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# Deadhead Minimization Problem in Multi-Depot Public Transport System

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*Abstract.* This paper addresses a vehicle scheduling problem in the public transport system of Krakow, Poland. The primary objective is to develop and evaluate a mathematical model for assigning bus schedules to depots in a way that minimizes non-revenue (deadhead) kilometers. The proposed model, referred to as the Deadhead Minimization Problem in Multi-Depot Public Transport System (DMPMDPTS), seeks to reduce the total distance that is traveled by vehicles from their home depots to the starting points of their first scheduled routes and from the final terminals back to their depots. The model assumes fixed-route structures and known deadhead distances between terminals. Real-world data that was based on the Krakow Municipal Transport (KKM) was used to validate and verify the model. The optimization model was implemented in AMPL and solved using the GLPK Integer Optimizer (v4.43). Computational experiments were conducted across multiple cases that differed in their constraints and parameters in order to assess the model's flexibility and performance. In all of the cases, optimal solutions were obtained in brief computation times. Compared to the existing operational schedules, the model consistently reduced deadhead kilometers. Case 1 achieved improvements without altering the numbers of vehicles per depot, while Case 2 led to further reductions of the costs of redistributing vehicles among depots, resulting in a less-balanced load structure. These findings demonstrated the model's potential for supporting decision-making in depot allocation within public transport operations.

*Keywords:* Vehicle Scheduling Problem, public transport system, crew rostering, deadhead minimization, deadhead kilometers, optimization, Operations Research

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## 1. INTRODUCTION

The primary goal of any enterprise is to maximize its profit through efficient operations. A prevailing trend in modern management is minimizing costs – particularly those that are associated with resource consumption. Lean Management principles advocate for companies to focus on what the customer truly values and to eliminate any components that do not add value to their product or service; any such non-value-adding elements represent a loss for the company. Modern Lean Management systems should be closely integrated with rational resource-management practices. It is essential for companies to assess whether changes in one area will lead to unintended consequences in others. Mathematical models and algorithms serve as valuable tools in this regard, allowing businesses to simulate changes and their potential impacts across different spheres (e.g., Agnetis et al., 2019; Gdowska et al., 2018; Korcyl et al., 2016; Książek et al., 2021; Villarreal et al., 2016). These tools' versatility allows for the calculations of possible costs or gains that would result from any proposed changes, making them essential in optimizing various aspects of operations. Moreover, mathematical models can be effectively applied in the designs of new systems or areas of operation (Csalódi et al., 2021).

In the context of a communications company (such as one that provides public transportation services), the price structure of the service – including fuel costs, vehicle depreciations, and employee salaries – becomes a key factor (Szczyrbak, 2016). A unit of measurement called "vehicle kilometers traveled" (VKT) is used to assess transportation efficiency and optimize operations. Companies typically develop operational plans based on policies that are aligned with contracts from public transport organizers. These plans detail the required numbers of vehicles and employees as well as their work schedules (Szczyrbak & Gdowska, 2017).

The research that is presented in this paper focuses on the scheduling of the bus brigades that serve daytime lines within the Krakow Municipal Transport (KKM) system; specifically, the study narrows its scope to the brigades of a single carrier that operates within the city's central transportation area. The objective of this research was to develop the Deadhead Minimization Problem in Multi-Depot Public Transport System (DMPMDPTS) and employ mixed-integer linear programming to solve it.

This paper is structured as follows. In Section 2, the research gap of insufficient research on deadhead kilometer minimization using exact methods is identified based on a systematic literature review. In Section 3, the Deadhead Minimization Problem in Multi-Depot Public Transport System (DMPMDPTS) is introduced together with the dedicated MILP model; then, the results are presented for three cases. The main conclusions are presented in Section 4.

## 2. PAPER POSITIONING

Planning in public transportation encompasses several broad areas – each involving decisions regarding the designs and operations of specific service components (Gdowska, 2018). From the passenger's perspective, the system should meet their needs by providing affordable and efficient public transportation. Additional criteria may include comfort, route selection, and the frequency and number of transfers that are required to reach their destination (e.g., work, school). From the transportation company's point of view, the system must also generate as much profit as possible. The primary challenge in its planning lies in finding a balance that satisfies both the passengers' and the company's needs. The complexity of transportation planning necessitates decisions at strategic, tactical, and operational levels. The communication system that is described in this paper distinguishes between the planning responsibilities of the organizer and the carrier (Borndörfer et al., 2017). The organizer is the entity that is responsible for managing the public transportation, while the carrier is the operator that is contracted to execute specific routes as per agreements with the organizer.

The relationship between objective service–performance indicators and passenger satisfaction in public transport is multifaceted. While measurable aspects such as punctuality, frequency, and travel time are important, they do not always align directly with perceived quality. Achieving high levels of user satisfaction requires a balanced approach that considers both operational efficiency and the subjective expectations of travelers (Friman & Fellesson, 2009). Mathematical optimization models for train timetabling can significantly reduce passenger waiting times at public transit terminals. By optimizing arrival and departure schedules, these models contribute to more-efficient service operations and improved passenger experience through better synchronization and reduced transfer delays (Hassannayebi et al., 2017).

The organizer's role encompasses strategic and tactical decision-making in public transport. Their responsibilities include determining how the network will operate and contracting carriers to run the selected routes. The key planning areas for the organizer include route and service planning, service-frequency planning, and timetable planning. Route and service planning involves creating a sequence of stops that a particular line will serve as well as determining the required type of fleet for each route. Service-frequency planning refers to deciding how many trips will be made on a given route at various times of the day (with the flexibility to adjust based on demand). Timetable planning involves scheduling the actual times of the departures and arrivals for each route, thus ensuring efficient service and meeting passenger needs. Intelligent bus routing relies on analyses of heterogeneous human mobility patterns to enhance route planning and service delivery. Integrating diverse data sources such as travel demands, temporal flow variations, and location-based behaviors enables more-adaptive and -efficient transit systems that better respond to passenger needs and urban dynamics (Iliopoulou & Kepaptsoglou, 2019).

Frequency planning (which is a critical part of service scheduling) is determined by the demand for the trips on a route at various times of the day. The operator has the flexibility to adjust the number of trips based on time-of-day demand. Once the frequency is set, the organizer can use this data to make further strategic decisions, such as adjusting the overall route structure or adding additional services. A time-dependent passenger demand-driven timetable synchronization and optimization model can effectively minimize total travel time in urban subway networks. By adjusting scheduling parameters in response to dynamic passenger flows (such as those that can be observed in large systems like the Beijing subway), such models enhance operational efficiency and improve the overall passenger experience (Shang et al., 2018).

## 3. DEADHEAD MINIMIZATION PROBLEM IN MULTI-DEPOT PUBLIC TRANSPORT SYSTEM

Within the Krakow agglomeration, public transport services (i.e., the Krakow Municipal Transport) encompass both day and night bus lines. These services are operated by two main carriers: Miejskie Przedsiębiorstwo Komunikacyjne S.A. (MPK Kraków), and Mobilis sp. z o.o. Given the operational complexities and the potential for a single vehicle to serve both day and night routes, this study focuses exclusively on the day brigades that are managed by MPK Kraków. By “brigade,” we understand a set of courses that a vehicle must navigate during one day; this set can includes several lines of a network, which is usually divided into two shifts. MPK Kraków deploys its bus fleet from three primary depots: Bieńczyce (PB), Płaszów (PP), and Wola Duchacka (PW). Each depot houses a specific assortment of bus models that reflect strategic allocations based on their operational requirements. For instance, the Bieńczyce depot primarily accommodates buses that are equipped with MAN and DAF engines, the Płaszów depot has historically housed Scania buses, and the Wola Duchacka depot includes vehicles that feature Mercedes-Benz engines (including gas-powered and electric buses). An estimated distribution of buses by type across these depots is presented in Table 1; this provides a foundational understanding of the resources allocated for day brigade operations within the MPK Kraków system.

**Table 1.** *MPK Kraków fleet (number of buses)*

| | PB | PP | PW |
|---|---|---|---|
| Mini | 18 | 0 | 29 |
| Midi | 0 | 0 | 13 |
| Electric | 0 | 0 | 6 |
| Maxi | 103 | 99 | 80 |
| Mega | 42 | 46 | 95 |

The primary criterion for the classification of the fleet is the lengths of the buses. Additionally, a distinct group of electric buses has been identified. The majority of the fleet consists of Maxi-class vehicles, which are characterized by a length of approximately 12 meters. Table 1 presents the distributions of the different bus types across all of the studied depots; of particular note is the deployment of the Mega-class buses – more than half of these are allocated to the Wola Duchacka depot. The data also indicates that the Płaszów depot operates only two classes of vehicles; this limited diversity may suggest an intentional specialization strategy that is aimed at operational efficiency and simplified maintenance. Tables 2 and 3 detail the allocations of the bus brigades to each depot as categorized by vehicle class and their utilization. An analysis of the data showed that the carrier did not fully utilize all of its resources, as at least one vehicle of each type remained unassigned at each depot. This approach likely served as a buffer against unexpected events such as mechanical failures or other disruptions (Szczyrbak, 2018).

**Table 2.** *Number of brigades of each type at given depot*

| | PB | PP | PW |
|---|---|---|---|
| Mini | 14 | 0 | 17 |
| Midi | 0 | 0 | 12 |
| Electric | 0 | 0 | 4 |
| Maxi | 89 | 76 | 52 |
| Mega | 31 | 35 | 77 |

**Table 3.** *Use of bus fleet*

| | Baseline [%] | | |
|---|---|---|---|
| | PB | PP | PW |
| Mini | 78 | n/d | 59 |
| Midi | n/d | n/d | 92 |
| Electric | n/d | n/d | 67 |
| Standard | 86 | 77 | 65 |
| Mega | 74 | 76 | 81 |

The distributions of the brigades by vehicle class varied among the studied depots. The Bieńczyce depot handled just over 40% of the Maxi-class brigades, whereas Wola Duchacka managed the smallest number of brigades of this type. In contrast, the majority of the Mega-class brigades were concentrated at the Wola Duchacka depot. Overall, this depot was responsible for the highest number of brigades, while the Płaszów depot accounted for just over one-quarter of the total brigade assignments. Maxi-class vehicles were used to operate the majority of the brigades across the network – particularly those that served high-demand agglomeration routes. At the Bieńczyce and Płaszów depots, most of the brigades were assigned to Maxi-class buses. In both cases, the ratio of the Maxi-class brigades to those that were operated by the other bus types was approximately two-to-one. In contrast, the Wola Duchacka depot utilized a more diverse vehicle portfolio, with the brigades being distributed across five bus classes; nearly half of these were Mega-class brigades, while Maxi-class vehicles were allocated to roughly one-third of the total. To support the analytical component of this study, it was necessary to develop a distance matrix; the values that were used for this were generated using the Google Maps tool.

Two matrices were created for the purposes of analysis: one representing the distances between each depot and the respective starting stop, and another indicating the distances from the final stop back to the home depot (Szczyrbak, 2018).

### 3.1. Mixed-integer model for the DMPMDPTS

The brigade-allocation problem describes the brigade to the specific type of vehicle that is located at one of the depots. In the DMPMDPTS, the optimization criterion is to minimize the sum of the kilometers that the vehicles must travel to reach the post-arrival stop and to return after the end of the work. The problem assumes that inter-arrival trips are fixed in the schedule. For the DMPMDPTS, a MIP program (1)–(5) was developed. The notation that is used in the DMPMDPTS is presented in Table 4.

**Table 4.** *Notation*

| **Sets** |
|---|
| $Z$ – depots |
| $T$ – types of vehicles |
| $B$ – brigades |
| $B^k$ – brigades that are served by vehicles of type $k$; $k \in T$, $B^k \subset B$ |
| **Parameters** |
| $a_{kp}$ – number of brigades of type $k$ assignable to depot $p$ |
| $d_{jp}$ – total deadhead distance in kilometers for brigade $j$ from depot $p$ (comprised of trip from depot to first service stop and return from final stop to depot) |
| **Decision variables** |
| $x_{jpk}$ – binary variable $x_{jpk} = 1$ if brigade $j$ is operated by vehicle of type $k$ from depot $p$; otherwise $x_{jpk} = 0$ |

The use of the binary variable assignment in the model was associated with the assignment of only one vehicle to a brigade. Compared to the model with an integer variable, the number of variables was much larger.

$$\min z = \sum_{j\in B}\sum_{k\in T}\sum_{p\in Z} d_{jp} x_{jpk} \tag{1}$$

$$\sum_{j\in B} x_{jpk} \le a_{kp},\ k \in T, p \in Z \tag{2}$$

$$\sum_{p\in Z} x_{jpk} = 1,\ j \in B^k,\ k \in T \tag{3}$$

$$x_{jpk} \in \{0, 1\}, \qquad j \in B,\ p \in Z,\ k \in T \tag{4}$$

The optimization criterion in the objective function (1) aimed to minimize the total distance that was traveled by the brigades. Formulated as an inequality, Constraint (2) ensured that the number of brigades of a given type from a single depot did not exceed the limit that was specified by parameter $a_{kp}$. Constraint (3) ensured that each brigade was assigned exactly one vehicle of a specified type regardless of the depot to which it was assigned. By splitting the set of brigades by vehicle type, the constraints were simplified, thus reducing the need for separate constraints for each decision variable that specified the vehicle type. While this approach was manageable for the small size of the task, it may not scale well in larger instances. Constraint (4) enforced the binary nature of the decision variable.

To exclude a depot from operation, Constraint (5) could be introduced into the model; in the case that was considered in this study, the Płaszów depot (PP) was excluded from use.

$$x_{jpk} = 0, \qquad j \in B,\ k \in T,\ p \in Z,\ p \in \{PP\} \tag{5}$$

3.2. Computational experiments

The DMPMDPTS model was implemented in AMPL and solved using the GLPK Integer Optimizer, (Version 4.43). The computational experiments were conducted on a Lenovo G50 laptop that was equipped with an Intel Core i7 2.4 GHz processor and 8 GB of RAM. The model was used to analyze three distinct cases. Parameter $a_{kp}$ is presented in Table 2, while parameter $d_{jp}$ was provided by MPK Kraków and is included in the documentation of the research to which this paper refers (Szczyrbak, 2018). Across all of the cases, the value of parameter $d_{jp}$ remained constant, whereas parameter $a_{kp}$ varied depending on the scenario. This variability enabled the flexible assignments of all of the brigades of a given type to a single depot.

In Case 1, the allocations of the brigades were based on actual operational data that was obtained from MPK Kraków; parameter $a_{kp}$ reflected these input values directly. The model results were compared against the real-world vehicle allocation. The number of decision variables in this scenario was 6105, with a memory usage of 4.7 MB. The model identified an optimal solution, and any key statistics that were related to the problem size and computational time were recorded. In Case 2, a new vehicle-to-brigade assignment was generated. Unlike the first scenario, the model allowed each depot to accept the maximum number of vehicles of each type. Although the assignment logic differed, the number of decision variables remained at 6105, while the memory usage increased to 6.2 MB. An optimal solution was again obtained. Case 3 followed the structure of Case 2 but introduced an additional constraint: the closure of one depot. This led to the reallocations of vehicles to those brigades that were under the new operational limitation. As in the previous scenarios, the number of decision variables remained unchanged (6105), while memory usage rose slightly (to 6.5 MB). The model successfully produced an optimal solution under this additional constraint.

Across all three cases, the DMPMDPTS model consistently identified optimal solutions. Based on the actual baseline data, the total distance that was traveled by the buses between their depots and their assigned routes – calculated as both departures from and returns to the depots – amounted to 6727 kilometers. In Case 1, this value was reduced to 6406.4 kilometers (representing a 4.76% decrease). This improvement was achieved under the constraint that no depot could accommodate more brigades than in the original allocation. In Case 2 (where the brigade-to-depot assignments were redesigned from scratch), the total distance dropped further – to 5820.9 kilometers (marking a 13.46% reduction as compared to the baseline). These favorable results motivated the development of Case 3, which explored the case of closing the Płaszów depot; however, this configuration led to a rise in the total distance to 5968 kilometers, which indicated that a depot's liquidation could negatively affect operational efficiency. A detailed comparison of the brigade assignments across the three variants is presented in Table 5.

**Table 5.** *Number of brigades assigned to depots after optimization*

| | Baseline | | | Case 1 | | | Case 2 | | | Case 3 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | PB | PP | PW | PB | PP | PW | PB | PP | PW | PB | PP | PW |
| Mini | 14 | 0 | 17 | 14 | 0 | 17 | 17 | 1 | 13 | 17 | 0 | 14 |
| Midi | 0 | 0 | 12 | 0 | 0 | 12 | 0 | 0 | 12 | 0 | 0 | 12 |
| Electric | 0 | 0 | 4 | 0 | 0 | 4 | 4 | 0 | 0 | 4 | 0 | 0 |
| Maxi | 89 | 76 | 52 | 89 | 76 | 52 | 67 | 26 | 124 | 75 | 0 | 142 |
| Mega | 31 | 35 | 77 | 31 | 35 | 77 | 56 | 1 | 86 | 56 | 0 | 87 |
| Sum of brigades | 134 | 111 | 162 | 134 | 111 | 162 | 144 | 28 | 235 | 152 | 0 | 255 |

The optimizations in Cases 2 and 3 led to significant changes in the allocations of the brigades across the three depots. In Case 2, the Płaszów depot was reduced to serving only 28 brigades, while Wola Duchacka depot took on 73 additional brigades as compared to the baseline. A notable shift in vehicle allocation occurred, with the Maxi vehicles serving substantial portions of the brigades. Furthermore, the Bieńczyce depot assumed responsibility for the operations of brigades with electric vehicles and saw an increase in the number of Mega-type vehicles. In Case 3 (with the closure of the Płaszów depot), the brigades that were previously assigned to Płaszów were redistributed to the other two depots. The Wola Duchacka depot then served 20 more brigades, while the Bieńczyce depot took on the remaining 8 brigades. Consequently, the Wola Duchacka depot was responsible for almost 60% of all of the brigades, while Płaszów depot handled only 7% of them. The changes in the depot allocations also influenced the distributions of the vehicles by class. In Case 2, a significant shift occurred in the Mini-class vehicles, with the Bieńczyce depot handling more than half of these brigades; this marked a noticeable deviation from the original data. The Maxi-class brigades were predominantly assigned to the Wola Duchacka depot (which experienced a considerable increase in its share), while the Płaszów depot saw a dramatic reduction in its role (now servicing only one brigade – a mere 1% of the total). The Mega-class brigades were distributed between the Wola Duchacka and Bieńczyce depots, with the Płaszów depot servicing only one brigade.

The brigade structure in the Bieńczyce depot became nearly balanced, with 46% of the brigades being served by Maxi vehicles and 39% by Mega vehicles. Notably, Case 2 also saw the electric bus brigades being allocated to the Bieńczyce depot – further contributing to the shift in depot responsibilities. The reduction in the role of the Płaszów depot was clearly illustrated by the graphical representation of the brigade share structures. As Case 2 progressed, a more balanced distribution emerged, with the Bieńczyce and Wola Duchacka depots increasingly absorbing the workload that was previously managed by Płaszów.

**Table 6.** *Summary of planned numbers of vehicles*

| | Baseline | | | Case 1 | | | Case 2 | | | Case 3 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | PB | PP | PW | PB | PP | PW | PB | PP | PW | PB | PP | PW |
| Mini | 18 | 0 | 29 | 18 | 0 | 29 | 22 | 2 | 23 | 22 | 0 | 25 |
| Midi | 0 | 0 | 13 | 0 | 0 | 13 | 0 | 0 | 13 | 0 | 0 | 13 |
| Electric | 0 | 0 | 6 | 0 | 0 | 6 | 6 | 0 | 0 | 6 | 0 | 0 |
| Maxi | 103 | 99 | 80 | 103 | 99 | 80 | 78 | 34 | 170 | 90 | 0 | 192 |
| Mega | 42 | 46 | 95 | 42 | 46 | 95 | 75 | 2 | 106 | 75 | 0 | 108 |
| Total | 163 | 145 | 223 | 163 | 145 | 223 | 181 | 38 | 312 | 193 | 0 | 338 |

In each of the developed variants that were analyzed, a lower total mileage was achieved as compared to the baseline case. In Case 1, the solution did not differ from the current state in terms of the number of vehicles that were used (see Table 6); the difference lay in the daily mileage (which was reduced by 320.6 kilometers). Assuming a cost of 6.6 PLN per truck kilometer, this translated into a daily savings of nearly 2000 PLN for the company. Case 2 introduced changes in the vehicle distribution; the newly developed allocation required the relocations of vehicles among the depots, which may have necessitated the expansions of one or more of the depots' facilities. The resulting savings amount to approximately 5500 kilometers per day. Additionally, Case 2 provided insights into the utilization levels of the individual depots, indicating that the Wola Duchacka depot experienced the highest load, with the lowest load being recorded at the Płaszów depot (whose closure was proposed in Case 3).

## 4. CONCLUSIONS

This study addresses the optimization of bus brigade utilization within public transportation by focusing on minimizing non-revenue-generating trips (commonly referred to as "deadhead kilometers"). A mathematical model was developed to achieve this objective; its effectiveness was evaluated using real-world data from the Krakow public transport system. The model demonstrated the capability of finding optimal solutions swiftly – even for complex cases – and it consistently outperformed the existing operational data in terms of reducing the total number of deadhead kilometers.

The findings aligned with previous research in the field. For instance, Mahadikar et al. (2015) developed a mixed-integer programming model to minimize dead kilometers in Bangalore's public transport by considering depot capacities and operational constraints. Similarly, Nasibov et al. (2013) applied various mathematical models to Izmir's bus system, achieving up to a 31.4% reduction in dead mileage under certain cases. These studies underscored the practical benefits of mathematical optimization in public transport operations

In the current study, case variants of the Deadhead Minimization Problem in Multi-Depot Public Transport System (DMPMDPTS) were tested. The key findings were as follows: (1) achieving a reduction in deadhead kilometers without altering the existing distribution of rolling stock among depots (Case 1); and (2) the additional minimization of deadhead kilometers by allowing the reassignments of vehicles between depots (Case 2). This indicated that strategic redistributions can lead to significant efficiency gains. These results demonstrated the DMPMDPTS's potential as a decision-support tool for those public transport authorities that are aiming to enhance their operational efficiency and reduce their unnecessary mileage.

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