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ARTICLE

POSSIBILITIES OF APPLYING THE EXTENDED EYRING RHEOLOGICAL MODEL IN THE TECHNOLOGY OF CEMENT SLURRIES USED IN OIL DRILLING

Krzysztof Skrzypaszek

AGH University of Krakow, Faculty of Drilling, Oil and Gas, Poland
ORCID: 0000-0003-2358-7361
e-mail: varna@agh.edu.pl

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Abstract: This study explores the feasibility of implementing an extended Eyring rheological model to describe the dependence between shear stress and shear rate in cement slurries used in drilling technologies. Advances in cement slurry technology have rendered traditional mathematical models, particularly the widely used linear Bingham model recommended by the API RP 13D (American Petroleum Institute Recommended Practice 13D) standard, insufficient for accurately predicting flow resistance during pumping operations. A misalignment between the model and the actual behavior of cement slurries can result in significant errors, potentially increasing operational costs. By identifying and applying the relevant rheological model, it is possible to optimize the system's performance, minimizing total pressure losses and thereby reducing overall drilling costs. This paper investigates the applicability of more sophisticated three-parameter rheological models, commonly utilized in other engineering disciplines, to address these challenges. Specifically, the extended Eyring model was adapted to the proprietary RheoSolution methodology developed by the Department of Drilling, Oil and Gas. To validate this approach, a series of laboratory tests were conducted on cement slurries widely used in the oil industry. The results were analyzed and compared against mathematical models recommended by the API standard. The findings confirm that the extended Eyring model offers superior accuracy in determining the rheological parameters of cement slurries for drilling applications, underscoring its potential as a robust tool for improving the efficiency of drilling operations.

Keywords: drilling, rheology, drilling fluids, cement slurries, rheological model, computer aided, numerical methods, gradient method

1. Introduction

The first attempts to use cement in drilling were made in 1903 when Frank Hill applied it to seal off water in a drilling well. By 1910, well cementing began to develop on a larger scale, initiating systematic advancements in drilling cement technology. Today, cement is primarily used in drilling to seal the space between the wellbore and casing, reduce mud leakage, and create stabilizing plugs. The requirements for cement slurries used in deep well drilling and for sealing and strengthening operations in challenging geological conditions are continuously increasing. In drilling and geoengineering, there is a growing need to develop new cement formulations that meet these demands. However, less emphasis has been placed on procedures for verifying their flow and rheological parameters. The widely used standard – API RP 13D (American Petroleum Institute Recommended Practice 13D) – recommends employing the linear Bingham model and the exponential Ostwald–de Waele model [1–3]. However, experience in analyzing the flow parameters of cement slurries for various drilling applications suggests that these models are far from optimal and, in many cases, they produce results with significant errors [4]. In order to minimize these errors, a methodology for selecting the optimal rheological model for technological drilling fluids – RheoSolution by Prof. Rafal Wiśniowski and the author of this article [5–9] – was developed at the Faculty of Drilling Oil and Gas. The idea was to create an algorithm to evaluate the correlation of the mathematical model with the actual values of shear stress as a function of shear rate [11–12]. Higher correlation results in higher accuracy in calculating the amount of pressure loss when pumping cement slurry during the cementing procedure. The consequence of this is the selection of appropriate cementitious aggregates, which has an economic dimension. The procedure for determining rheological parameters (API RP 13D) can be improved by replacing simple mathematical models with more advanced ones (often with higher correlation) and by replacing more advanced viscosity meters with the Fann and Chan models widely used in the petroleum industry [1]. This article will present the application of the extended Eyring rheological model (so far not used in drilling practice) for the purpose of describing the dependence of shear stresses as a function of shear rate occurring during the pumping of sample cement slurries of various applications [8].

2. Rheological models used in the oil industry

The most popular used rheological models in the oil industry are [5, 9, 13–15].

– Bingham model:

$$\tau = \tau_y + \eta_{pl}\left(-\frac{dv}{dr}\right) \tag{1}$$

A linear model of a plastic body, which at rest has a three-dimensional structure with a certain elasticity. Once this elasticity is exceeded – the flow limit or yield point – the body flows. The flow limit is a measure of the intermolecular forces present in a fluid. Once it is exceeded, a Bingham body takes on the characteristics of a Newtonian fluid, in which stresses propagate in direct proportion to the forces causing them – linearly. The body is described by two parameters. This model is the first modification of Newton's model and was widely used as the basic rheological model because of its simplicity. It is now being replaced in most calculations by more complex models that better describe the dependence between shear stress and shear rate in a fluid.

– Ostwald–de Waele model:

$$\tau = k\left(-\frac{dv}{dr}\right)^n \tag{2}$$

Exponential pseudoplastic body model. This model was introduced when it was noticed that in the graph of shear stress from shear rate with both axes logarithmic, the plotted curve is close to a straight line. The equation is two-parameter and well describes the behavior of most fluids especially shear-thinning fluids, but it lacks consideration of the flow boundary in the form of a free expression.

– Herschel–Bulkley model:

$$\tau = \tau_y + k\left(-\frac{dv}{dr}\right)^n \tag{3}$$

Three-parameter model, an extension of the power model by supplementing it with the value of the flow limit. Currently, the model that is most widely used due to its high accuracy, versatility and the ability to adapt the model to the parameters. It describes pseudoplastic shearthinning fluids very well and describes dilatant shearthickening fluids well [3, 4].

Less commonly used models and not included in the API methodology are:

– Casson's model modified to a nonlinear form of Bingham's model. This is a pseudoplastic fluid model which is quite accurate in the low shear rate range while less accurate in the shear rate range >60 s^{-1}. It is rarely used in drilling practice [5, 13, 14].
– Newton's model – an equation describing a Newtonian fluid. For approximating the properties of non-Newtonian fluids, it is used occasionally, but

only when it is a sufficiently accurate approximation or in situations where accuracy is not required but simplicity of calculation is [4].

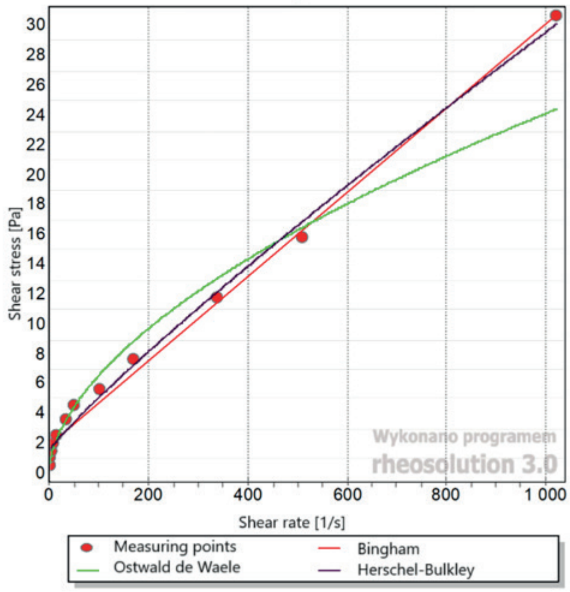

Fig. 1. Summary of rheological models for a sample test fluid recommended by API RP 13D standard: Bingham, Ostwald–de Waele, Herschel–Bulkley

The aforementioned models are successfully used in drilling practice, but it has been noted that linear models perform poorly in the low shear rate range and the process fluids used in drilling are now being modified with chemical additives essentially changing their initial flow parameters. Often during research there is a situation where some models better describe the dependence of shear stress as a function of shear rate for low shear rates and others perform better for high shear rates. Therefore, the author analyzed several rheological models used in other industries with the idea of adapting a multi-parameter rheological model for the drilling industry. The primary criterion was comparable or better correlation than the Herschel–Bulkley model for test data, which would provide a good reference for the applicability of the models in question. The Herschel–Bulkley power model gives the best fit in most situations and most accurately reflects the rheological parameters of the technological drilling fluids tested at the Faculty of Drilling, Oil and Gas. As a result of this analysis, the Eyring model extended with a linear element was selected for further study (4).

$$\tau = A\left(-\frac{dv}{dr}\right) + B\sinh^{-1}\left(\frac{-\frac{dv}{dr}}{C}\right) \tag{4}$$

The Eyring model gave very satisfactory results for simple fluids and suspensions, and when extended with a linear element also for cement slurries.

The Eyring model was chosen after analyzing rheological models used in other areas of engineering that are applied to fluids similar to cement slurries. The following rheological models were analyzed:
- Robertson – Stiff,
- Sisko,
- Vom Berg,
- Eyring,
- Ellis,
- Cross,
- Mizrahi – Berka,
- Vocaldo,
- Shangrawa – Grim – Mattocks.

3. Methodology for selecting the optimal rheological model – RheoSolution 5.0

The result of laboratory tests conducted with FANN-type rotational viscometers is a set of measuring points, where the values of shear stresses in the technological fluid arising under different shear rates are given. Regression is used to fit the rheological parameters of the model to the values of the measurement points. For the Bingham model, which is a linear model, and for the exponential Ostwald–de Waele model, which can be linearized, linear regression analysis is used [6]. Another recommended rheological model of Herschel–Bulkley cannot be linearized, because when determining the equations of the parameters, an entangled equation of one variable will be obtained. Also, the linearization of equations is not possible for the postulated Eyring model, the notation of these models is as follows.
- Herschel–Bulkley [5, 8]:

$$\tau = \tau_y + k\left(-\frac{dv}{dr}\right)^n \tag{5}$$

Simplified formulation for the Herschel–Bulkley model:

$$y = a + bx^c \tag{6}$$

Least squares method for the Herschel–Bulkley model:

$$\begin{aligned} U &= \sum_{i=1}^{m}(y_i - (a + bx_i^c))^2 \\ &= \sum_{i=1}^{m}(y_i^2 - 2y_i - 2bx_i^c + a^2 + 2abx_i^c + b^2x^{2c}) \to \min \end{aligned} \tag{7}$$

The partial derivatives of the parameters a, b and c form the following system of equations:

$$\begin{cases} \dfrac{\partial U}{\partial a} = -2\sum_{i=1}^{m} y_i + 2b\sum_{i=1}^{m} x_i^c + 2am = 0 \\ \dfrac{\partial U}{\partial b} = -2\sum_{i=1}^{m} y_i x_i^c + 2a\sum_{i=1}^{m} x_i^c + 2b\sum_{i=1}^{m} x_i^{2c} = 0 \\ \dfrac{\partial U}{\partial c} = -2\sum_{i=1}^{m}(y_i x_i^c \ln x_i) + 2ab\sum_{i=1}^{m}(x_i^c \ln x_i) \\ \quad + 2b^2\sum_{i=1}^{m}(x_i^{2c}\ln x_i) = 0 \end{cases} \tag{8}$$

– Eyring:

$$\tau = A\left(-\frac{dv}{dr}\right) + B\sinh^{-1}\left(\frac{-\frac{dv}{dr}}{C}\right) \tag{9}$$

Simplified formulation for the Eyring model:

$$y = ax + b\sinh^{-1}\left(\frac{x}{C}\right) \tag{10}$$

Least squares method for the Eyring model:

$$\begin{aligned} U &= \sum_{i=1}^{m}\left(y_i - \left(ax_i + b\sinh^{-1}\left(\frac{x_i}{c}\right)\right)\right)^2 \\ &= \sum_{i=1}^{m}\left(y_i^2 - 2ax_i y_i - 2y_i b\sinh^{-1}\left(\frac{x_i}{c}\right) + a^2x_i^2 \right. \\ &\left. + 2ax_i b\sinh^{-1}\left(\frac{x_i}{c}\right) + b^2\left(\sinh^{-1}\left(\frac{x_i}{c}\right)\right)^2\right) \end{aligned} \tag{11}$$

The system of equations is formed by the partial derivatives of the parameters a, b and c:

$$\begin{cases} \dfrac{\partial U}{\partial a} = \sum_{i=1}^{m}(-2x_i\left(-ax_i - b\sinh^{-1}\left(\frac{x_i}{c}\right) + y_i\right)) = 0 \\ \dfrac{\partial U}{\partial b} = \sum_{i=1}^{m}\left(-2\sinh^{-1}\left(\frac{x_i}{c}\right)\left(-ax_i - b\sinh^{-1}\left(\frac{x_i}{c}\right) + y_i\right)\right) = 0 \\ \dfrac{\partial U}{\partial c} = \sum_{i=1}^{m}\dfrac{2bx_i\left(-ax_i - b\sinh^{-1}\left(\frac{x_i}{c}\right) + y_i\right)}{c^2\sqrt{1+\frac{x_i^2}{c^2}}} = 0 \end{cases} \tag{12}$$

The systems of derived equations are unsolvable analytically, as attempts result in a tangled equation involving a single variable. To address this, nonlinear regression techniques are essential for estimating equation parameters. One commonly used method is the gradient-based approach [7, 10, 15]. This technique identifies local minima within a function and selects the smallest one as the global minimum. The procedure involves starting from an initial point in the function's domain, where the fit of the model is evaluated as the sum of squared differences. A vector is then calculated, moving in the reverse direction of the function's gradient. The step size is determined by a unit value adjusted by a scaling factor, which can be tuned to improve the method's performance.

$$U(a,b,c) = \sum_{i=1}^{n}(f(a,b,c) - y_i)^2 \to \min \tag{13}$$

$$\hat{v} = \frac{-\vec{\nabla}U(a,b,c)}{\left|-\vec{\nabla}U(a,b,c)\right|}\cdot\alpha_k \tag{14}$$

The parameters of the model are adjusted based on the vector values, with the algorithm treating the modified point as a fresh starting location. This process repeats iteratively until a local minimum is achieved with the specified precision. Once this is done, the initial point is relocated, and the search for another local minimum begins. After executing the algorithm a set number of times, the smallest local minimum among those discovered is identified and accepted as the global minimum. A depiction of this process can be found in Figure 2 [11, 12, 15].

To simplify the notation, individual parameters are assigned to variables $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and constants $\boldsymbol{y_i}$ and $\boldsymbol{x_i}$ are data from successive measurement points [10]. With each step, only the variables are modified (Fig. 3). Assignment of the model equations to the scheme:

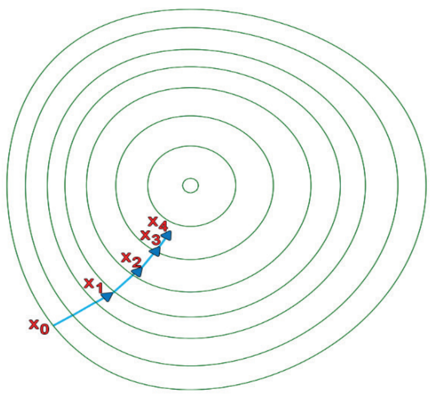

Fig. 3. Graphical representation of the gradient method

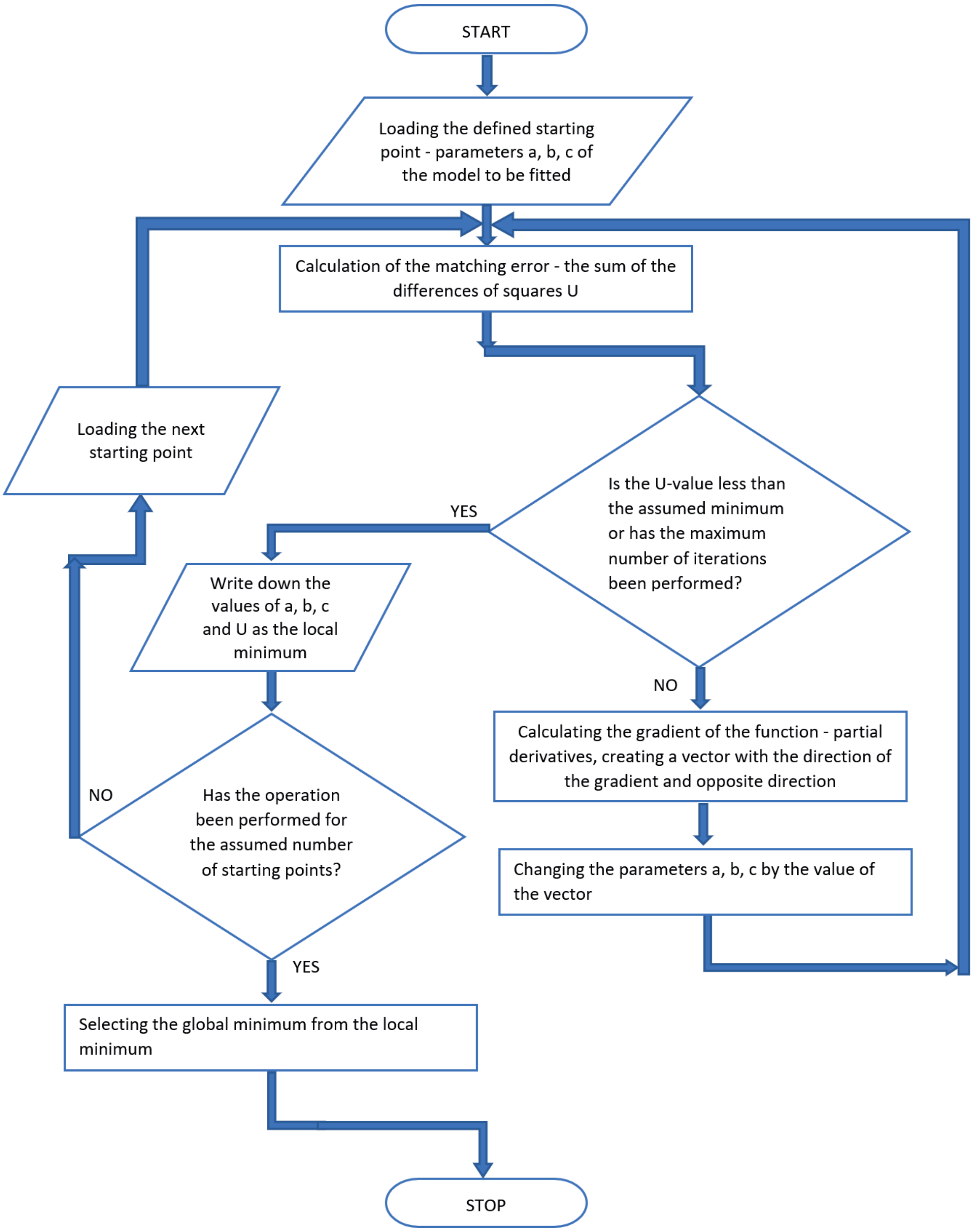

Fig. 2. Algorithm diagram for the simple gradient method

To streamline the notation, individual parameters are represented as variables *a*, *b*, and *c*, while the constants y_i and x_i correspond to data from consecutive measurement points (10). At each iteration, only the variables are updated.

The model equations are then incorporated into the framework as follows:

$$\tau = A\left(-\frac{dv}{dr}\right) + B\sinh^{-1}\left(\frac{-\frac{dv}{dr}}{C}\right) \tag{15}$$

$$
\begin{aligned}
a &= A \\
b &= B \\
c &= C \\
y_i &= \tau_i \\
x_i &= -\frac{dv}{dr_i}
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
U(a,b,c) &= \sum_{i=1}^{n} \left(f(a,b,c) - y_i \right)^2 \\
&= \sum_{i=1}^{n} \left(a x_i + b \sinh^{-1}\left(\frac{x_i}{c} \right) - y_i \right)^2 \rightarrow \min
\end{aligned} \tag{17}
$$

This method is universal and, once the formulas for the partial derivatives (6) are derived, it can be applied to any model with any number of parameters [12, 15], which greatly facilitates the process of implementing subsequent models into tools such as the RheoSolution 5.0 program developed at the Department of Drilling and Geoengineering of the Faculty of Drilling, Oil and Gas based on the proprietary RheoSolution methodology.

4. Laboratory testing

To verify the applicability of the extended Eyring model in determining the rheological parameters of cement slurries, laboratory tests were conducted on sample cement slurries. Samples 1 and 2 are relatively simple cement slurries modified with additives used in geothermal drilling. Samples 3, 4, 5 are cement slurries with w/c ratios = 0.5 that differ in the type of cement used in their preparation [4]. The tests were performed using a 12-range viscosity meter of the FANN type recommended by the API standard [3]. Then, using the RheoSolution methodology presented earlier, the rheological parameters, statistical parameters and correlation of the Bingham, Ostwald–de Waele and the Herschel–Bulkley models with the laboratory fluid tested were determined [2]. The results of these calculations served as a reference to the results obtained with the extended Eyring model and are shown in Tables 1–10 and Figures 4–8.

Sample No. 1: cement slurry w/c = 0.6 (CEM I 42.5R) with the addition of 20% diatomite (BWOC).

Table 1. Results of rheological measurements for sample No. 1

Rotor speed [rot/min]	Angle [°]	Shear rate [s^{-1}]	Shear stress [Pa]
600	107	1 022.040	54.643
300	69	511.020	35.237
200	56	340.680	28.598
100	37	170.340	20.427
60	28	102.204	16.852
30	24	51.102	13.278
20	21	34.068	11.746
10	18	17.034	9.192
6	13	10.220	6.639
3	8	5.110	4.085
2	7	3.406	3.575
1	4	1703	2.043

Table 2. Summary of correlation coefficients of the analyzed rheological models for sample No. 1

Rheological model	Pearson correlation coefficient, *R*	Fischer-Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.978	1 021.92	54.64
Ostwald–de Waele	0.992	510.96	35.23
Herschel–Bulkley	0.995	340.64	28.59
Eyring	0.998	2 848.47	9.42

The determined rheological parameters of the Eyring model for sample No. 1: parameter A – 0.03834 [Pa·s], parameter B – 2.10052 [Pa·s], parameter C – 0.87642 [–].

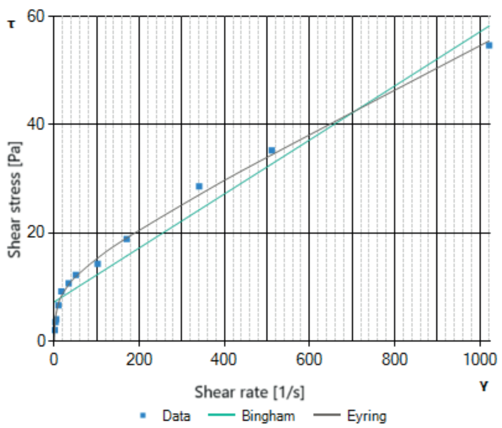

Fig. 4. Comparison of the Eyring model with the Bingham model (API) for sample No. 1

Sample No. 2: cement slurry w/m = 1.1 (CEM I 42.5R) with the addition of 20% graphite (BWOC).

Table 3. Results of rheological measurements for sample No. 2

Rotor speed [rot/min]	Angle [°]	Shear rate [s^{-1}]	Shear stress [Pa]
600	39	1 022.040	21.448
300	26	511.020	13.278
200	17	340.680	8.682
100	9	170.340	4.596
60	6	102.204	3.066
30	4	51.102	2.044
20	3	34.068	1.533
10	3	17.034	1.533
6	2	10.220	1.022
3	2	5.110	1.022
2	1	3.406	0.511
1	1	1.703	0.511

Table 4. Summary of correlation coefficients of the analyzed rheological models for sample No. 2

Rheological model	Pearson correlation coefficient, *R*	Fischer-Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.995	1 040.49	4.39
Ostwald–de Waele	0.948	87.95	47.09
Herschel–Bulkley	0.997	2 273.39	2.02
Eyring	0.997	1 485.24	3.08

Determined rheological parameters of the Eyring model for sample No. 2: parameter A – 0.01901 [Pa · s], parameter B – 0.3336 [Pa · s], parameter C – 1.1391 [–].

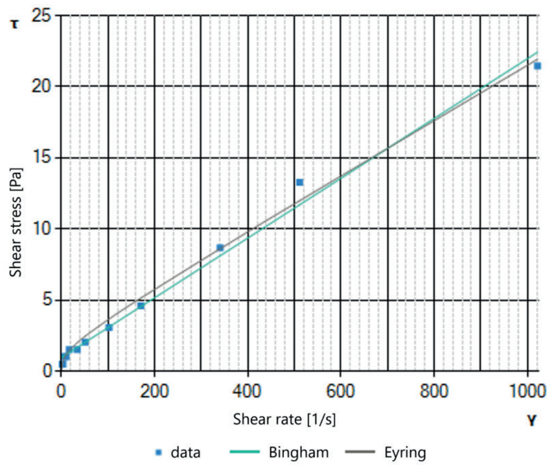

Fig. 5. Comparison of the Eyring model with the Bingham model (API) for sample No. 2

Sample No. 3: cement slurry w/c = 0.5 (CEM III/A 32.5R) without additives.

Table 5. Results of rheological measurements for sample No. 3

Rotor speed [rot/min]	Angle [°]	Shear rate [s^{-1}]	Shear stress [Pa]
600	Out of range	–	–
300	Out of range	–	–
200	112	340.680	57.196
100	85	170.340	43.408
60	72	102.204	36.769
30	57	51.102	29.109
20	49	34.068	25.023
10	34	17.034	17.363
6	24	10.220	12.256
3	15	5.110	7.660
2	11	3.406	5.617
1	8	1.703	4.085

Table 6. Summary of correlation coefficients of the analyzed rheological models for sample No. 3

Rheological model	Pearson correlation coefficient, *R*	Fischer-Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.917	42.34	453.29
Ostwald–de Waele	0.961	126.54	169.59
Herschel–Bulkley	0.984	266.41	83.14
Eyring	0.973	139.57	154.61

The determined rheological parameters of the Eyring model for sample No. 3: parameter A – 0.1081 [Pa · s], parameter B – 3.1171 [Pa · s], parameter C – 0.2797 [–].

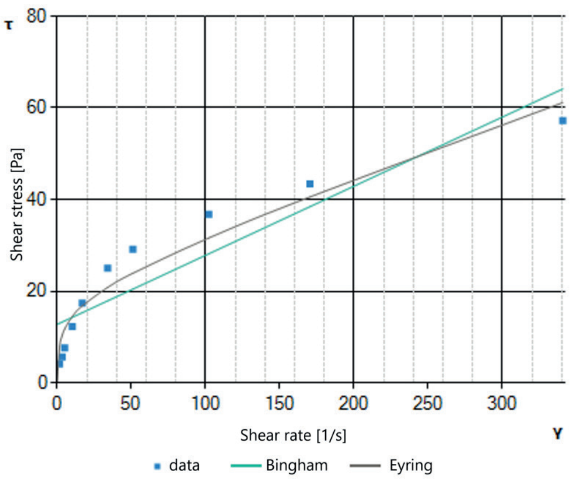

Fig. 6. Comparison of the Eyring model with the Bingham model (API) for sample No. 3

Sample No. 4: cement slurry w/c = 0.5 (CEM I 42.5R) without additives.

Table 7. Results of rheological measurements for sample No. 4

Rotor speed [rot/min]	Angle [°]	Shear rate [s^{-1}]	Shear stress [Pa]
600	144	1 022.040	73.538
300	103	511.020	52.600
200	77	340.680	39.322
100	57	170.340	29.109
60	46	102.204	23.491
30	35	51.102	17.874
20	30	34.068	15.320
10	23	17.034	11.746
6	15	10.220	7.660
3	11	5.110	5.617
2	9	3.406	4.596
1	7	1.703	3.575

Table 8. Summary of correlation coefficients of the analyzed rheological models for sample No. 4

Rheological model	Pearson correlation coefficient, *R*	Fischer-Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0.964	42.34	360.64
Ostwald–de Waele	0.997	2 299.09	22.49
Herschel–Bulkley	0.998	2 843.365	18.21
Eyring	0.994	644.233	79.38

Determined rheological parameters of the Eyring model for sample No. 4: parameter A – 0.0544 [Pa · s], parameter B – 2.5926 [Pa · s], parameter C – 0.4705 [–].

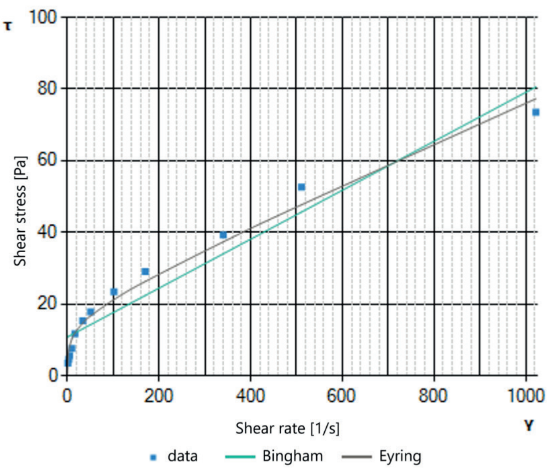

Fig. 7. Comparison of the Eyring model with the Bingham model (API) for sample No. 4

Sample No. 5: cement slurry w/c = 0.5 (CEM G HSR).

Table 9. Results of rheological measurements for sample No. 5

Rotor speed [rot/min]	Angle [°]	Shear rate [s^{-1}]	Shear stress [Pa]
600	118	1 022.040	60.260
300	78	511.020	39.833
200	61	340.680	31.151
100	45	170.340	22.981
60	37	102.204	18.895
30	29	51.102	14.810
20	26	34.068	13.278
10	20	17.034	10.214
6	15	10.220	7.660
3	10	5.110	5.107
2	8	3.406	4.085
1	6	1.703	3/064

Table 10. Summary of correlation coefficients of the analyzed rheological models for sample No. 5

Rheological model	Pearson correlation coefficient, *R*	Fischer-Sneadecor coefficient, *F*	Sum of squares, *U*
Bingham	0,971	168.567	180.58
Ostwald–de Waele	0.993	795.722	40.02
Herschel–Bulkley	0.996	1 355.444	23.61
Eyring	0.999	7 014.752	4.59

Determined rheological parameters of the Eyring model for sample No. 5: parameter A – 0.0388 [Pa · s], parameter B – 2.842 [Pa · s], parameter C – 1.2355 [–].

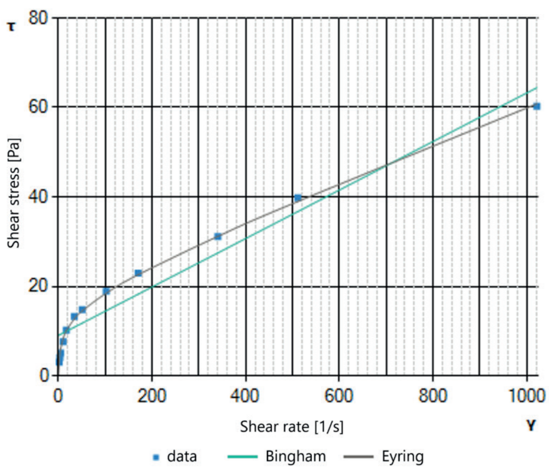

Fig. 8. Comparison of the Eyring model with the Bingham model (API) for sample No. 5

5. Conclusions

Analysis of the laboratory results obtained showed almost full correlation of the Eyring rheological model with all samples. The average correlation is 0.992 proving that the Eyring model is very suitable for describing cement slurries with basic formulations. Significant differences can be observed between the linear Bingham model and the Eyring model, particularly in the low shear rate range, where the Eyring model demonstrates better accuracy. In samples with a high yield stress, the Eyring model shows a much stronger correlation compared to the Ostwald–de Waele pseudoplastic fluid model. When compared to the Herschel–Bulkley model, the results are generally comparable, with the Eyring model showing a slight advantage. When studying the rheology of cement slurries used in the drilling industry, it is recommended to take into account the extended Eyring model, laboratory tests have shown its usefulness in this regard and the RheoSolution methodology is a tool that effectively helps select the appropriate rheological model for the cement slurry used in industrial practice. At the AGH Faculty of Drilling, Oil and Gas, further development work is being carried out on the usefulness of this model for other, more chemically complex technological drilling fluids. The Eyring model is implemented in the RheoSolution version 5 software, based on the proprietary RheoSolution methodology, which is commonly used by academics and students of the Faculty of Drilling, Oil and Gas when studying technological fluids used in drilling.

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Conflicts of Interest: The author of this paper declares no conflicts of interest.

References

[1] Bourgoyne A.T., Milheim K.K., Chenevert M.E., Young F.S.: *Applied Drilling Engineering.* Society of Petroleum Engineers, SPE Textbook Series, Houston 1986, Section 4.8.
[2] Klotz J.A., Brigham W.E.: *Determining Parameters for the Herschel–Bulkley Model.* Journal of Petroleum Technology, 50, 1998, pp. 80–81.
[3] American Petroleum Institute. *Rheology and Hydraulics in Oil-Well Drilling Fluids (API RP 13D).* API Standards, Washington D.C. 2006.
[4] Pellegrino C., Faleshini F.: *Improving Sustainability in Concrete Industry by Using Recycled Materials.* Springer International Publishing, Cham 2016, pp. 37–43.
[5] Wiśniowski R., Skrzypaszek K., Toczek P.: *Analysis of Rheological Models in Drilling Fluid Dynamics: Vom Berg and Hahn–Eyring Approaches.* Energies, 15, art. 5583, 2022. http://doi.org/10.3390/en15155583.
[6] Kutner M.H., Nachtsheim C.J., Neter J., Li W.: *Applied Linear Statistical Models.* 5th ed., McGraw-Hill – Irwin, New York 2004.
[7] Chapra S.C.: *Applied Numerical Methods with MATLAB for Engineers and Scientists.* 3rd ed., McGraw-Hill Education, New York 2012.
[8] Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P.: *Numerical Recipes: The Art of Scientific Computing.* 3rd ed., Cambridge University Press, Cambridge, 2007.
[9] Wiśniowski R.: *Application of Nonlinear Models in Drilling Fluid Hydraulics.* Modern No-Dig Techniques and Technologies, 2, 2000, pp. 20–28.
[10] Wiśniowski R., Skrzypaszek K., Małachowski T.: *Numerical Analysis for Selecting Rheological Models of Drilling Fluids.* Energies, 13, art. 3192, 2020.
[11] Sharma M.M., Zhang R., Chenevert M.E.: *Novel Nanoparticle-Based Drilling Fluids.* SPE Annual Technical Conference and Exhibition, San Antonio, October 8–10, 2012, pp. 1–13.
[12] Nocedal J., Wright S.J.: *Numerical Optimization.* 2nd ed., Springer, New York 2006.
[13] Boyd S., Vandenberghe L.: *Convex Optimization.* Cambridge University Press, Cambridge, 2004.
[14] Papo A.: *Rheological models for cement pastes.* Materials and Structures, 21, 1988, pp. 41–46. https://doi.org/10.1007/BF02472527.
[15] Stryczek S., Wiśniowski R., Skrzypaszek K.: *Kierunki rozwoju badań nad reologią płynów wiertniczych.* Wiertnictwo, Nafta, Gaz, 24, 1, pp. 243–250.
[16] Wiśniowski R., Skrzypaszek K.: *Analiza modeli reologicznych stosowanych w technologiach inżynierskich.* Wiertnictwo, Nafta, Gaz, 23, 1, pp. 293–299.

Nomenclature

Symbol	Explanations	Unit
a, b, c	regression coefficient	[–]
A, B, C	rheological parameter in the extended Eyring model	[–]
U	sum of residuals squared	[–]
F	Fisher-Snedecor index	[–]
dv/dr	gradient of shear rate	[s^{-1}]
η_{pl}	plastic viscosity	[Pa·s]
$\dot{\gamma}_i$	shear rate measured at i^{th} rotate speed	[s^{-1}]
k	consistency	[$Pa \cdot s^n$]
m	measurements number	[–]
n	exponential index	[–]
R	correlation coefficient	[–]
τ	shear stress	[Pa]
τ_{i-}	shear stress measured at i-th rotational speed	[Pa]
τ_y	YP	[Pa]
$\bar{\tau}$	average value of shear stress	[Pa]
w/c	water/cement ratio	[–]
w/m	water/mixture ratio	[–]