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ARTICLE

POTENTIAL APPLICATIONS OF THE VOM BERG RHEOLOGICAL MODEL IN RESEARCH ON POLYMER-MODIFIED DRILLING MUDS

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Abstract: This article examines the potential applications of the Vom Berg rheological model in the technology of polymer-modified drilling muds. In recent years, intensive research has been conducted at the Faculty of Drilling, Oil, and Gas to optimize procedures for selecting rheological models for technological fluids used in drilling operations. One of the key outcomes of this research is the proprietary RheoSolution methodology, applied in this study to assess the adaptability of the Vom Berg rheological model. Originally developed for analyzing the rheology of cement slurries in civil engineering, this model has been utilized here to describe the relationship between shear stress and shear rate in polymer-modified drilling muds. As part of the research, laboratory experiments were conducted at the Drilling Fluids Laboratory of the Faculty of Drilling, Oil, and Gas, focusing on drilling muds modified with xanthan biopolymer. The obtained results served as the basis for a comparative analysis of the classical API methodology and the proposed RheoSolution approach in determining the rheological parameters of the tested fluids. Special attention was given to the applicability of the Vom Berg model as a tool for a more precise characterization of drilling mud behavior under dynamic conditions. This article is part of a broader series of publications aimed at demonstrating the utility and potential advantages of the RheoSolution methodology in studies on the rheological properties of technological drilling muds.

Keywords: drilling, drilling muds, drilling fluids, rheology, Vom Berg, RheoSolution

1. Introduction

Rheological modeling plays a fundamental role in the study of drilling fluids, as it directly influences the accuracy of flow resistance calculations in a closed-loop circulation system. Proper determination of the relationship between shear stress and shear rate is essential for assessing the hydraulic parameters of drilling operations, particularly in terms of fluid transport efficiency, wellbore stability, and cuttings removal. The precise estimation of flow resistance enables the optimization of key drilling infrastructure elements, such as mud pumps, and contributes to the overall efficiency and safety of drilling operations. The ability to accurately characterize the rheological behavior of drilling fluids is therefore a crucial aspect of optimizing their formulation and ensuring their effective performance under various wellbore conditions.

In practical drilling applications, rheological models recommended by the American Petroleum Institute Recommended Practice 13 (API RP 13) are widely used. These models range from linear models, such as the Bingham Plastic model, to power-law models, such as the Ostwald-de Waele model [1]. While these models provide useful approximations, their ability to accurately describe the complex behavior of modified drilling fluids remains limited. Many modern drilling fluids, particularly those enhanced with polymeric additives or nanoparticles, exhibit non-Newtonian behavior that deviates from the assumptions underlying these conventional models. As a result, there is a growing need for alternative approaches that can offer more accurate and flexible representations of drilling fluid rheology, particularly for fluids that undergo structural modifications under different shear conditions. One of the key rheological parameters that play a significant role in the evaluation of drilling fluids is yield point. This parameter defines the minimum shear stress required to initiate fluid flow and is particularly crucial in drilling applications, as it determines the ability of the fluid to suspend and transport cuttings, maintain wellbore stability, and prevent solid phase sedimentation. Traditional models, such as the Bingham Plastic model, incorporate yield point as a primary characteristic, however, their simplified nature often leads to inaccuracies in predicting the behavior of complex drilling fluids, especially those exhibiting structural transformations under dynamic conditions. A more advanced rheological model capable of capturing these changes could significantly improve the accuracy of hydraulic calculations and contribute to more effective drilling fluid design.

The Vom Berg rheological model originates from extensive research conducted in the 1990s on cement slurries used in construction, which naturally led to its application in cement slurry technologies for wellbore cementing [2]. The need for a more accurate description of cementitious materials, which often exhibit complex flow properties due to their time-dependent structural evolution, motivated the development of multi-parameter rheological models such as the Vom Berg equation. Given the similarities between cement slurries used in construction and those employed in oil and gas well cementing, the model gained interest in the drilling industry. However, the author of this article has observed that the Vom Berg model also provides excellent correlation when applied to polymer-modified drilling fluids, in contrast to conventional bentonite-based drilling fluids, where linear models such as the Bingham Plastic model (recommended by API) have been successfully used. This suggests that the Vom Berg model may have broader applicability in the study of advanced, chemically modified drilling fluids.

In the following sections, the author aims to demonstrate that, by utilizing the classical methodology for selecting rheological models based on rheological measurements performed with a FANN 12-speed viscometer, it is possible to successfully apply the Vom Berg equation to improve the accuracy of describing the relationship between shear stress and shear rate [3]. By comparing the results obtained from traditional models with those provided by the Vom Berg model, this study highlights its potential advantages in refining the characterization of drilling fluid behavior. This improvement in rheological characterization justifies the use of the Vom Berg model as a reliable tool for analyzing the flow properties of modified drilling fluids, particularly those enhanced with polymeric additives. The findings presented in this paper aim to contribute to the development of more precise methodologies for rheological analysis in drilling fluid technology, ultimately leading to better fluid performance and enhanced drilling efficiency.

2. The RheoSolution methodology and its application in research on the rheology of technological drilling fluids

The methodology employed regression analysis to model the relationship between shear stress and shear rate, while the Pearson linear correlation coefficient was used as a comparative criterion. In the case of mathematically simple rheological models: Bingham's linear model and Ostwald–de Waele's exponential model, the rheological parameters can be determined in a relatively straightforward analytical manner.

In the case of linear regression, the optimal function is predicted in the form of a linear equation:

$$\hat{y} = ax + b \tag{1}$$

The least squares condition is specified in the following form:

$$U = \sum_{i=1}^{m} (y_i - \hat{y})^2 \to \min \tag{2}$$

After incorporating the dependencies, the following expression is obtained:

$$U = \sum_{i=1}^{m} (y_i^2 - 2ax_i y_i - 2by_i + a^2 x_i^2 + 2abx_i + b^2) \to \min \tag{3}$$

The above equation is a function of the coefficients a and b. Therefore, the condition for minimizing the function U can be expressed in the form of the following equations:

$$\begin{cases} \dfrac{\partial U}{\partial a} = -2\sum_{i=1}^{m} x_i y_i + 2a\sum_{i=1}^{m} x_i^2 + 2b\sum_{i=1}^{m} x_i = 0 \\ \dfrac{\partial U}{\partial b} = -2\sum_{i=1}^{m} y_i + 2a\sum_{i=1}^{m} x_i + 2bm = 0 \end{cases} \tag{4}$$

Solving this system of equations yields the well-known formulas for linear regression coefficients found in the literature [4, 5]:

$$a = \frac{m\sum_{i=1}^{m} x_i y_i - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} y_i}{m\sum_{i=1}^{m} x_i^2 - \left(\sum_{i=1}^{m} x_i\right)^2} \tag{5}$$

$$b = \frac{\sum_{i=1}^{m} y_i - a\sum_{i=1}^{m} x_i}{m} \tag{6}$$

The use of regression analysis (1)–(6) to determine the reological parameters of selected rheological models is as follows:

– **The Bingham model** is a linear model of a plastic body that, in a resting state, maintains a three-dimensional structure with a certain degree of elasticity. Once this elasticity – known as the yield stress or plastic limit – is exceeded, the material begins to flow. The yield stress represents the intermolecular forces present in the fluid. After surpassing this threshold, a Bingham body exhibits characteristics of a Newtonian fluid, where stress propagates in direct proportion to the forces causing it, following a linear relationship [6].

This model is described by two parameters and represents the first modification of Newton's model. Due to its simplicity, it has been widely used as a fundamental rheological model for decades.

Assuming the rheological model of Bingham fluid in the form of:

$$\tau = \tau_y + \eta \cdot \dot{\gamma} \tag{7}$$

and substituting $\eta = a$, $\tau_y = b$, $\dot{\gamma} = x$ and $\tau = y$ the following relationship is obtained:

$$\hat{y} = ax + b \tag{8}$$

next, the rheological parameters of the Bingham fluid can be determined as follows:

$$\eta = \frac{m\sum_{i=1}^{m} x_i y_i - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} y_i}{m\sum_{i=1}^{m} x_i^2 - \left(\sum_{i=1}^{m} x_i\right)^2} \tag{9}$$

$$\boldsymbol{\tau_y} = \frac{\sum_{i=1}^{m} \boldsymbol{y_i} - \boldsymbol{a}\sum_{i=1}^{m} \boldsymbol{x_i}}{m} \tag{10}$$

– **The power-law model**, also known as the Ostwald–de Waele model, describes a pseudoplastic body. This model was introduced after it was observed that the resulting curve closely resembles a straight line on a plot of shear stress versus shear rate with both axes in logarithmic scale, the resulting curve closely resembles a straight line. It is a two-parameter model that effectively describes the behavior of most fluids, particularly shear-thinning fluids. However, it does not account for the yield stress, which is a limitation of the model [7].

Approximating the given fluid using the Ostwald–de Waele model:

$$\tau = k \cdot \dot{\gamma}^n \tag{11}$$

the calculation procedure requires linearization of the above relationship into the following form:

$$\hat{y} = ax + b \tag{12}$$

this is achieved by taking the logarithm of both sides of the equation:

$$\ln \tau = \ln k + n \ln \dot{\gamma} \tag{13}$$

and substituting: $n = a$, $\ln k = b$, $\ln \dot{\gamma} = x$, $\ln\tau = y$.

Next, the rheological parameters of the Ostwald–de Waele model are determined as follows:

$$n = \frac{m\sum_{i=1}^{m} x_i y_i - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} y_i}{m\sum_{i=1}^{m} x_i^2 - \left(\sum_{i=1}^{m} x_i\right)^2} \quad (14)$$

$$k = e^{\left(\frac{\sum_{i=1}^{m} y_i - a\sum_{i=1}^{m} x_i}{m}\right)} \quad (15)$$

– **The multiparameter Vom Berg model** is presented in the form of the following equation:

$$\tau = \tau_y + B\sinh^{-1}\left(\frac{-\frac{dv}{dr}}{C}\right) \quad (16)$$

the least squares method applied to the Vom Berg model is formulated as follows:

$$\begin{aligned} U &= \sum_{i=1}^{m}\left(y_i - \left(a + b\sinh^{-1}\left(\frac{x_i}{c}\right)\right)\right)^2 = \\ &= \sum_{i=1}^{m}\left(y_i^2 - 2ay_i - 2y_i b\sinh^{-1}\left(\frac{x_i}{c}\right) + a^2 + \right. \\ &\left. +2ab\sinh^{-1}\left(\frac{x_i}{c}\right) + b^2\left(\sinh^{-1}\left(\frac{x_i}{c}\right)\right)^2\right) \to \min \end{aligned} \quad (17)$$

the partial derivatives of the parameters *a*, *b*, and *c* form the following system of equations:

$$\begin{aligned} \frac{\partial U}{\partial a} &= 2\sum_{i=1}^{m}\left(b\sinh^{-1}\left(\frac{x_i}{c}\right)\right) - 2\sum_{i=1}^{m} y_i + 2am = 0 \\ \frac{\partial U}{\partial b} &= \sum_{i=1}^{m}\left[-2\sinh^{-1}\left(\frac{x_i}{c}\right)\left(-a - b\sinh^{-1}\left(\frac{x_i}{c}\right) + y_i\right)\right] = 0 \\ \frac{\partial U}{\partial c} &= \sum_{i=1}^{m}\frac{2bx_i\left(-a - b\sinh^{-1}\left(\frac{x_i}{c}\right) + y_i\right)}{c^2\sqrt{1 + \frac{x_i^2}{c^2}}} = 0 \end{aligned} \quad (18)$$

The above system of equations cannot be solved analytically, as such an attempt leads to an implicit equation involving a single variable. Consequently, a numerical approach was developed to determine the parameters of the Vom Berg model. For this purpose, the simple gradient method, previously described in the author's earlier works, was applied. This method is relatively straightforward to implement and computationally efficient for solving nonlinear equations. It is based on defining and computing a partial derivative vector, which aligns with the error function gradient but moves in the opposite direction. The algorithm iterates along this direction as long as the error function value decreases. Once the function ceases to decrease, a new vector is computed [8–11].

In this case, the vector is three-dimensional, with a unit length set to 1 to accelerate calculations. The error function, where the parameters *x*, *y*, and *z* represent the rheological parameters of the model, is formulated as follows:

$$U(x,y,z) = \sum_{i=0}^{n}\left(f(x,y,z) - y_i\right)^2 \to \min \quad (19)$$

the vector follows the direction of the error function gradient but has an opposite orientation, with a length equal to 1:

$$\hat{v} = \frac{-\vec{\nabla}U}{\left|-\vec{\nabla}U\right|} \quad (20)$$

as a result of numerical calculations, the final vector is obtained:

$$\vec{v} = (x, y, z) \quad (21)$$

whose values are:

$$\begin{gathered} \tau = \tau_y + B\sinh^{-1}\left(\frac{-\frac{dv}{dr}}{C}\right) \\ \tau_y = v_x \\ B = v_y \\ C = v_z \end{gathered} \quad (22)$$

In the RheoSolution methodology, statistical criteria are applied to assess the quality of the model's fit to the measurement data:

– the sum of squared differences:

$$U = \sum_{i=1}^{m}\left(y_i - \hat{y}\right)^2 \quad (23)$$

– Pearson linear correlation coefficient:

$$R = \sqrt{1 - \frac{U}{\sum_{i=1}^{m}\left(y_i - \bar{y}\right)^2}} = \sqrt{1 - \frac{\sum_{i=1}^{m}\left(y_i - \hat{y}\right)^2}{\sum_{i=1}^{m}\left(y_i - \bar{y}\right)^2}} \quad (24)$$

– Fischer–Sneadecor coefficient:

$$F = \frac{R^2}{1 - mR^2} \tag{25}$$

The above methodology is implemented in the RheoSolution software and has been used in the interpretation of laboratory results presented in the next chapter (Tabs. 1–18, Figs. 1–9).

3. Laboratory studies conducted at the Faculty of Drilling, Oil, and Gas

To evaluate the applicability of the Vom Berg rheological model in describing the rheological properties of polymer-modified drilling muds, a series of laboratory experiments were conducted at the Drilling Fluids Laboratory of the Faculty of Drilling, Oil, and Gas. The primary objective was to compare the effectiveness of the Vom Berg model with the widely used Bingham and Ostwald–de Waele models recommended by API.

The compositions of the tested drilling muds were developed in collaboration with Polski Serwis Płynów Wiertniczych Sp. z o.o. (PSPW Krosno), a company specializing in drilling fluid services. This cooperation ensured that the tested mud formulations closely resembled real-world drilling fluid systems used in industrial applications.

The study involved nine drilling mud samples, each with a different composition:

– Sample 1 – A base bentonite mud containing only water and 5% Bentopol Żebiec bentonite, serving as the control sample.
– Sample 2 – The base mud with 0.1% xanthan biopolymer, used to assess the impact of biopolymer addition on rheology.
– Sample 6 – The base mud with 0.3% xanthan biopolymer, allowing evaluation of the effect of increased biopolymer concentration.
– Samples 3, 4, 5, 7, 8, and 9 – Mud formulations incorporating additional rheological additives such as Alcomer and Desco [12, 13], commonly used in the drilling industry to enhance stability and control filtration properties. These additives were included to replicate industrial drilling mud formulations and assess their influence on rheological model fitting.

Rheological measurements were conducted using a FANN 12-speed rotational viscometer, recommended by API [1]. The data obtained from these experiments were processed and analyzed using RheoSolution 5.0, a proprietary software based on the RheoSolution methodology developed by Prof. Rafał Wiśniowski and the author of this study [14]. This advanced tool facilitated precise rheological model fitting and comprehensive evaluation of their effectiveness in describing the properties of the tested drilling muds. The results, presented in the following sections in the form of tables and graphs, highlight the extent to which the Vom Berg model outperforms API-recommended models in capturing the rheological behavior of polymer-modified drilling muds. Special emphasis was placed on comparing the performance of each model across different shear rate ranges, providing insight into their suitability for various operational conditions in drilling technology.

For simplicity and better readability of the graphs, their axes are labeled with the Greek letters **τ**, representing shear stress [Pa], and **γ**, denoting shear rate (–*dv*/*dr*) with the unit 1/s.

Table 1. Results of rheological measurements for sample No. 1 (Bentonite 5% without the addition)

Rotor speed [rot/min]	Angle [o]	Shear rate [1/s]	Shear stress [Pa]
600	28	1 022.040	14.299
300	20	511.020	10.214
200	17	340.680	8.682
100	14	170.340	7.149
60	12	102.204	6.128
30	11	51.102	5.617
20	10	34.068	5.107
10	10	17.034	5.107
6	9	10.220	4.596
3	9	5.110	4.596
2	8	3.406	4.085
1	8	1.703	4.085

Table 2. Summary of correlation coefficients of the analyzed rheological models for sample No. 1

Rheological model	Pearson correlation coefficient, *R*	Fischer–Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.987	377.93	2.66
Ostwald–de Waele	0.921	54.99	15.88
Vom Berg	0.953	102.99	9.14

The determined rheological parameters of the Vom Berg model for sample No. 1: parameter *B* = 2.22 Pa·s, parameter *C* = 45.36 [–], parameter *YP* = 3.74 Pa.

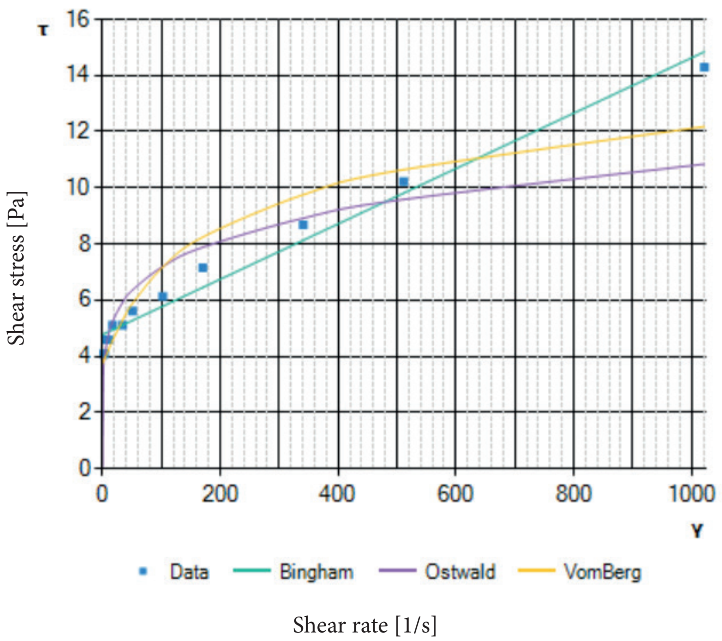


Fig. 1. Comparison of the Vom Berg model with the Bingham and Ostwald–de Waele model (API) for sample No. 1

Table 3. Results of rheological measurements for sample No. 2
(Bentonite 5% with the addition of 0.1% xanthan gum biopolymer (BWOC))

Rotor speed [rot/min]	Angle [o]	Shear rate [1/s]	Shear stress [Pa]
600	54	1 022.040	27.577
300	42	511.020	21.448
200	35	340.680	17.874
100	26	170.340	13.278
60	22	102.204	11.235
30	19	51.102	9.703
20	18	34.068	9.192
10	16	17.034	8.171
6	15	10.220	7.661
3	14	5.110	7.149
2	14	3.406	7.149
1	13	1.703	6.639

Table 4. Summary of correlation coefficients of the analyzed rheological models for sample No. 2

Rheological model	Pearson correlation coefficient, *R*	Fischer–Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.973	177.91	26.09
Ostwald–de Waele	0.944	81.89	53.36
Vom Berg	0.998	2 257.38	2.16

Determined rheological parameters of the Vom Berg model for sample No. 2: parameter B = 8.15 Pa·s, parameter C = 177.45 [–], parameter YP = 7.01 Pa.

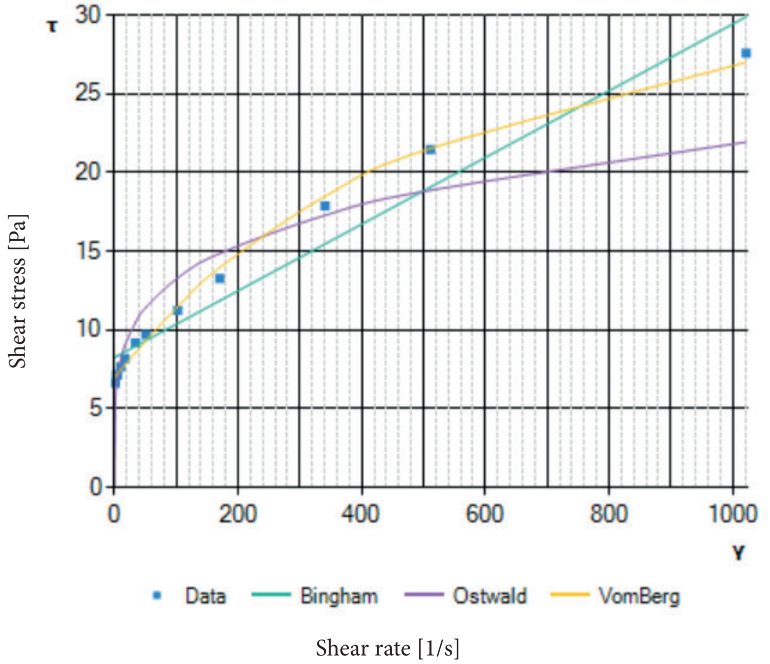


Fig. 2. Comparison of the Vom Berg model with the Bingham and Ostwald–de Waele model (API) for sample No. 2

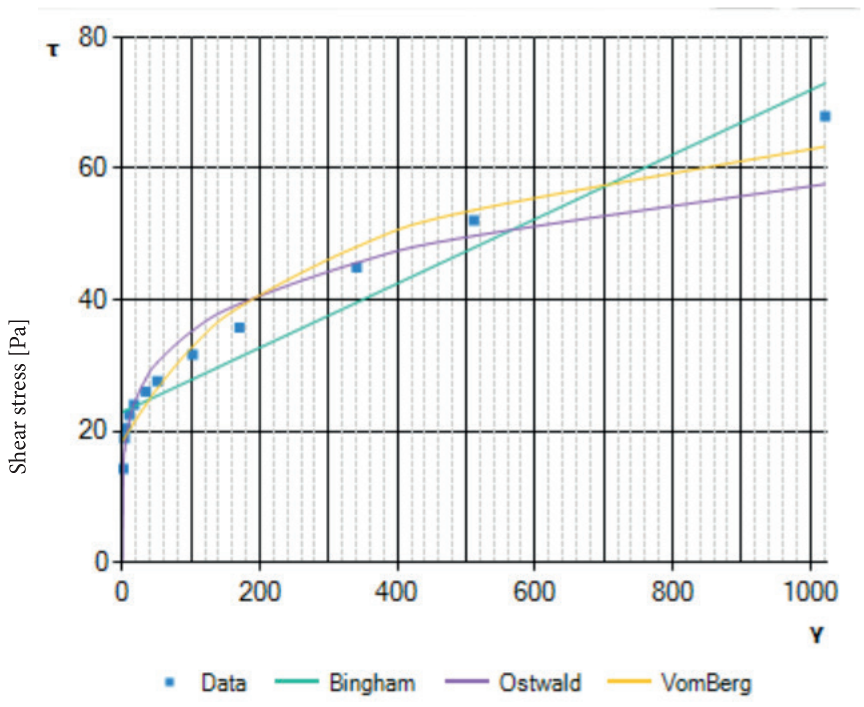
Table 5. Results of rheological measurements for sample No. 3

Rotor speed [rot/min]	Angle [o]	Shear rate [1/s]	Shear stress [Pa]
600	133	1 022.040	67.920
300	102	511.020	52.089
200	88	340.680	44.940
100	70	170.340	35.747
60	62	102.204	31.662
30	54	51.102	27.577
20	51	34.068	26.045
10	47	17.034	24.002
6	44	10.220	22.470
3	40	5.110	20.427
2	37	3.406	18.895
1	28	1.703	14.299

Table 6. Summary of correlation coefficients of the analyzed rheological models for sample No. 3 (Bentonite 5% with the addition of 0.1% xanthan gum biopolymer and Alcomer 0.02% (BWOC))

Rheological model	Pearson correlation coefficient, R	Fischer–Sneadecor coefficient, F	Sum of squares, U
Bingham	0.950	116.453	213.82
Ostwald–de Waele	0.970	161.603	157.56
Vom Berg	0.985	312.740	83.77

Determined rheological parameters of the Vom Berg model for sample No. 3: parameter B = 14.06 Pa·s, parameter C = 84.03 [–], parameter YP = 18.43 Pa.



Shear rate [1/s]

Fig. 3. Comparison of the Vom Berg model with the Bingham and Ostwald–de Waele model (API) for sample No. 3

Table 7. Results of rheological measurements for sample No. 4
(Bentonite 5% with the addition of 0.1% xanthan gum biopolymer and Alcomer 0.02% + Desco 0.1% (BWOC))

Rotor speed [rot/min]	Angle [o]	Shear rate [1/s]	Shear stress [Pa]
600	84	1 022.040	42.897
300	62	511.020	31.662
200	53	340.680	27.066
100	41	170.340	20.938
60	35	102.204	17.874
30	30	51.102	15.320
20	28	34.068	14.299
10	24	17.034	12.256
6	22	10.220	11.235
3	20	5.110	10.214
2	19	3.406	9.703
1	17	1.703	8.682

Table 8. Summary of correlation coefficients of the analyzed rheological models for sample No. 4

Rheological model	Pearson correlation coefficient, *R*	Fischer–Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.969	155.354	73.18
Ostwald–de Waele	0.965	137.213	82.20
Vom Berg	0.980	238.580	48.68

Determined rheological parameters of the Vom Berg model for sample No. 4: parameter $B = 7.96$ Pa·s, parameter $C = 49.20$ [–], parameter $YP = 8.49$ Pa.

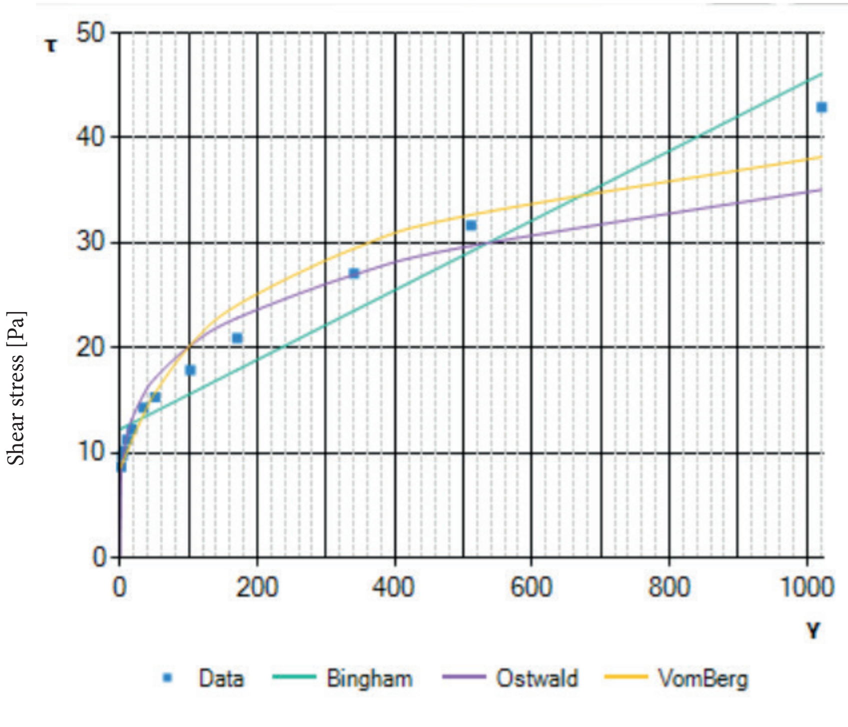


Fig. 4. Comparison of the Vom Berg model with the Bingham and Ostwald–de Waele model (API) for sample No. 4

Table 9. Results of rheological measurements for sample No. 5
(Bentonite 5% with the addition of 0.1% xanthan gum biopolymer and Alcomer 0.02% + Desco 0.2% (BWOC))

Rotor speed [rot/min]	Angle [o]	Shear rate [1/s]	Shear stress [Pa]
600	74	1 022,040	60,260
300	53	511,020	39,833
200	44	340,680	31,151
100	33	170,340	22,981
60	27	102,204	18,895
30	22	51,102	14,810
20	19	34,068	13,278
10	17	17,034	10,214
6	15	10,220	7,660
3	14	5,110	5,107
2	13	3,406	4,085
1	11	1,703	3,064

Table 10. Summary of correlation coefficients of the analyzed rheological models for sample No. 5

Rheological model	Pearson correlation coefficient, *R*	Fischer–Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.973	183.920	55.63
Ostwald–de Waele	0.965	135.854	73.97
Vom Berg	0.979	219.938	46.92

Determined rheological parameters of the Vom Berg model for sample No. 5: parameter B = 7.57 Pa·s, parameter C = 50.64 [–], parameter YP = 5.15 Pa.

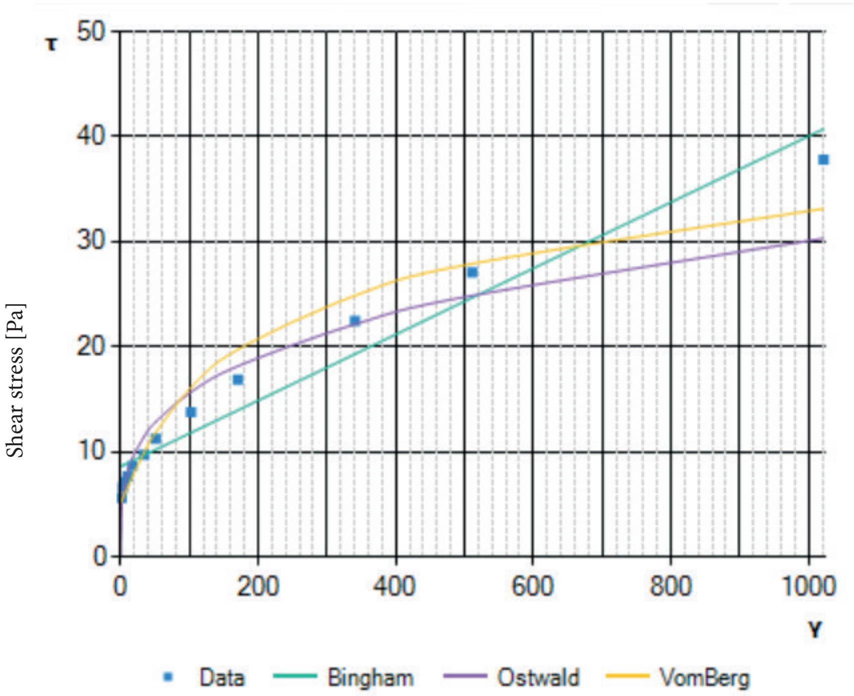


Fig. 5. Comparison of the Vom Berg model with the Bingham and Ostwald–de Waele model (API) for sample No. 5

Table 11. Results of rheological measurements for sample No. 6
(Bentonite 5% with the addition of 0.3% xanthan gum (BWOC))

Rotor speed [rot/min]	Angle [o]	Shear rate [1/s]	Shear stress [Pa]
600	100	1 022.040	51.068
300	73	511.020	37.279
200	62	340.680	31.662
100	48	170.340	24.513
60	41	102.204	20.938
30	35	51.102	17.874
20	32	34.068	16.342
10	28	17.034	14.299
6	26	10.220	13.278
3	24	5.110	12.256
2	22	3.406	11.235
1	20	1.703	10.214

Table 12. Summary of correlation coefficients of the analyzed rheological models for sample No. 6

Rheological model	Pearson correlation coefficient, *R*	Fischer–Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.972	174.430	92.85
Ostwald–de Waele	0.961	122.132	129.61
Vom Berg	0.994	734.043	23.01

Determined rheological parameters of the Vom Berg model for sample No. 6: parameter $B = 14.13$ Pa·s, parameter $C = 151.03$ [–], parameter $YP = 11.73$ Pa.

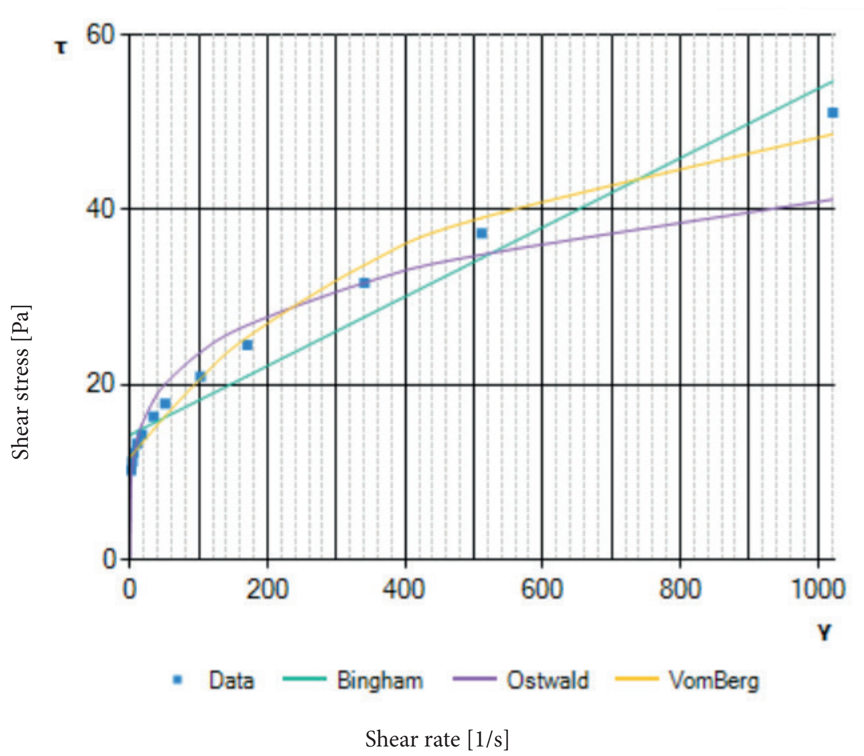


Fig. 6. Comparison of the Vom Berg model with the Bingham and Ostwald–de Waele model (API) for sample No. 7

Table 13. Results of rheological measurements for sample No. 7
(Bentonite 5% with the addition of 0.3% xanthan gum and Alcomer 0.01% (BWOC)

Rotor speed [rot/min]	Angle [o]	Shear rate [1/s]	Shear stress [Pa]
600	146	1 022.040	74.559
300	115	511.020	58.728
200	99	340.680	50.557
100	82	170.340	41.876
60	75	102.204	38.301
30	68	51.102	34.726
20	63	34.068	32.173
10	58	17.034	29.619
6	53	10.220	27.066
3	46	5.110	23.491
2	41	3.406	20.938
1	37	1.703	18,895

Table 14. Summary of correlation coefficients of the analyzed rheological models for sample No. 7

Rheological model	Pearson correlation coefficient, *R*	Fischer–Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.946	85.680	317.48
Ostwald–de Waele	0.979	241.592	120.74
Vom Berg	0.982	261.863	111.74

Determined rheological parameters of the Vom Berg model for sample No. 7: parameter B = 13.71 Pa·s, parameter C = 65.38 [–], parameter YP = 22.42 Pa.

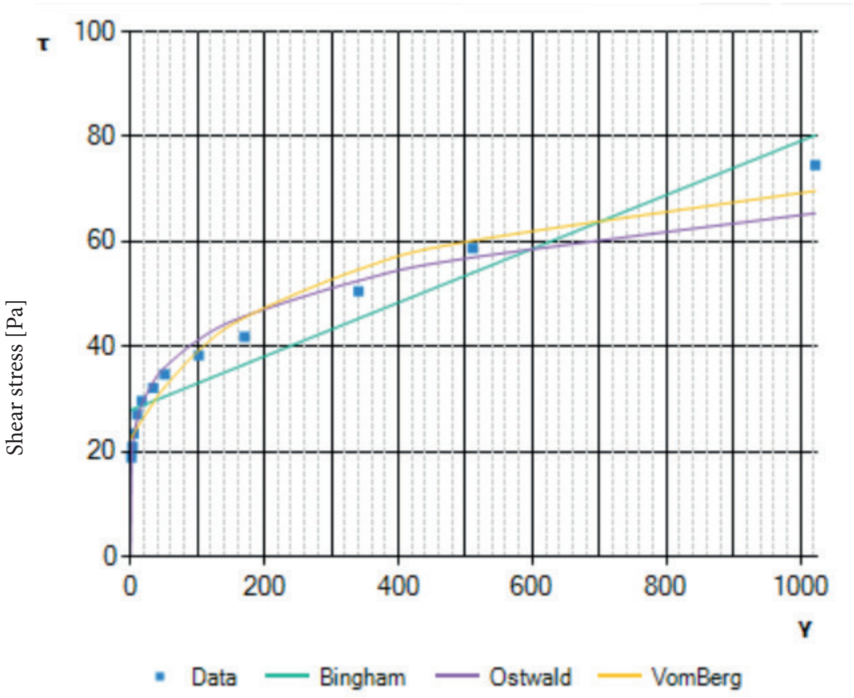


Fig. 7. Comparison of the Vom Berg model with the Bingham and Ostwald–de Waele model (API) for sample No. 7

Table 15. Results of rheological measurements for sample No. 8
(Bentonite 5% with the addition of 0.3% xanthan gum and Alcomer 0.02% + Desco 0.1% (BWOC))

Rotor speed [rot/min]	Angle [o]	Shear rate [1/s]	Shear stress [Pa]
600	118	1 022.040	60,260
300	91	511.020	46.472
200	78	340.680	39.833
100	62	170.340	31.662
60	54	102.204	27.577
30	46	51.102	23.491
20	43	34.068	21.959
10	38	17.034	19.406
6	35	10.220	17.874
3	33	5.110	16.852
2	31	3.406	15.831
1	28	1.703	14.299

Table 16. Summary of correlation coefficients of the analyzed rheological models for sample No. 8

Rheological model	Pearson correlation coefficient, *R*	Fischer–Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.963	129.356	159.93
Ostwald–de Waele	0.966	143.513	145.18
Vom Berg	0.996	997.096	22.13

Determined rheological parameters of the Vom Berg model for sample No. 8: parameter $B = 16.23$ Pa·s, parameter $C = 152.80$ [–], parameter $YP = 16.54$ Pa.

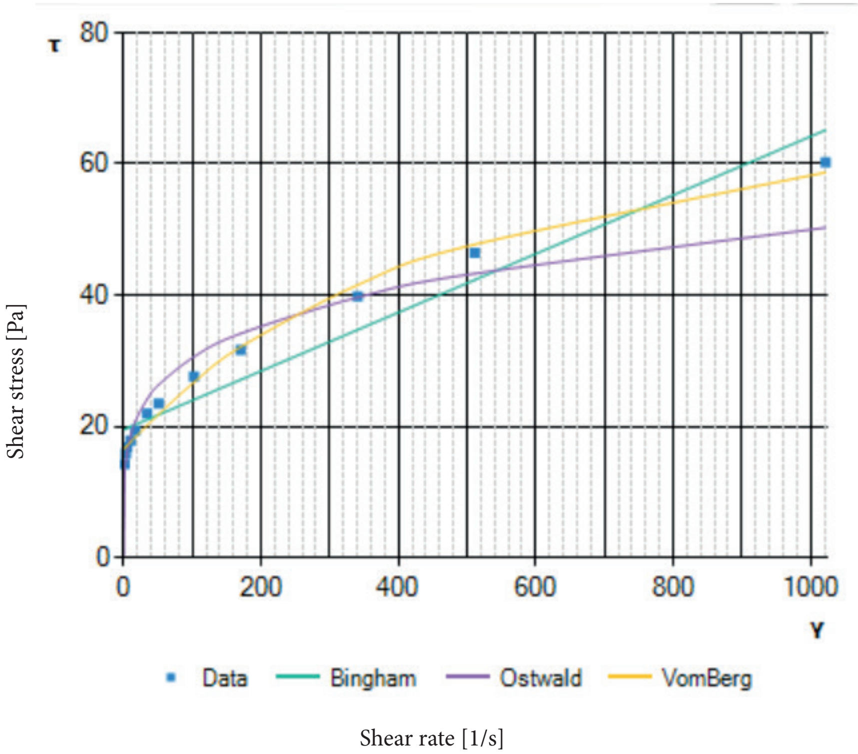


Fig. 8. Comparison of the Vom Berg model with the Bingham and Ostwald–de Waele model (API) for sample No. 8

Table 17. Results of rheological measurements for sample No. 9
(Bentonite 5% with the addition of 0.3% xanthan gum and Alcomer 0.02% + Desco 0.2% (BWOC))

Rotor speed [rot/min]	Angle [o]	Shear rate [1/s]	Shear stress [Pa]
600	92	1 022.040	46.982
300	67	511.020	34.215
200	55	340.680	28.087
100	42	170.340	21.448
60	36	102.204	18.384
30	29	51.102	14.810
20	26	34.068	13.278
10	22	17.034	11.235
6	20	10.220	10.214
3	17	5.110	8.682
2	16	3.406	8.171
1	15	1.703	7.660

Table 18. Summary of correlation coefficients of the analyzed rheological models for sample No. 9

Rheological model	Pearson correlation coefficient, R	Fischer–Sneadecor coefficient, F	Sum of squares, U
Bingham	0.969	157.452	98.03
Ostwald–de Waele	0.971	167.507	92.48
Vom Berg	0.992	599.254	26.94

Determined rheological parameters of the Vom Berg model for sample No. 9: parameter B = 12.19 Pa·s, parameter C = 110.302 [–], parameter YP = 8.301 Pa.

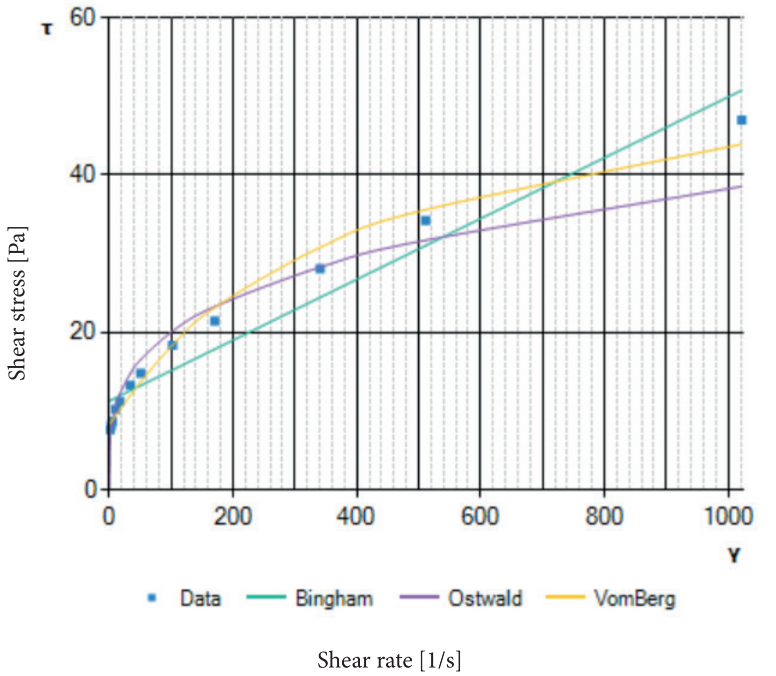


Fig. 9. Comparison of the Vom Berg model with the Bingham and Ostwald–de Waele model (API) for sample No. 9

Conclusions

The laboratory studies conducted at the Drilling Fluids Laboratory of the Faculty of Drilling, Oil, and Gas aimed to assess the applicability of the Vom Berg rheological model in describing the relationship between shear stress and shear rate in polymer-modified drilling muds. The results indicate that this model exhibits significantly higher correlation with experimental data compared to the Bingham and Ostwald–de Waele models, which are widely used in API methodology, particularly in the case of polymer-enhanced drilling muds. In the control sample, which consisted of a simple bentonite-based mud (5% Bentopol Zębiec bentonite) [15], the highest correlation was obtained for the Bingham model (R = 0.987), suggesting that in simple water-bentonite systems, its linear nature provides a better fit than the nonlinear Vom Berg model (R = 0.953). However, the situation changes with the introduction of polymer modifications. Upon adding 0.1% xanthan biopolymer in sample 2, the correlation coefficient for the Vom Berg model increased to 0.998, whereas for the Bingham model, it dropped to 0.973, and for the Ostwald–de Waele model, it was 0.944. In sample 6, where the biopolymer concentration was increased to 0.3%, this trend persisted – the Vom Berg model achieved a correlation of 0.994, outperforming both the Bingham model (0.972) and the Ostwald–de Waele model (0.961). The analysis of the remaining samples (3, 4, 5, 7, 8, and 9), which were further modified with Alcomer and Desco, confirmed the superior correlation of the Vom Berg model in describing their rheological behavior. Alcomer is widely used as a hydration inhibitor and filtration control agent, while Desco acts as a dispersant, improving viscosity control [12, 13]. The inclusion of these additives aimed to replicate real-world drilling fluid compositions, ensuring a more practical evaluation of the effectiveness of the tested rheological models.

The findings indicate that while API-recommended models, particularly the Bingham model, provide a good correlation (with coefficients not falling below 0.94), the Vom Berg model consistently exhibits a higher level of accuracy for polymer-modified drilling muds. The Ostwald–de Waele model, with correlation values not lower than 0.95, proves most effective in the low shear rate range, whereas the Bingham model, though more versatile across a wide range of shear rates encountered in drilling operations, does not fully capture the complex flow behavior of polymer-enhanced drilling muds.

The key conclusion of this study is that while API-recommended rheological models ensure a satisfactory fit to experimental data for laboratory-tested drilling muds, the application of the RheoSolution methodology and the use of the Vom Berg model significantly enhance the correlation, achieving values close to unity. These results suggest that the Vom Berg model may serve as a more appropriate tool for describing the rheology of polymer-modified drilling muds, offering potential benefits for improving the design and performance control of drilling fluids used in the oil and gas industry.

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Nomenclature

a, b, c – regression model coefficients [–]
B, C – rheological parameter in the Eyring model [–]
F – Fisher–Snedecor index [–]
U – sum of squared residuals [–]
dv/dr – shear rate gradient [s^{-1}]
η_{pl} – plastic viscosity [Pa·s]
$\dot{\gamma}_I$ – shear rate measured at *i*-th rotational speed [s^{-1}]
k – coefficient of consistency [Pa·s^n]
m – number of measurements with viscometer [–]
n – exponential index [–]
R – Pearson's correlation coefficient [–]
τ – shear stress [Pa]
τ_i – shear stress measured at *i*-th rotational speed [Pa]
YP, τ_y – yield point [Pa]
$\bar{\tau}$ – average value of shear stress [Pa]