-values > 10%).

**Table 2**  
Average return spreads of portfolio formed on maximal and minimal values of FUN, NUM, DEN, BV/MV, and CAP

Portfolios are formed on:	FUN	NUM	DEN	BV/MV	CAP
M1: 1995–2005, 36 quarters $\bar{r}$ (p-value [%]) <sup>a</sup>	0.09 (0.00)	0.07 (0.17)	-0.00 (0.49)	0.05 (13.98)	-0.00 (97.62)
M1: 2005–2012, 28 quarters $\bar{r}$ (p-value [%]) <sup>a</sup>	0.04 (2.31)	0.03 (7.69)	-0.06 (0.83)	0.05 (2.65)	-0.03 (28.93)
M2: 1995–2012, 64 quarters $\bar{r}$ (p-value [%]) <sup>a</sup>	0.11 (0.00)	0.08 (0.00)	-0.08 (0.00)	0.03 (6.83)	-0.01 (74.84)

**Table 2 cont.**

$\bar{r}_{\text{variant}}$	$\bar{r}_{\text{variant}_1} = 0.09$			$\bar{r}_{\text{variant}_2} = 0.01$	
(p-value [%]) <sup>b</sup>	0.00				
M2: 1995–2005, 36 quarters $\bar{r}$ (p-value [%]) <sup>a</sup>	0.09 (0.00)	0.07 (0.17)	−0.05 (0.49)	0.05 (13.98)	−0.00 (97.62)
M2: 2005–2012, 28 quarters $\bar{r}$ (p-value [%]) <sup>a</sup>	0.12 (0.00)	0.10 (0.00)	−0.13 (0.00)	0.04 (2.02)	−0.02 (48.47)
M3: 1995–2005, 36 quarters $\bar{r}$ (p-value [%]) <sup>a</sup>	0.12 (0.00)	0.10 (0.06)	−0.06 (3.77)	0.04 (19.53)	−0.02 (58.84)
M3: 2005–2012, 28 quarters $\bar{r}$ (p-value [%]) <sup>a</sup>	0.06 (2.15)	0.03 (6.50)	−0.05 (1.49)	0.04 (4.52)	−0.05 (11.27)

$\bar{r}$  is average spread value, <sup>a</sup> $H_0: \bar{r} = 0, H_1: \bar{r} \neq 0$ ; <sup>b</sup> $H_0: \bar{r}_{\text{variant}_1} = \bar{r}_{\text{variant}_2}, H_1: \bar{r}_{\text{variant}_1} > \bar{r}_{\text{variant}_2}$ .  
 In Mode M1, negative-BV stocks are excluded from portfolios. Mode M2 eliminates speculative stocks meeting one of the following boundary conditions: 1) MV/BV > 100; 2) ROE < 0 and BV > 0 and  $r_{it} > 0$ ; 3) MV/BV > 30 and  $r_{it} > 0$ , where MV is the stock market value, ROE is return on book value (BV),  $r_{it}$  is return of portfolio  $i$  during period  $t$ . Mode M3 eliminates speculative stocks meeting additional condition 4) MV/E < 0, where E is average earning for last four quarters. Spread values in M1 and M3 for whole investigated period (1995–2012) are available from authors upon request.

Source: own research

### 3. Stock pricing under conditions of CAPM

The statistical model testing the classic CAPM can be described by Equations (1) and (2). The regressions of time series (1) are analyzed in the first pass. Equation (2) is analyzed in the second pass as cross-section regressions ( $\forall t = 1, \dots, T$ ; see Fama-MacBeth [1973], whose procedure is used) and the time-cross-section regression using panel data:

$$r_{it} - RF_t = \alpha_i + \beta_{i,M}(RM_t - RF_t) + e_{it}, \quad t = 1, \dots, T; \quad \forall i = 1, \dots, 15 \quad (1)$$

$$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \varepsilon_{it}; \quad i = 1, \dots, 15; \quad t = 1, \dots, T \quad (2)$$

The response variable of the above regressions is the excess of return ( $r_{it} - RF_t$ ) of 15 test portfolios constructed on FUN, NUM, and DEN as well as the excess of returns of 10 portfolios built on BV/MV and CAP. Risk-free rate of return (RF) is evaluated by the 91-day Treasury bill rate of return. Explanatory variable of regression (1) is a market factor defined as an excess market return over risk-free rate

$(RM_t - RF_t)$ . Market return (RM) is evaluated by the return on the WIG index of the WSE. Explanatory variable of regression (2) constitutes the loading of market factor (beta) estimated in the first pass.

The values of parameters of regressions (1) are determined by means of the GLS method with the application of the Prais-Winsten procedure with first-order autocorrelation. Table 3 presents the values of parameters of regression (1) for the full-sample and for the portfolios of the Mode-M1 type (the parameter values for the sub-periods and for Modes M2 and M3 are similar and are available upon request).

Beta values are estimators of systematic risk connected with the market factor. The betas are significantly different from zero for the all of the tested cases (p-values = 0.00). The beta values are similar for the different modes and variants of portfolio building. They change as follows: in the first sub-period – from 0.61 to 1.37; in the second sub-period – from 0.69 to 1.4; and for the whole sample – from 0.75 to 1.26. Coefficients  $R^2$  seem to be independent of portfolio forming, ranging from 32% to 92%.

If speculative stocks are eliminated, the intercepts of regressions (1) are equal to zero for all portfolios formed on FUN, NUM, and DEN. This is confirmed by the GRS-F statistic (see Gibbons et al., 1989) equal for the whole sample to 3.65 (p-value = 0.03%) for Mode M1, 1.18 (p-value = 32.19%) for Mode M2, and 0.78 (p-value = 69.20%) for Mode M3. The changes of the GRS-F statistic for Modes M1, M2, and M3 in sub-periods are similar. This proves that the tested CAPM generates mean-variance-efficient portfolios.

The changes of the GRS-F statistic for portfolios formed on BV/MV and CAP are more difficult to interpret.

**Table 3**

Time-series regression of excess stock returns on stock-market factor (RM – RF)

$$r_{it} - RF_t = \alpha_i + \alpha_{i,M}(RM_t - RF_t) + e_{it}, \quad t = 1, \dots, 64; \quad \forall i = 1, \dots, n$$

Response variable								
<b>Panel A:</b> Excess returns on $n = 15$ stock portfolios formed on FUN, NUM, and DEN								
<b>Mode M1:</b> GRS-F = 3.65, p-value(GRS) = 0.03%; $R^2 = 62\% - 89\%$ .								
<b>Mode M2:</b> GRS-F = 1.18, p-value(GRS) = 32.19%; $R^2 = 52\% - 87\%$ .								
<b>Mode M3:</b> GRS-F = 0.78, p-value(GRS) = 69.20%; $R^2 = 43\% - 86\%$ .								
<b>Panel B:</b> Excess returns on $n = 10$ stock portfolios formed on BV/MV and CAP								
<b>Mode M1:</b> GRS-F = 1.68, p-value(GRS) = 10.93%; $R^2 = 32\% - 92\%$ .								
<b>Mode M2:</b> GRS-F = 0.89, p-value(GRS) = 54.53%; $R^2 = 42\% - 92\%$ .								
<b>Mode M3:</b> GRS-F = 1.74, p-value(GRS) = 9.56%; $R^2 = 38\% - 90\%$ .								
GLS method	Mode M1; Panel A				Mode M1; Panel B			
Portfel	$\alpha_i$	p-value [%]	$\beta_{i,M}$	p-value [%]	$\alpha_i$	p-value [%]	$\beta_{i,M}$	p-value [%]

Table 3 cont.

Portfel	Portfolios formed on FUN				Portfolios formed on BV/MV			
MIN, Quintile <sub>1</sub>	-0.05	0.27	1.10	0.00	-0.02	0.24	1.00	0.00
Quintile <sub>2</sub>	-0.05	0.03	0.89	0.00	-0.01	22.53	0.93	0.00
Quintile <sub>3</sub>	-0.02	4.41	0.83	0.00	0.01	19.44	1.07	0.00
Quintile <sub>4</sub>	-0.00	60.30	0.97	0.00	0.01	72.99	0.75	0.00
MAX, Quintile <sub>5</sub>	0.03	0.19	1.03	0.00	0.02	27.18	0.89	0.00
Portfel	Portfolios formed on NUM				Portfolios formed on CAP			
MIN, Quintile <sub>1</sub>	-0.04	3.42	1.10	0.00	0.01	50.57	1.26	0.00
Quintile <sub>2</sub>	-0.04	0.13	0.84	0.00	0.00	89.88	0.99	0.00
Quintile <sub>3</sub>	-0.02	2.05	0.75	0.00	-0.00	96.76	1.09	0.00
Quintile <sub>4</sub>	-0.00	77.09	1.06	0.00	-0.01	50.42	1.11	0.00
MAX, Quintile <sub>5</sub>	0.02	4.23	1.02	0.00	-0.00	54.86	0.97	0.00
Portfel	Portfolios formed on DEN				-			
MIN, Quintile <sub>1</sub>	0.03	0.20	0.90	0.00				
Quintile <sub>2</sub>	0.00	90.63	1.00	0.00				
Quintile <sub>3</sub>	-0.02	5.04	0.87	0.00				
Quintile <sub>4</sub>	-0.02	8.62	0.95	0.00				
MAX, Quintile <sub>5</sub>	-0.03	2.22	1.18	0.00				
<p>RF is 91-day Treasury bill rate of return. RM is evaluated by return on WIG index of WSE. GRS-F is F-statistic of Gibbons et al. (1989). Prais-Winsten algorithm is used for correction of autocorrelation. In Mode M1, negative-BV stocks are excluded from portfolios. Mode M2 eliminates speculative stocks meeting one of the following boundary conditions: 1) <math>MV/BV &gt; 100</math>; 2) <math>ROE &lt; 0</math> and <math>BV &gt; 0</math>; and 3) <math>MV/BV &gt; 30</math> and <math>r_{it} &gt; 0</math>, where <math>MV</math> is stock market value, <math>ROE</math> is return on book value (BV), <math>r_{it}</math> is return of portfolio <math>i</math> during period <math>t</math>. Mode M3 eliminates speculative stocks meeting additional condition 4) <math>MV/E &lt; 0</math>, where <math>E</math> is average earning for last four quarters. Sample period is from 1995 to 2012, 64 Quarters.</p>								

Source: own research

In the second pass, the value of beta loading is estimated. Beta loading defines the risk premium for the market factor. The beta (for portfolio  $i$ ) is constant for all periods, while response variables constitute the returns that should by nature be random (see Cochrane 2001, p. 231). Therefore, in the time-cross-section estimation, the lack of autocorrelation of the residual component

may be presumed. The impact of heteroskedasticity is taken into account by means of the change of the variables method. If the Fama-MacBeth procedure is run, the Prais-Winsten method is used. In each tested period (for cross-section data), first-order autocorrelation of the residual component is taken into account.

The impact of estimation errors of the true beta values in the first pass is taken into account by correcting the standard errors of the beta loadings estimated in the second pass. With this purpose in mind, application is made of Shanken's estimator (see Shanken, 1992). In order to assess the risk premium values (in keeping with the proposal of Jagannathan and Wang [1998]), t-statistics are analyzed without consideration and with consideration (SH-*t* statistics) to Shanken's corrections.

Tables 4 and 5 present the values of estimated parameters of regressions (2), the values of informal determination coefficient  $R^2_{LL}$  applied by Lettau and Ludvigson (2001), and the values of the  $Q^A(F)$  statistic for the test of Shanken (1985) that the pricing errors in the model are jointly zero ( $R^2_{LL}$  is a measure following Lettau and Ludvigson [2001] that show the fraction of the cross-sectional variation in average returns that are explained by a tested model and is calculated as follows:  $R^2_{LL} = [\sigma_c^2(\bar{r}_i) - \sigma_c^2(\bar{e}_i)] / \sigma_c^2(\bar{r}_i)$ , where  $\sigma_c^2$  denotes a cross-sectional variance, and the variables with bars over them denote time-series averages).

Comparing the results placed in Tables 4 and 5, it can be stated that, if portfolios are built on BV/MV and CAP, the tested application of CAPM does not price risk on WSE. The values of risk premium  $\gamma_M$  is insignificantly different from zero for all of the tested cases (p-values > 37% after correction of error in the variables).

**Table 4**

Values of risk premium ( $\gamma_m$ ) estimated from second-pass regression for portfolios formed on FUN, NUM, and DEN

$$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \varepsilon_{it}; \quad i = 1, \dots, 15; \quad t = 1, \dots, T$$

Parameter	1995–2012			1995–2004		2005–2012		
	Conditions of forming portfolios							
	M1	M2	M3	M1 = M2	M3	M1	M2	M3
<b>Time-cross-sectional regression</b>								
$\gamma_0$	-0.03 <sup>b</sup>	-0.14 <sup>b</sup>	-0.19 <sup>b</sup>	-0.10	-0.16	0.02	-0.22	-0.07
p-value [%]	40.61	0.42	0.00	2.09	0.01	56.39	0.18	25.84



Table 4 cont.

p-value [%] <sup>a</sup>	40.94	3.16	1.10	4.11	0.56	57.52	9.36	30.55
$\gamma_M$	0.02 <sup>b</sup>	0.12 <sup>b</sup>	0.19 <sup>b</sup>	0.07	0.15	-0.03	0.22	0.07
p-value [%]	70.18	1.71	0.01	9.48	0.05	44.67	0.41	28.72
p-value [%] <sup>a</sup>	70.35	6.61	1.36	12.76	0.94	45.54	10.76	32.64
$R^2_{LL}$ [%]	0.99 <sup>b</sup>	11.48 <sup>b</sup>	56.74 <sup>b</sup>	9.36	44.74	16.10	26.09	6.18
$Q^A(F)$ (p-value [%])	4.05 <sup>b</sup> (0.02)	1.28 <sup>b</sup> (25.69)	0.81 <sup>b</sup> (64.73)	2.92 (1.29)	2.23 (4.73)	2.60 (0.77)	5.37 (0.18)	1.09 (43.30)
<b>Fama-MacBeth cross-sectional regressions</b>								
$\gamma_0$	-0.02	-0.13	-0.19	-0.09	-0.16	0.04	-0.24	-0.05
p-value [%]	61.66	0.01	0.00	0.66	0.04	11.56	0.00	22.18
p-value [%] <sup>a</sup>	61.68	0.17	0.63	1.35	0.96	14.52	1.89	24.95
$\gamma_M$	-0.01	0.11	0.19	0.07	0.14	-0.06	0.23	0.05
p-value [%]	89.61	0.20	0.00	9.35	0.32	16.34	0.04	32.93
p-value [%] <sup>a</sup>	89.61	1.03	0.88	11.66	2.38	18.34	3.22	35.50
$R^2_{LL}$ [%]	0.05	11.50	56.56	9.66	44.99	19.29	26.10	7.15
$Q^A(F)$ (p-value [%])	4.05 (0.02)	1.28 (25.69)	0.81 (64.73)	2.92 (1.29)	2.23 (4.73)	2.60 (0.77)	5.37 (0.18)	1.09 (43.30)
<p>In Mode M1, negative-BV stocks are excluded from portfolios. Mode M2 eliminates speculative stocks meeting one of the following boundary conditions: 1) MV/BV &gt; 100; 2) ROE &lt; 0 and BV &gt; 0; and 3) MV/BV &gt; 30 and <math>r_{it} &gt; 0</math>, where MV is stock market value, ROE is return on book value (BV), <math>r_{it}</math> is return of portfolio i during period <math>t</math>. Mode M3 eliminates speculative stocks meeting additional condition 4) MV/E &lt; 0, where E is average earning for last four quarters. <math>R^2_{LL}</math> is measure following Lettau and Ludvigson (2001) showing fraction of cross-sectional variation in average returns that are explained by each model. <math>Q^A(F)</math> reports F-statistic and its corresponding p-value (indicated below in brackets) for test of Shanken (1985) that pricing errors in model are jointly zero. SH t-stat statistic of Shanken (1992) adjusting for errors-in-variables.<sup>a</sup> After adjusting for errors-in-variables, according to Shanken (1992).</p>								

Source: own research, <sup>b</sup> Urbański et al. (2014)

Forming portfolios on FUN, NUM, and DEN reflects investment strategies used by investors, which is confirmed by the high return spreads (see Tab. 2). The values of risk premium  $\gamma_M$  for these portfolios are significantly different from zero for Modes M2 and M3 in the whole sample, for Mode M3 in the first sub-period, and for Mode M2 in the second sub-period. Coefficient  $R^2_L$  grows if speculative stocks are eliminated from the portfolios, assuming for the whole sample values less than 1% for Mode M1, about 11.5% for Mode M2, and more than 56% for Mode 3. Also, pricing errors decrease after the elimination of speculative stocks. This is documented by the values of the  $Q^A(F)$  statistic (see Tab. 4). This proves that mean-variance-efficient portfolios are generated if speculative stocks are excluded from consideration.

Table 5

Values of risk premium ( $\gamma_m$ ) estimated from second-pass regression for portfolios formed on BV/MV and CAP

$$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \varepsilon_{it}; \quad i = 1, \dots, 15; \quad t = 1, \dots, T$$

Parameter	1995–2012			1995–2004		2005–2012			
	Conditions of forming portfolios								
	M1	M2	M3	M1=M2	M3	M1	M2	M3	
<b>Time-cross-sectional regression</b>									
$\gamma_0$	-0.00 <sup>b</sup>	-0.00 <sup>b</sup>	-0.08 <sup>b</sup>	0.01	-0.05	0.04	-0.06	0.02	
p-value [%]	98.39	94.81	46.97	86.13	56.74	54.71	34.83	7.50	
p-value [%] <sup>a</sup>	98.40	94.82	53.21	86.44	58.21	56.09	36.31	7.54	
$\gamma_M$	-0.00 <sup>b</sup>	-0.01 <sup>b</sup>	0.08 <sup>b</sup>	-0.03	0.04	-0.04	0.05	-0.01	
p-value [%]	94.28	89.05	45.52	61.91	67.25	58.94	36.28	38.29	
p-value [%] <sup>a</sup>	94.28	89.07	51.73	62.59	68.38	60.04	37.08	37.70	
$R^2_L$ [%]	0.23 <sup>b</sup>	0.39 <sup>b</sup>	10.70 <sup>b</sup>	16.08	2.07	0.07	30.66	0.81	
$Q^A(F)$ (p-value [%])	1.98 (6.65)	1.00 (44.61)	1.32 (25.85)	1.33 (27.04)	0.84 (57.98)	1.33 (28.78)	0.76 (64.29)	2.05 (9.54)	
<b>Fama-MacBeth cross-sectional regressions</b>									
$\gamma_0$	0.01	0.00	-0.11	0.02	-0.01	-0.02	-0.04	0.01	
p-value [%]	88.36	98.01	24.22	72.76	84.85	51.31	27.71	93.46	

Table 5 cont.

p-value [%] <sup>a</sup>	88.39	98.02	35.84	73.56	84.85	52.18	28.63	93.52
$\gamma_M$	-0.01	-0.01	0.11	-0.04	0.00	0.03	0.04	0.02
p-value [%]	80.45	81.65	23.04	72.76	96.95	55.28	39.91	76.16
p-value [%] <sup>a</sup>	80.50	81.70	34.28	73.56	96.95	55.85	40.22	76.32
$R_{LL}^2$ [%]	0.36	0.97	23.55	16.43	0.34	0.17	31.58	6.52
$Q^A(F)$ (p-value [%])	1.98 (6.65)	1.00 (44.61)	1.32 (25.85)	1.33 (27.04)	0.84 (57.98)	1.33 (28.78)	0.76 (64.29)	2.05 (9.54)

In Mode M1, negative-BV stocks are excluded from portfolios. Mode M2 eliminates speculative stocks meeting one of the following boundary conditions: 1) MV/BV > 100; 2) ROE < 0 and BV > 0; and 3) MV/BV > 30 and  $r_{it} > 0$ , where MV is stock market value, ROE is return on book value (BV),  $r_{it}$  is return of portfolio  $i$  during period  $t$ . Mode M3 eliminates speculative stocks meeting additional condition 4) MV/E < 0, where E is average earning for last four quarters.  $R_{LL}^2$  is measure following Lettau and Ludvigson (2001) showing fraction of cross-sectional variation in average returns that are explained by each model.  $Q^A(F)$  reports F-statistic and its corresponding p-value (indicated below in brackets) for test of Shanken (1985) that pricing errors in model are jointly zero. SH t-stat is statistic of Shanken (1992) adjusting for errors-in-variables. <sup>a</sup> After adjusting for errors-in-variables, according to Shanken (1992).

Source: own research, <sup>b</sup> Urbański et al. (2014)

#### 4. Influence of feature of constructed portfolios on classic CAPM application

Jagannathan and Wang (1998) argue that taking into account a characteristic of the formed portfolios is necessary in testing the CAPM applications, while Urbański (2011) presents the predictive possibilities of FUN, NUM, and DEN on the basis of which portfolios are formed. Therefore, it seems necessary to verify the validity of the tested CAPM application in the presence of the characteristics of the built portfolios. Tests are conducted for regression (2) supplemented with portfolio characterizing factors. The testing procedure is shown by Equations (3) through (7):

$$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_Z Z_{i,t-1} + \varepsilon_{it}; \quad i = 1, \dots, 15 \quad t = 1, \dots, T \quad (3)$$

where  $Z_{i,t-1}$  are  $FUN_i$ ,  $NUM_i$ , or  $DEN_i$  for period  $t - 1$ , and null hypothesis is  $H_0: \gamma_Z = 0$ .

Practically, the following regressions are analyzed:

$$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_Z \text{FUN}_{i,t-1} + \varepsilon_{it}; \quad i = 1, \dots, 15; t = 1, \dots, T \quad (4)$$

$$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_Z \text{NUM}_{i,t-1} + \varepsilon_{it}; \quad i = 1, \dots, 15; t = 1, \dots, T \quad (5)$$

$$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_Z \text{DEN}_{i,t-1} + \varepsilon_{it}; \quad i = 1, \dots, 15; t = 1, \dots, T \quad (6)$$

$$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_Z \text{FND}_{i,t-1} + \varepsilon_{it}; \quad i = 1, \dots, 15; t = 1, \dots, T \quad (7)$$

In regression (4),  $\text{FUN}_{i,t-1}$  is a vector with coordinates:  $\text{FUN}_{1,t-1}, \dots, \text{FUN}_{5,t-1}, \text{FUN}_{1,t-1}, \dots, \text{FUN}_{5,t-1}, \text{FUN}_{1,t-1}, \dots, \text{FUN}_{5,t-1}$ . Similarly, in regressions (5–7),  $\text{NUM}_{i,t-1}$  is a vector:  $\text{NUM}_{1,t-1}, \dots, \text{NUM}_{5,t-1}, \text{NUM}_{1,t-1}, \dots, \text{NUM}_{5,t-1}, \text{NUM}_{1,t-1}, \dots, \text{NUM}_{5,t-1}$ .  $\text{DEN}_{i,t-1}$  is a vector:  $\text{DEN}_{1,t-1}, \dots, \text{DEN}_{5,t-1}, \text{DEN}_{1,t-1}, \dots, \text{DEN}_{5,t-1}, \text{DEN}_{1,t-1}, \dots, \text{DEN}_{5,t-1}$ . Variable  $\text{FND}_{i,t-1}$  is a vector with coordinates:  $\text{FUN}_{i,t-1}, i = 1, \dots, 5, \text{NUM}_{i,t-1}, i = 1, \dots, 5$  and  $\text{DEN}_{i,t-1}, i = 1, \dots, 5$ .

The estimated parameter values of regressions (4–7) are presented in Table 6.

**Table 6**

Time-cross-section regressions showing effect of portfolio characteristics representing specification tests of CAPM for whole sample

	M1	M2	M3	M1	M2	M3	M1	M2	M3
$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_{\text{FUN}} \text{FUN}_{i,t-1} + \varepsilon_{it}; \quad i = 1, \dots, 15; t = 1, \dots, 64$									
Panel A	$\gamma_0$			$\gamma_M$			$\gamma_{\text{FUN}}$		
Parameter	-0.03	-0.10	-0.18	0.01	0.07	0.16	0.01	0.01	0.01
p-value [%]	43.14	5.86	0.01	89.74	26.37	0.09	20.54	13.72	18.44
$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_{\text{NUM}} \text{NUM}_{i,t-1} + \varepsilon_{it}; \quad i = 1, \dots, 15; t = 1, \dots, 64$									
Panel B	$\gamma_0$			$\gamma_M$			$\gamma_{\text{NUM}}$		
Parameter	-0.03	-0.10	-0.18	0.01	0.07	0.17	0.01	0.01	0.00
p-value [%]	40.85	4.36	0.00	88.58	22.26	0.05	19.81	16.19	42.37
$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_{\text{DEN}} \text{DEN}_{i,t-1} + \varepsilon_{it}; \quad i = 1, \dots, 15; t = 1, \dots, 64$									
Panel C	$\gamma_0$			$\gamma_M$			$\gamma_{\text{DEN}}$		
Parameter	-0.01	-0.12	-0.17	-0.04	0.10	0.19	0.02	0.00	-0.01
p-value [%]	72.80	1.95	0.02	44.76	11.13	0.01	1.12	70.62	64.63

**Table 6 cont.**

$r_{it} - RF_t = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_{FND} FND_{i,t-1} + \varepsilon_{it}; \quad i = 1, \dots, 15; t = 1, \dots, 64$									
Panel D	$\gamma_0$			$\gamma_M$			$\gamma_{FND}$		
Parameter	-0.04	-0.10	-0.17	0.00	0.06	0.15	0.01	0.01	0.01
p-value[%]	37.06	4.53	0.01	92.63	26.65	0.14	5.59	4.33	25.45
<p>Time-cross-section estimation is applied using panel data. Beta parameters are estimated (in first pass) by GLS using Prais-Winsten procedure while, heteroskedasticity is corrected (in second pass) by means of change of variables method. Panels A, B, C, and D show whether lagged FUN, NUM, DEN, and FND add new information to classical CAPM. Variable <math>FND_{i,t-1}</math> is vector with the following coordinates: <math>FUN_{i,t-1}, i = 1, \dots, 5, NUM_{i,t-1}, i = 1, \dots, 5</math> and <math>DEN_{i,t-1}, i = 1, \dots, 5</math>. In Mode M1, negative-BV stocks are excluded from portfolios. Mode M2 eliminates speculative stocks meeting one of the following boundary conditions: 1) <math>MV/BV &gt; 100</math>; 2) <math>ROE &lt; 0</math> and <math>BV &gt; 0</math>; and 3) <math>MV/BV &gt; 30</math> and <math>r_{it} &gt; 0</math>, where <math>MV</math> is stock market value, <math>ROE</math> is return on book value (BV), <math>r_{it}</math> is return of portfolio <math>i</math> during period <math>t</math>. Mode M3 eliminates speculative stocks meeting additional condition 4) <math>MV/E &lt; 0</math>, where <math>E</math> is average earning for last four quarters. Sample period is from 1995 to 2012, 64 Quarters.</p>									

Source: own research

Panel A shows whether lagged FUN adds new information to classical CAPM. Panels B, C, and D show whether lagged NUM, DEN, and FND (respectively) add new information to CAPM. The test results show that FUN and NUM added separately to the regressions (see Panels A and B) have insignificant influence on the estimation results in three tested cases (M1, M2, and M3). Function DEN added to the regression has a significant influence on estimation if the portfolios are formed according to Modes M1 and M2. However, if the speculative stocks, defined in Mode M3, are excluded from portfolios, FUN, NUM, and DEN have not significant influence on estimation.

Table 7 shows the values of informal determination coefficient  $R^2_{LL}$  computed for regressions (4–7) as compared to  $R^2_{LL}$  for regression (2).

The values of the  $R^2_{LL}$  coefficients for regressions supplemented with portfolio characterizing factors  $Z_{i,t-1}$  are higher as compared to  $R^2_{LL}$  computed for regression (2). However, the increases are the highest for Mode M1 (where only negative-BV stocks are eliminated) – reaching 1599%. The lower increases are in the case of Mode M2, and the lowest (about 3%) if the speculative stocks, defined in Mode M3, are excluded from portfolios. This means that, in the case of Mode M3, the characteristics of the formed portfolios have the lowest impact

on the value of risk premium  $\gamma_M$ . Also, in this case, the average returns of the formed portfolios are best explained by the tested specification representing the classic CAPM.

**Table 7**

Measures showing fraction of cross-sectional variation in average returns explained by tested specification of CAPM

Mode	$r_{it}^{ex} = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \varepsilon_{it}$			$r_{it}^{ex} = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_Z Z_{i,t-1} + \varepsilon_{it}$			$Z_{i,t-1}$
	M1	M2	M3	M1	M2	M3	
$\bar{R}_{LL}^2$ [%]	0.99	11.48	56.74	11.81	14.42	57.41	FUN
				11.18	14.45	58.12	NUM
				20.05	12.25	57.51	DEN
				24.23	24.28	61.29	FND
$\bar{R}_{LL}^2$ [%]				16.82	16.35	58.58	-
$d\bar{R}_{LL}^2$ [%]				1,599.0	42	3	

$\bar{R}_{LL}^2$  is informal determination coefficient following Lettau and Ludvigson (2001), showing fraction of cross-sectional variation in average returns that are explained by each specification.  $\bar{R}_{LL}^2$  is average value of  $\bar{R}_{LL}^2$  for regressions with added FUN, NUM, DEN, and FND, respectively.  $d\bar{R}_{LL}^2$  is increase of  $\bar{R}_{LL}^2$  after added of  $Z_{i,t-1}$  into regression. In Mode M1, negative-BV stocks are excluded from portfolios. Mode M2 eliminates speculative stocks meeting one of the following boundary conditions: 1)  $MV/BV > 100$ ; 2)  $ROE < 0$  and  $BV > 0$ ; and 3)  $MV/BV > 30$  and  $r_{it} > 0$ , where  $MV$  is stock market value,  $ROE$  is return on book value (BV),  $r_{it}$  is return of portfolio  $i$  during period  $t$ . Mode M3 eliminates speculative stocks meeting additional condition 4)  $MV/E < 0$ , where  $E$  is average earning for last four quarters.  $r_{it}^{ex}$  is excess of returns over risk free rate:  $r_{it}^{ex} = r_{it} - RF_t$ . Sample period is from 1995 to 2012, 64 Quarters.

Source: own research

## 5. Conclusions

In this paper, we examine the influence of speculative stocks on pricing that would result from the correctness of CAPM assumptions. The study leads to the following conclusions:

1. The values of systematic risk are significantly different from zero for all of the tested cases, and they are similar for the different modes and variants of portfolio building.
2. However, if speculative stocks are eliminated, the classic CAPM generates mean-variance-efficient portfolios formed on FUN, NUM, and DEN in all of the tested periods. This is confirmed by the GRS-F statistic being less than 1.18 (p-value = 32.19%).
3. If portfolios are built on BV/MV and CAP, the classic CAPM does not price the risk premium on WSE.
4. The return spreads for portfolios formed on FUN, NUM, and DEN are significantly higher than the spreads for portfolios formed on BV/MV and CAP.
5. If speculative stocks are excluded from the analysis, the values of risk premium for portfolios formed on FUN, NUM, and DEN are significantly different from zero in all of the tested periods.
6. If speculative stocks are excluded from the portfolios formed on FUN, NUM, or DEN, the values of informal determination coefficient  $R^2_{LL}$  (showing the explained fraction of the cross-sectional variation in average returns) grows from 1% to 56%.
7. If speculative stocks are excluded from the portfolios formed on FUN, NUM, or DEN, pricing errors decrease; the values of  $Q^A(F)$  statistic fall from 4.05 (p-value = 0.02%) to 0.81 (p-value = 64.73%). This confirms that the classic CAPM generates mean-variance-efficient portfolios.
8. The correctness of the tested CAPM application in the presence of characteristics of the built portfolios is verified; if speculative stocks are excluded from the portfolios, FUN, NUM, and DEN do not have a significant influence on the estimation results.
9. Research results are in line with Conjectures 1 and 2.

The research results may be a contribution to explaining the incorrect stock pricing in highly developed capital markets in light of the classic CAPM.

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## Appendix. The model of portfolio management

The algorithm that groups companies into portfolios is based on functional FUN, defined by Equations (1), (2), and (3). The comprehensive economic interpretation of FUN is presented in the work of Urbański (2011). The investment is more attractive if the FUN value is higher. Functional FUN provides the characteristics of the companies that are assessed well by NUM and at the same time priced lowly by DEN:

$$\text{FUN} = \frac{\text{nor}(\text{ROE}) \cdot \text{nor}(\text{AS}) \cdot \text{nor}(\text{APO}) \cdot \text{nor}(\text{APN})}{\text{nor}(\text{MV}/\text{E}) \cdot \text{nor}(\text{MV}/\text{BV})} \quad (1\text{A})$$

where:

$$\begin{aligned} \text{ROE} = F_1; \text{AS} = F_2 &= \frac{\sum_{t=1}^i S(Q_t)}{\sum_{t=1}^i S(nQ_t)}; \text{APO} = F_3 = \frac{\sum_{t=1}^i \text{PO}(Q_t)}{\sum_{t=1}^i \text{PO}(nQ_t)}; \\ \text{APN} = F_4 &= \frac{\sum_{t=1}^i \text{PN}(Q_t)}{\sum_{t=1}^i \text{PN}(nQ_t)}, \text{MV}/\text{E} = F_5; \text{MV}/\text{BV} = F_6 \end{aligned} \quad (2\text{A})$$

$F_j$  variables are functions of company evaluation indicators (for  $j = 1, \dots, 4$ ) and functions of pricing indicators (for  $j = 5, 6$ ). Functions  $F_j$  ( $j = 1, \dots, 6$ ) are transformed to normalized areas  $\langle a_j; b_j \rangle$  according to Eq. (3A):

$$\text{nor}(F_j) = \left[ a_j + (b_j - a_j) \cdot \frac{F_j - c_j \cdot F_j^{\min}}{d_j \cdot F_j^{\max} - c_j \cdot F_j^{\min} + e_j} \right] \quad (3\text{A})$$

In Equations (1A), (2A), and (3A), the corresponding indications are as follows: ROE is a return on book equity;  $\sum_{t=1}^i S(Q_t)$ ,  $\sum_{t=1}^i \text{PO}(Q_t)$ ,  $\sum_{t=1}^i \text{PN}(Q_t)$  are values that are accumulated from the beginning of the year as net sales revenue ( $S$ ), operating profit ( $\text{PO}$ ), and net profit ( $\text{PN}$ ) at the end of the “ $i^{\text{th}}$ ” quarter ( $Q_i$ );  $\sum_{t=1}^i \overline{S(nQ_t)}$ ,  $\sum_{t=1}^i \overline{\text{PO}(nQ_t)}$ ,  $\sum_{t=1}^i \overline{\text{PN}(nQ_t)}$  are average values, accumulated from the beginning of the year as  $S$ ,  $\text{PO}$ , and  $\text{PN}$  at the end of  $Q_i$  over the last  $n$  years (the

present research assumes that  $n = 3$  years);  $MV/E$  is the market-to-earning value ratio;  $E$  is the average earning for the last four quarters;  $MV/BV$  is the market-to-book value ratio;  $a_j, b_j, c_j, d_j, e_j$  are variation parameters. Functions  $F_j$  ( $j = 1, \dots, 6$ ) are transformed into equal normalized area  $\langle 1; 2 \rangle$  (if  $\sum_{t=1}^i PN(Q_t), \sum_{t=1}^i PO(Q_t), \sum_{t=1}^i \overline{PN(nQ_t)}$  or  $\sum_{t=1}^i \overline{PO(nQ_t)}$  in equation (2A) is negative, Functions  $F_j$  ( $j = 1, 3, 4$ ) are transformed into area  $(0, 1)$ ).

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