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ARTICLE

MATHEMATICAL MODEL TO ANALYZE THE GEOMETRIC LIMITATIONS OF MECHATRONIC DEVICES MOVING IN CURVED PIPE SECTIONS

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Abstract: This article presents a method enabling the determination of the minimum radius of pipe bending in which it is possible to move freely a cylinder with defined dimensions. A respective mathematical model has been presented. The below-described method can be useful in the future, while designing mechatronic tools for working in lateral bores starting from a vertical bore, and also while designing inspection robots moving in pipes and pipelines.

Keywords: drilling, radial drilling, pipeline inspection robots, mechatronic downhole assemblies, robotics, mobile robots

1. Introduction

Over the last twenty years, we have observed [1–5] the dynamic development of technologies in the drilling industry enabling the construction of small-profile lateral bores starting from vertical bores. This technology enables low-cost reaming and testing of production layers. It also enables the increased productivity of existing deposits. Methods developed all over the world use the phenomenon of rock hydro-quarrying, where a tool with a specially constructed nozzle is introduced through a deflector, by means of which a lateral borehole is made with the use of a stream of liquid under high pressure.

We have also seen the development of mechatronic subsurface systems in the drilling industry, which enable different kinds of operations when run to the borehole. An example of such devices may be drilling rigs for drilling in a space environment [6]. We can imagine that similar devices will also be available for lateral bores in the future and they could be used for conducting local geophysical analyses, local deposit tests, and many other kinds of operations. In the case of horizontal bores, the section where the trajectory changes is counted in terms of many meters, but with the lateral bores described herein, the change occurs at lengths no bigger than several meters, and frequently at even less than one meter. The geometric dimensions of such mechatronic subsurface systems have to be adapted in such a way that they are able to go through a deflector from a vertical borehole to a lateral borehole and back.

We have also been observing the development of mobile robots for pipe inspection and cleaning. The geometric limitations described above also concern this kind of devices, since they have to be able to freely deal with bends of pipes with a radius frequently smaller than one meter, or even much less.

For these reasons, a method enabling the verification of whether a device modelled with a cylinder shape with the radius r and length H, may go freely through a pipe with a bore with a radius R and a bend radius ρ, has been developed. A mathematical model has been provided for the method under consideration. Its implementation will enable the determination of a given value of parameters r, H, R (and more precisely $a = R - r$) the minimum value of the bend radius ρ, ensuring the free movement of that device.

2. Definition of the scientific problem

The issue described in this article is a problem of running (pushing) a cylinder with the radius r and height H in a curved pipe with an internal radius R, the internal pass-through part of which is first a cylinder, next 1/4 torus, and then a cylinder again. The internal radius (of the borehole) is equal to $r + a$, where "a" is a given clearance value. Minimum pipe bending radiuses (torus radius) ρ_{min}, through which the cylinder of a given size will go through for a specific r value and different "a" and H values, have to be determined.

This problem can be presented mathematically. To this end, it is necessary to define a set describing the geometric limitations of the pipe inside which a cylinder moves. It should be defined against the selected main coordinate system indicated as $O(x_0, y_0, z_0)$. Next, other cross-sectional fields are created in the agreed pipe section, which determine planes perpendicular to the axis of that pipe. It is then checked whether it is possible to find on its area the position of the center of gravity of the cylinder with a specific radius r and height H for each of those cross-sectional fields, so that it is possible to establish the orientation of the said cylinder so that it does not go beyond the pipe area in which it runs. Failure to meet this condition in at least one cross-sectional field of the pipe means that it is impossible to move a cylinder through it with the required geometric dimensions. In order to conduct such analysis, it is necessary to define a set to describe geometric limitations of the cylinder being moved. It should be remembered that the position of the center of gravity of the cylinder may differ, like its orientation, and that is why it is best to describe it in the local coordinate system (related to that cylinder), and subsequently transform it to the $O(x_0, y_0, z_0)$ system, in which the geometric limitations of the pipe are described mathematically. The mathematical model presented in the section below enables mathematical notation of ideas to be described. It is worth remembering that subsequent cutting planes are indexed by the α parameter. Since the value of that parameter is included in the continuous set of real numbers, the number of possible cross-sectional areas of the pipe is indefinite and discretization of this parameter is necessary. In practice, it suffices to conduct the above-described analysis for subsequent cross-sectional areas of the pipe with values of the α parameter changing by an adequately small step so that it is still accurate and corresponds to real conditions.

3. Results

A mathematical model was created using the description employed in robotics to describe manipulators. Figures 1–3 present the coordinate systems, angles, and symbols applied in the mathematical model.

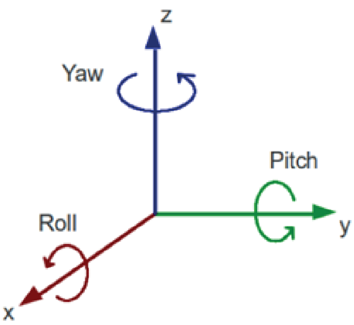

Fig. 1. RPY angles – Roll (ϕ), Pitch (θ), Yaw (ψ)

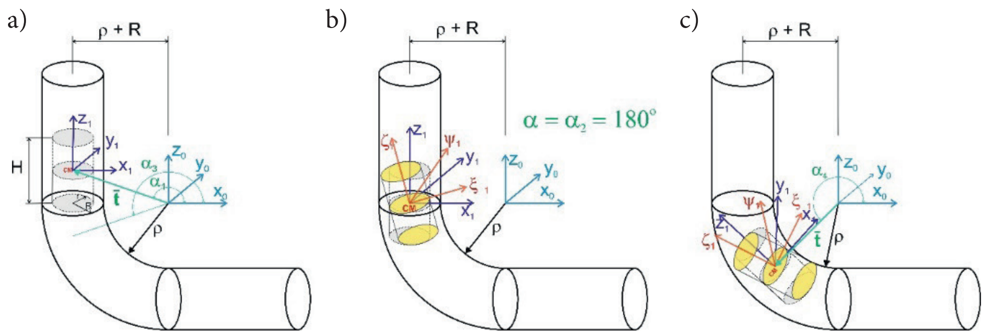

Fig. 2. Conceptual drawings for the mathematical model

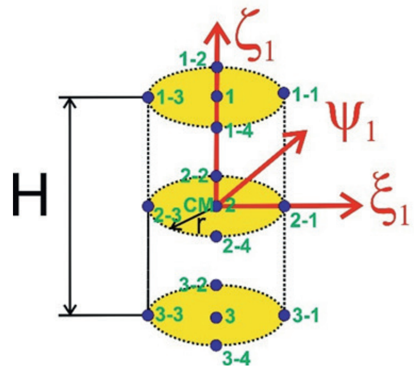

Fig. 3. Cylinder moving through a pipe hole and main points on it

3.1. Mathematical notations used

The following mathematical notations were used in the created mathematical model:

$O(x_0, y_0, z_0)$ – a coordinate system against which the geometric limitations of the pipe through which a cylinder moves are described. This is a basic coordinate system to which coordinates described against other coordinate systems are transformed,

$O(x_1, y_1, z_1)$ – a coordinate system related to the cylinder. The circle being the cross-section of the cylinder through the plane parallel to both its bases and going through the center of gravity, *CM*, is normally set to the axis of the pipe hole through which this cylinder moves,

$O(\xi_1, \psi_1, \zeta_1)$ – a coordinate system related to the cylinder rotated around the center of gravity by the Roll (ϕ) and Pitch (θ) angles,

CM – the center of gravity of the cylinder,

t – the vector connecting the center of the $O(x_0, y_0, z_0)$ system with the center of gravity of the cylinder,

α – the angle between the 0-x_0 axis and the $\boldsymbol{t}$ vector,
ϕ – the angle of rotation of the cylinder around the 0-x_1 axis (Roll) beginning in the center of gravity, CM, of this cylinder,
ϕ_{min} – the minimum value of the angle of rotation of the cylinder around the 0-x_1 axis (Roll) beginning in the center of gravity, CM, of this cylinder,
ϕ_{max} – the maximum value of the angle of rotation of the cylinder around the 0-x_1 axis (Roll) beginning in the center of gravity, CM, of this cylinder,
$\Delta\phi$ – a change (step) of the angle of rotation of the cylinder around the 0-x_1 axis (Roll) beginning in the center of gravity, CM, of this cylinder,
θ – the angle of rotation of the cylinder around the 0-y_1 axis (Pitch) beginning in the center of gravity, CM, of this cylinder,
θ_{min} – the minimum value of the angle of rotation of the cylinder around the 0-y_1 axis (Pitch) beginning in the center of gravity, CM, of this cylinder,
θ_{max} – the maximum value of the angle of rotation of the cylinder around the 0-y_1 axis (Pitch) beginning in the center of gravity, CM, of this cylinder,
$\Delta\theta$ – a change (step) of the angle of rotation of the cylinder around the 0-y_1 axis (Pitch) beginning in the center of gravity, CM, of this cylinder
$A1$ – matrix transforming coordinates described in the $O(x_1, y_1, z_1)$ system to coordinates in the $O(x_0, y_0, z_0)$ system,
$A2$ – matrix transforming coordinates described in the $O(\xi_1, \psi_1, \zeta_1)$ system to coordinates in the $O(x_1, y_1, z_1)$ system,
ρ – the bend radius of the pipe,
ρ_{min} – the minimum bend radius of the pipe,
ρ_{max} – the maximum bend radius of the pipe,
$\Delta\rho$ – a change in the bend radius of the pipe (step),
r – the radius of the cylinder,
r_{min} – the minimum radius of the cylinder,
r_{max} – the maximum radius of the cylinder,
Δr – a change (step) of the cylinder radius,
H – height of the cylinder moving through the pipe,
H_{min} – the minimum height of the cylinder moving through the hole,
H_{max} – the maximum height of the cylinder moving through the hole,
ΔH – a change in the height of the cylinder moving through the hole,
R – the radius of the hole in the pipe: $R = r + 2a$,
a – clearance between the cylinder and the internal wall of the pipe hole,
a_{min} – the minimum clearance between the cylinder and the internal wall,
a_{max} – the maximum clearance between the cylinder and the internal wall of the pipe hole,
Δa – a change (step) of clearance between the cylinder and the internal wall of the pipe hole.

3.2. Mathematical model

The $O(x_0, y_0, z_0)$ system is a basic coordinate system. In this system, geometric limitations of the pipe (through which the cylinder moves) have been described. The pipe is first a cylinder, then it constitutes 1/4 of a torus and a cylinder again, which has been described in the below formulas (1), (2) and (3), respectively. The sum of S_1, S_2, S_3 sets gives an S set, which represents the whole pipe (4).

$$S_1 = \{(x,y,z): x \le 0 \wedge z \ge 0 \wedge (x + (\rho + R))^2 + y^2 \le R^2 \wedge z \le 2H\} \quad (1)$$

$$S_2 = \{(x,y,z): x \le 0 \wedge z \le 0 \wedge (\sqrt{x^2 + z^2} - (\rho + R))^2 + y^2 \le R^2\} \quad (2)$$

$$S_3 = \{(x,y,z): x \ge 0 \wedge z \le 0 \wedge y^2 + (z + (\rho + R))^2 \le R^2 \wedge x \le 2H\} \quad (3)$$

$$S = \bigcup_{i=1}^{3} S_i = S_1 \cup S_2 \cup S_3 \quad (4)$$

S_1, S_2, S_3 sets are sets of points in the $\boldsymbol{R^3}$ space of real numbers. Those sets depend on parameters ρ, R, $\boldsymbol{H}$, which signify the bend radius of the pipe, the radius of the pipe hole and cylinder height, respectively. The $2H$ value was introduced in order to limit S_1 and S_3 sets. The adopted limitation assumes arbitrarily that the height of cylinders represented by S_1 and S_3 sets is twice as big as the height of the cylinder moving through the pipe. There is dependence between the radius of the hole in the pipe and the radius of the cylinder and clearance (5).

$$\boldsymbol{R} = \boldsymbol{r} + 2\boldsymbol{a} \quad (5)$$

The cylinde, which moves through a curved pipe described with the S set is described against the $O(\xi_1, \psi_1, \zeta_1)$ system. As mentioned in Item 2, the method proposed in this article assumes that for subsequent cross-sections of the pipe included in planes perpendicular to the pipe axis it is checked whether there is such a position of the center of gravity of the cylinder (located within the area of a given cross-section) and its orientation which ensures that the cylinder is included in the pipe. From the mathematical point of view, such an operation boils down to checking whether for subsequent centers of gravity of the cylinder, located within the above-mentioned cross-section, there is at least one set of points (x, y, z) which describes this cylinder (one cylinder location), which is included in the S set. If this condition is not met, it means that the cylinder projects beyond the pipe and it is impossible to move it. Other section planes are indexed by the α parameter, and possible centers of gravity of the cylinder within the cross-section area are determined by $((x_1,CM)_i, (y_1,CM)_j)$. The condition described is expressed here by the following formula (6) and (7).

$$C \subset S \tag{6}$$

$$C_{\alpha,((x_{1,CM})_i,(y_{1,CM})_j)} = \{(\xi_1,\psi_1,\zeta_1): \left(\zeta_1 \geq -\frac{H}{2}\right) \wedge \left(\zeta_1 \leq \frac{H}{2}\right) \wedge ((\xi_1)^2+(\psi_1)^2 \leq r^2)\} \tag{7}$$

The formula (7) is very general. In the designation of this set C, and more precisely in the index, there is information that the center of gravity of the cylinder described with the C set is in the point at $((x_1, CM)_i, (y_1, CM)_j)$, located on the cutting plane described with the α parameter. Those symbols will be explained below. Checking of the condition (6) is difficult when applying a general notation of C sets. However, one may use the property of the solid, which a cylinder is, that it is a convex set. For a given set to be convex, each section made up by two points being set elements has to be included in this set. Owing to this property of the C set, if an adequately dense grid is defined on its edge, all of the points will belong to the S set, it may be assumed that the C set is included in the S set. Formulas from (8) to (11) define a set of subsequent points in the grid on the edge of the cylinder being the set described with the formula (7).

$$\partial(C_{\alpha,((x_{1,CM})_i,(y_{1,CM})_j)})_1 = \{((\xi_1)_{klm},(\psi_1)_{klm},(\zeta_1)_{klm}): k = 0, \ldots, [2\pi r/0.5];\ l = 0;\ m = 0, \ldots, (r/0.5);\ \Delta\varphi = 0.5/r; (\xi_1)_{klm} = 0.5 \cdot m \cdot \cos(k \cdot \Delta\varphi);\ (\psi_1)_{klm} = 0.5 \cdot m \cdot \sin(k \cdot \Delta\varphi); (\zeta_1)_{klm} = -(H/2) + 0.5 \cdot l\} \tag{8}$$

$$\partial(C_{\alpha,((x_{1,CM})_i,(y_{1,CM})_j)})_2 = \{((\xi_1)_{klm},(\psi_1)_{klm},(\zeta_1)_{klm}): k = 0, \ldots, [2\pi r/0.5];\ l = 1; \ldots, ((H/0.5) - 1);\ m = (r/0.5); \Delta\varphi = 0.5/r;\ (\xi_1)_{klm} = 0.5 \cdot m \cdot \cos(k \cdot \Delta\varphi); (\psi_1)_{klm} = 0.5 \cdot m \cdot \sin(k \cdot \Delta\varphi);\ (\zeta_1)_{klm} = -(H/2) + 0.5 \cdot l\} \tag{9}$$

$$\partial(C_{\alpha,((x_{1,CM})_i,(y_{1,CM})_j)})_3 = \{((\xi_1)_{klm},(\psi_1)_{klm},(\zeta_1)_{klm}): k = 0, \ldots, [2\pi r/0.5];\ l = (H/0.5);\ m = 0, \ldots, (r/0.5);\ \Delta\varphi = 0.5/r; (\xi_1)_{klm} = 0.5 \cdot m \cdot \cos(k \cdot \Delta\varphi);\ (\psi_1)_{klm} = 0.5 \cdot m \cdot \sin(k \cdot \Delta\varphi); (\zeta_1)_{klm} = -(H/2) + 0.5 \cdot l\} \tag{10}$$

$$\partial(C_{\alpha,((x_{1,CM})_i,(y_{1,CM})_j)}) = \bigcup_{n=1}^{3} \partial(C_{\alpha,((x_{1,CM})_i,(y_{1,CM})_j)})_n \tag{11}$$

Although formulas from (8) to (11) are long, they are not complicated. This is a notation enabling recording of all points which might belong to the ∂C set, being the edge of the C set, in a general way. ∂C_1, ∂C_2, ∂C_3 sets are sets of the above-mentioned grid located on the lower base of the cylinder, the side wall of the cylinder and on the upper base of the cylinder, respectively. The sum of those sets (11) determines the ∂C set, being a set of grid points located on the edge of the C set. The k index relates to the angle growth. The precision of the angle dimension is such that the corresponding length of the arch with the radius equal to the radius of the cylinder r is 0.5 mm. Thus, the k index has to change from zero to the value of the ceiling function from the expression: $2pr/0.5$. The expression $2pr/0.5$, in a general case, is not the total multiple of 0.5, so it was necessary to apply the ceiling function in order to make a full circle. It causes adding some points, as the ceiling function rounds up a real number to the nearest higher integer, but it does not cause any problems with calculations or any errors. The l index relates to the z_1 coordinate of points, which may change from $-H/2$ to $H/2$. Since the precision of the linear dimension is 0.5 mm, the l index may change from 0 to $H/0.5$. The m index relates to the radius growth. It may change from 0 to the value equal to r (cylinder radius). Since the precision of the linear dimension is 0.5 mm, the m index may change from 0 to $r/0.5$.

It should be noted that the ∂C_1, ∂C_2, ∂C_3 sets only depend on two parameters: cylinder radius, r and cylinder height, H. The $O(x_1, y_1, z_1)$ system is permanently related to the cylinder and hooked in the point which is its center of gravity. A set of grid points for the ∂C edge of the cylinder with established dimensions of r and H will be the same towards the $O(x_1, y_1, z_1)$ system, regardless of whether the cylinder will be tilted (Roll, Pitch) or moved towards the $O(x_1, y_1, z_1)$ system. That is why, in the implementing program presenting the model it will be enough to set coordinates of all grid points on the edge of the cylinder only once (i.e. to determine the ∂C set), and next, in subsequent steps, to transform them to the $O(x_1, y_1, z_1)$ system and further to the $O(x_0, y_0, z_0)$ system.

In planes of subsequent pipe cross-sections described by the α parameter, circles with the radius expressed by the formula (12) are determined. The center of those circles is always the intersection of the center line of the pipe with a given cutting plane. This point is the point where the beginning of the $O(x_1, y_1, z_1)$ coordinate system is hooked. If the center of the cylinder is within the area of this circle, angles of the cylinder rotation (roll, pitch) are zero, the area being the cross-section of this cylinder (circle) is included in the area being the cross-section of the pipe. It may turn out that for a given bend radius of the pipe ρ, if the center of gravity of the cylinder located in the cutting plane of the pipe moves from the center of the O_α circle to a different point included in this circle, it will be possible to find such cylinder orientation that it will be included in the pipe (which might not be possible if the cylinder's center of gravity overlaps with the center of the circle, O_α). Obviously, it is not possible to check each point of the O_α circle, as it contains infinitely many points. Discretization of that set is necessary. To this end, a rectangular grid of points located in the cutting plane, described against the $O(x_1,y_1,z_1)$ system, whose $Dx_1 = Dy_1 = 0.5$ mm, is created. All points of this grid, which belong to the O_α circle, create a discrete set, $O_{\alpha,dis}$, described with the equation (13). Vectors of translation of the center of gravity of the cylinder into subsequent points of the $O_{\alpha,dis}$ set have been described with the formula (14).

$$R - r = r + a - r = a \tag{12}$$

$$\begin{gathered} O_{\alpha,dis} = \{((x_1)_i, (y_1)_j): i, j = 0, \ldots, (2 \cdot (a/(0.5)); \\ (x_1)_i = -a + 0.5 \cdot i; (y_1)_j = -a + 0.5 \cdot j; \\ ((x_1)_i)^2 + ((y_1)_j)^2 \le a^2\} \end{gathered} \tag{13}$$

$$\vec{k}_{i,j} = [(x_1)_i, (y_1)_j]; \; ((x_1)_i, (y_1)_j) \in O_\alpha \tag{14}$$

As mentioned above, a set of points of the grid at the edge of the cylinder, ∂C, is described against the $O(x_1, y_1, z_1)$ system, which is permanently related to the cylinder and hooked in the point which is its center of gravity. It is necessary to describe those points against the $O(x_0, y_0, z_0)$ system. However, first transformation of coordinates of those points to the $O(x_1, y_1, z_1)$ system has to be done. Transformation from the $O(x_1, y_1, z_1)$ system to the $O(x_1, y_1, z_1)$ system requires three steps:

$$T(\vec{k}_{i,j}) \to R(x_1, \phi) \to R(y_2, \theta) \tag{15}$$

The first step is translation by the vector k given in the formula (14), the second is rotation around the x'_1 axis by the ϕ angle, the third one is rotation around the y''_1 axis by the θ angle. In order to perform inverse transformation, it is necessary to find inverse matrixes of the described transformations. A transformation matrix has the form given with the equation (17).

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = A_2 \begin{bmatrix} \xi_1 \\ \psi_1 \\ \zeta_1 \\ 1 \end{bmatrix} \tag{16}$$

$$\begin{gathered} A_2 = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & (x_1)_i \\ 0 & 1 & 0 & (y_1)_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{gathered} \tag{17}$$

Next, the transformation has to take place from the $O(x_1, y_1, z_1)$ system to the $O(x_0, y_0, z_0)$ system. Figure 2a presents the initial position of the cylinder. Its lower base is on the Ox_0y_0 plane, the center of gravity of the cylinder is on the center line of the pipe. The location of the center of gravity of the cylinder, CM, located on the center line of the pipe against the $O(x_0, y_0, z_0)$ system determines the radial vector $\boldsymbol{t}$. The angle between the Ox_0 axis and the radial vector $\boldsymbol{t}$ is determined by α. As mentioned before, this angle parameterizes (or numbers) subsequent cutting planes of the pipe, which are normal to the axis of this pipe. For the situation presented in Figure 2a the α parameter is given with the formula (18)

$$\alpha = \alpha_1 = \pi - \mathrm{arctg}\left(\frac{H/2}{\rho + R}\right) \tag{18}$$

The location of the cylinder, which $\alpha = a_1$ corresponds to, is the initial location from which checking whether the cylinder moving through the pipe with a given value of its bend radius ρ (of that pipe) is included in this pipe begins. Importantly, it is not necessary to check this condition along the whole pipe length. It is enough to do it only:

1. At the section of the pipe in which it leaves the part of the pipe described with the S_1 set and enters the part described with the S_2 set (torus). For $\alpha = \alpha_2 = \pi$ there is a change in the pipe curvature. The problem is to determine the α_3 angle for which the cylinder has left the S_1 set and may be only in the S_2 set. It was assumed that it is an angle for which the distance of the projection on the bottom (negative) part of the Oz_0 axis of the point corresponding to this angle and located on the circle with the radius $\rho + R$ (negative coordinate z_0) from the center of the $O(x_0, y_0, z_0)$ coordinate system is higher than or equal to $3 \cdot H$.

   If the value of the bend radius of the pipe is so small that the said value is smaller than $3 \cdot H$, in such case, the inclusion of the cylinder in the pipe at the section from $\alpha = \alpha_1$ to $\alpha = 3\pi/2$ occurs;
2. For $\alpha = \alpha_4 = \pi + \pi/4$ (torus center). Due to the torus rotational symmetry, if the cylinder with the center of gravity located within the circle O_α, being in the cutting plane corresponding to the desired value of the angle $\alpha = \alpha_4$ may be included in the pipe, then for other values of the α angle (higher than a_3 and smaller than $(3\pi/2 - (a_3 - \pi))$) it will be also possible.

In order to move from the $O(x_0, y_0, z_0)$ system to the $O(x_1, y_1, z_1)$ system, it is necessary to perform a string of operations, as given in the formula (19). The first one is translation by vector $\boldsymbol{t}_\alpha$, the second one is a rotation around the y'_0 axis by the α angle. However, the orientation of the $O(x_1, y_1, z_1)$ system and the vector of the t_α translation depend on the value of the α angle and in what range of values the current value of that angle is. It has been described in detail in Table 1. For each case, transformation matrix A_1 was determined (formulas (21)–(23)), transforming coordinates in the $O(x_1, y_1, z_1)$ system to coordinates in the $O(x_0, y_0, z_0)$ system, in accordance with the formula (20).

Table 1. Ranges of the α angle and corresponding cutting planes of the pipe in which $O\alpha$ circles and $O(x_1, y_1, z_1)$ coordinate systems are determined

Range of α angle	Cutting plane	$O(x_1, y_1, z_1)$ coordinate system	Vector of translation ***t***/Notes
$\alpha\in[\alpha_1 ; \alpha_2=p]$ Center of gravity of the cylinder, *CM*, in the S_1 set Angle α is set	Parallel to the Ox_0y_0 plane	Orientation identical as of the $O(x_0, y_0, z_0)$ system Beginning of the system is moved against the beginning of the $O(x_0, y_0, z_0)$ system by vector ***t***	$\vec{t}=\left[-(\rho+R);\ 0;\ H/2-0.5m\right]$ $m=0,\cdots,\frac{H/2}{0.5}$ $\alpha=\pi-arctg\left(\frac{H/2-0.5m}{\rho+R}\right)$
$\alpha\in(\pi ; 3\pi/2]$ Center of gravity of the cylinder, *CM*, in the S_2 set	Plane overlapped with the Ox_0y_0 plane, rotated by the angle α	System rotated by the angle (α – π) around the y_0 axis of the $O(x_0, y_0, z_0)$ system Beginning of the system is moved against the beginning of the $O(x_0, y_0, z_0)$ system by vector ***t***	$\vec{t}=\left[(\rho+R)\cos\alpha;\ 0;\ (\rho+R)\sin\alpha\right]$ The step by which angle α should be changed is an angle for which the arch length is 0.5 mm $\Delta\alpha=\frac{0.5}{\rho+R}$
α > 3π/**2** Center of gravity of the cylinder, *CM*, in the S_3 set	Parallel to the Oy_0z_0 plane	System rotated by the π/2 angle around the y_0 axis of the $O(x_0, y_0, z_0)$ system Beginning of the system is moved against the beginning of the $O(x_0, y_0, z_0)$ system by vector ***t***	$\vec{t}=\left[0.5m;\ 0;\ -(\rho+R)\right]$ $m=1,\ \cdots,\ \frac{H/2}{0.5}$ In this part of the pipe it is not checked, whether the cylinder being moved will be included inside the pipe, because due to the symmetry (of the pipe), it corresponds geometrically to the first range of the α angle Minimum value of the parameter m equal to one results from α > 3π/2

$$T(\vec{t}_\alpha)\rightarrow R(y_0,\alpha) \tag{19}$$

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = A_1 \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \tag{20}$$

$$\alpha\in\left[\alpha_1;\ (\alpha_2=\pi)\right]\Rightarrow A_1=\begin{bmatrix} 1 & 0 & 0 & -(\rho+R) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & H/2-0,5m \\ 0 & 0 & 0 & 1 \end{bmatrix};\ m=0,\ \cdots,\ \frac{H/2}{0.5\text{ mm}} \tag{21}$$

$$\alpha\in\left[\alpha_1;(\alpha_2=\pi)\right]\Rightarrow A_1=\begin{bmatrix} \cos(\alpha-\pi) & 0 & \sin(\alpha-\pi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\alpha-\pi) & 0 & \cos(\alpha-\pi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0 & (\rho+R)\cos\alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & (\rho+R)\sin\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

$$\alpha > \frac{3\pi}{2} \cdot A_1 = \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0.5m \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -(\rho+R) \\ 0 & 0 & 0 & 1 \end{bmatrix}; \; m = 1, \cdots, \frac{H/2}{0.5 \text{ mm}} \tag{23}$$

If transformations described with $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ matrixes are now made, full transformation is acquired, which enables moving from the $O(x_1, y_1, z_1)$ system to the $O(x_0, y_0, z_0)$ system, in which geometric limitations of the pipe (S set) are described by formula (24):

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = A_1 \cdot A_2 \cdot \begin{bmatrix} \xi_1 \\ \psi_1 \\ \zeta_1 \\ 1 \end{bmatrix} \tag{24}$$

Now, with regard to the subsequent values of the α angle (for which it is checked whether the cylinder is included in the pipe), it should be checked whether the points belonging to ∂C_α being the edge of the cylinder, whose center of gravity belongs to some of the points of the $O_{\alpha,dis}$ set (described with the formula (4–14)), are included in the S set. We now search for the smallest value of the bend radius ρ, for which the described condition is satisfied for all the examined values of the α angle.

4. Conclusions

This article presents an original model to solve the problem of running the cylinder in a bent pipe. This model and method may be implemented in the form of a computer program which will enable to determine tables informing, for specific geometrical dimensions: r (radius) and H (height) of the cylinder, clearance a, what should be the minimum bend radius of a rigid pipe so that it is possible to move this cylinder through it. This problem is particularly important in the context of designing miniature mechatronic devices to operate in lateral boreholes going out from the main vertical borehole. It is also significant for mobile robotics in terms of designing robots moving inside pipes.

The proposal contained in the article ensures a high precision of calculation by adopting a suitably dense grid of points creating the edge of the cylinder. The proposed precision of the linear dimension is 0.5 mm, whereas the precision of the angle dimension is the precision corresponding to an angle for which an arch with the radius equal to the cylinder radius r has the length equal to 0.5 mm. In the opinion of the author, such detailed analysis will enable valuable numeric analysis. However, it relates to high computational complexity, which requires the application of a supercomputer for this purpose and using many of its nodes. To this end, it will be necessary to implement the model and method presented here in the form of a computational program using the so-called parallel computing. Works on the preparation of such a program are ongoing.

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