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Multi-Criteria Optimization Approach to Modeling Negotiation Process

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Abstract. This paper presents a multi-criteria optimization approach for modeling the negotiation process. The negotiation process is modeled as a special multi-criteria problem. The method of finding solutions involves a process of the interactive selection of certain proposals. The parties submit proposals concerning the subjects of the negotiations; these proposals constitute the parameters of the multi-criteria optimization problem. Selecting the solutions is accomplished by solving the optimization problem using the parameters that define the aspirations of each party to the negotiations. Finally, the solutions reached by the parties are evaluated.

Keywords: negotiation process, multi-criteria optimization, equitably efficient decision, achievement function, set of negotiations, method for solution selection

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1. INTRODUCTION

This paper presents a multi-criteria optimization approach for modeling the negotiation process. The negotiation process is modeled as a special multi-criteria problem. The solution to this problem has properties that the parties consider to be justified in the negotiations. The solution obtained in such a way is acceptable by the parties. Negotiations are used to reach decisions in cases where the interests of the participants differ; they are carried out in order to achieve a more favorable result than could be achieved without negotiation. The negotiating parties benefit by reaching a mutual agreement as opposed to acting separately. A well-negotiated agreement is better for the parties than no agreement at all; however, some agreements are more favorable than others for both parties. In complex negotiations, the parties not only want to reach an agreement but an optimal agreement; i.e., that will be the best possible for both parties.

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Negotiations are characterized by a lack of clear solutions and the necessity of taking the preferences of the parties into account. The negotiating process can be modeled using the game theory; the solution is then the Nash cooperative solution or the Raiffy–Kalai–Smorodinsky solution (Luce, Raiffa, 1996; Malawski *et al.*, 1997; Straffin, 2004; Raiffa *et al.*, 2003; Roszkowska, 2011).

The present paper is devoted to applications of multi-criteria optimization for decision selection in the negotiating process. The process of negotiations is modeled as a special multi-criteria optimization problem whose solution is an equitably efficient decision. The method of decision selection is based on the interactive selection of certain proposals for solutions; that is, the algorithm requires the responses of the parties during this process. The parties submit their proposals concerning the subjects of negotiations; these proposals become the parameters of the multi-criteria optimization problem (thus, the problem is solved). Then, the parties evaluate the solution: accepting or rejecting it. In the case of rejection, the parties submit new proposals including new values for the parameters, and the problem is solved again for these new parameters. The process of selecting a solution is not a one-off but an iterative process of learning by the parties about the negotiated problem.

2. MODELING OF NEGOTIATION PROCESS

The negotiation process is modeled as an interactive decision-making process. Each party presents its proposals for the solutions. The negotiating process then becomes one of seeking a mutual decision that reconciles the interests of both parties. The parties try to find a mutual compromise solution. The decisions require the voluntary consent of both parties and are taken jointly, not unilaterally. Such decisions must be accepted by both parties.

During the negotiation process, many different functions are executed using the same set of feasible solutions. The negotiation process is modeled by means of the introduction of a decision variable describing the solution along with two evaluation functions evaluating the solution from the point of view of each party. During the negotiations, each proposal is evaluated by each party using the appropriate evaluation function. This function is a measure of the party's satisfaction with a given solution and evaluates the degree to which each party realizes its goals for each subject of the negotiations. The higher the value of the function, the greater satisfaction of the party; thus, each the function is maximized. The basis for evaluation and solution selection are these two functions of evaluation (i.e., the criteria of both parties).

We assume the following terms:

- Party 1 and Party 2 are the parties in the negotiations,
- n is the number of subjects for the negotiations,
- $x \in X_0$ is a solution (i.e., a decision to which both parties are to agree) belonging to set of feasible decisions $X_0 \subset R^n$, $x = (x_1, x_2, \ldots, x_n)$, and each coordinate $x_i,\ i = 1, \ldots, n$ defines the $i - th$ subject of the negotiations,
- $f_1 : X_0 \to R$ is the evaluation function of decision x by Party 1,
- $f_2 : X_0 \to R$ is the evaluation function of decision x by Party 2.

The problem of decision selection possesses a multi-criteria character. Each party wants to maximize its own evaluation function; however, each must take into account the existence of the other party. The selection of a solution is done by using both evaluation functions.

The negotiation process is considered to be a multi-criteria optimization problem with the function of purpose $f = (f_1, f_2)$:

$$\max_x \{ \big(f_1(x), f_2(x) \big) : \quad x \in X_0 \}, \tag{1}$$

where:

$x \in X$ – a vector of the decision variables,
$f = (f_1, f_2)$ – the vector function that maps decision space X into evaluation space Y_0,
X_0 – the set of feasible decisions.

Model (1) specifies that we are interested in the maximization of two evaluation functions (f_1 and f_2).

Problem (1) is considered in the evaluation space; i.e., the following problem is considered:

$$\max_x \{ y = (y_1, y_2) : \qquad y \in Y_0 \}, \tag{2}$$

where:

$x \in X$ – a vector of the decision variables,
$y = (y_1, y_2)$ – the function vector, individual coordinates
$y_i = f_i(x),\ i = 1, 2$ – represent the single scalar criteria, the first coordinate represents the evaluation criterion of a solution offered by Party 1, and the second is the evaluation criterion of a solution offered by Party 2,
$Y_0 = (f_1, f_2)(X_0)$ – the set of achievement vectors of the evaluation.

The set of achievements results Y_0 is given in implicit form; i.e., through set of feasible decisions X_0 and the mapping of model $f = (f_1, f_2)$. To determine value y, a simulation of the model is necessary:

$$y = (f_1, f_2)(x) \text{ for } x \in X_0$$

.

The purpose of problem (1) is to assist in the selection of a decision that takes into account the best interests of both parties (Lewandowski, Wierzbicki, 1989; Łodziński, 2007; Ogryczak *et al.*, 2008).

3. EQUITABLY EFFICIENT DECISION

Any solution in the negotiation process should satisfy certain properties that the parties accept as reasonable.

The solution should be as follows:

- a non-dominated solution (Pareto-optimal solution); i.e., one in which the solution for one party cannot be improved without impairing the solution for the other party,
- a symmetric solution; i.e., that does not depend on the way the parties are numbered, in which neither party is more important, and where the parties are treated in the same way in the sense that the solution does not depend on the names of the party or other factors specific to a given party,
- an equalizing solution; i.e., where a lesser variation of the coordinates of an evaluation is preferable to a vector with the same sum of the coordinates but with a greater diversity of the coordinates.
- one that takes into account the relative strength of the parties to the negotiations.

A decision that satisfies the first three conditions is an equitably efficient decision. This is an efficient decision that satisfies additional conditions: anonymity and the axiom of an equalizing solution.

Non-dominated solutions (Pareto-optimal solutions) are defined as follows:

$$\hat{Y}_0 = \{\hat{y} \in Y_0 : (\hat{y} + \tilde{D}) \cap Y_0 = \emptyset\}, \tag{3}$$

where $\tilde{D} = D \setminus \{0\}$ is a positive cone without the top. As a positive cone, $\tilde{D} = R^m_+$ can be adopted.

In the decision space, the appropriate feasible decisions are specified. Decision $\hat{x} \in X_0$ is called an efficient decision if the corresponding vector of evaluation $\hat{y} = f(\hat{x})$ is a non-dominated vector (Lewandowski, Wierzbicki, 1989; Wierzbicki, 1982).

In the multi-criteria problem (1) used to select a decision in the negotiation process, the preference relationship should satisfy additional properties: one of anonymity and one of an equalizing solution.

This preference relationship is called an anonymous relationship if, for each assessment $y = (y_1, y_2, \ldots, y_m) \in R^m$ and for any permutation P of set $\{1, \ldots, m\}$, the following property holds:

$$\big(y_{P(1)}, y_{P(2)}, \ldots, y_{P(m)}\big) \approx \big(y_1, y_2, \ldots, y_m\big). \tag{4}$$

No distinction is made between the results that differ in their arrangement of the coordinates. Evaluation vectors that have the same coordinates but organized in a different manner are identified.

The preference relationship satisfies the axiom of equalizing transfer if the following condition is satisfied.

For evaluation vector $y = (y_1, y_2, \ldots, y_m) \in R^m$:

$$y_{i'} > y_{i''} \Rightarrow y - \varepsilon \cdot e_{i'} + \varepsilon \cdot e_{i''} \succ y \text{ for } 0 < y_{i'} - y_{i''} < \varepsilon. \tag{5}$$

An equalizing transfer represents a slight deterioration of a better coordinate of the evaluation vector and, simultaneously, an improvement of a poorer coordinate, giving the strictly preferred evaluation vector as compared to the initial evaluation vector. This is a structure of equalizing; i.e., the evaluation vector with less diversity of the coordinates is preferred as related to the vector with the same sum of the coordinates but with a greater diversity of the coordinates.

A non-dominated vector satisfying the anonymity property and the axiom of equalizing transfer is called an equitably non-dominated vector. The set of equitably non-dominated vectors is denoted by $\hat{Y}_{0e}$. In the decision space, the equitably efficient decisions are specified. Decision $\hat{x} \in X_0$ is called an equitably efficient decision if corresponding evaluation vector $\hat{y} = f(\hat{x})$ is an equitably non-dominated vector. The set of equitably efficient decisions is denoted by $\hat{X}_{0e}$.

The relationship of equitable domination can be expressed as the relationship of inequality for the cumulative ordered evaluation vectors. This relationship can be determined with the use of transformation $\bar{T} : R^m \to R^m$ that cumulates the non-increasing coordinates of the evaluation vector.

Transformation $\bar{T} : R^m \to R^m$ is defined as follows:

$$\bar{T}_i(y) = \sum_{j=1}^{i} T_j(y) \text{ for } i = 1, 2, \ldots, m, \tag{6}$$

where $T(y)$ is a vector with non-increasing ordered coordinates of vector y; i.e., $T(y) = (T_1(y), T_2(y), \ldots, T_m(y))$, where $T_1(y) \leqslant T_2(y) \leqslant \ldots \leqslant T_m(y)$, and there is permutation P of set $\{1, \ldots, m\}$ such that $T_i(y) = y_{P(i)}$ for $i = 1, \ldots, m$.

Evaluation vector y^1 dominates vector y^2 in an equitable way if the following condition is satisfied:

$$y^1 \succ_e y^2 \Leftrightarrow \bar{T}(y^1) \geqslant \bar{T}(y^2), \tag{7}$$

The relationship of equitable domination $\succ_e$ is a simple vector domination for the evaluation vectors with coordinates that are the accumulated values of the ordered evaluation vector (Kostreva *et al.*, 2005; Ogryczak *et al.*, 2008; Yager, 1988).

Solving the problem of decision selection in the negotiating process consists of determining an equitably efficient decision that satisfies the preferences of both parties.

4. SCALARING THE PROBLEM

For determining an equitably efficient solution to a multi-criteria problem (1), a specific multi-criteria problem is solved – a problem with the vector function of the cumulative ordered evaluation vectors (i.e., the following problem):

$$\max_y \{ \big(\bar{T}_1(y), \bar{T}_2(y), \ldots, \bar{T}_m(y)\big) : \quad y \in Y_0 \}, \tag{8}$$

where:

y – an evaluation vector; $y = (y_1, y_2, \ldots, y_m)$,
$\bar{T}(y)$ – a cumulative ordered evaluation vector; $\bar{T}(y) = \big(\bar{T}_1(y), \bar{T}_2(y), \ldots, \bar{T}_m(y)\big)$,
Y_0 – the set of the achievement evaluation vectors.

An efficient solution to a multi-criteria optimization problem (8) is an equitably efficient solution to a multi-criteria problem (1).

To determine the solution to a multi-criteria problem (8), the scalaring of this problem with scalarizing function $s : Y \times \Omega \to R^1$ is introduced:

$$\max_x \{s(y, \bar{y}) : x \in X_0\}, \tag{9}$$

where:

y – an evaluation vector; $y = (y_1, y_2, \ldots, y_m)$,

$\bar{y}$ – the control parameters for the individual evaluations; $\bar{y} = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m)$.

This is a problem of single-objective optimization using a specially created scalarizing function of two variables – evaluation vector $y \in Y$ and control parameter $\bar{y} \in \Omega \in R^m$. This function is $s : Y \times \Omega \to R^1$. Parameter $\bar{y}$ is available to the parties, enabling them to review the set of equitably efficient solutions.

The optimal solution to the problem (9) should be a solution to the multi-criteria problem (8). The scalarizing function should satisfy certain properties (i.e., of completeness and of sufficiency). The property of sufficiency means that, for each control parameter $\bar{y}$, the solution for the scalaring problem is the equitably an efficient solution (i.e., $\hat{y} \in \hat{Y}_{0e}$). The property of completeness means that, through the appropriate changes of parameter $\bar{y}$, any solution $\hat{y} \in \hat{Y}_{0e}$ can be achieved. This function completely characterizes the set of the equitably efficient solutions. Inversely, each maximum of such a function is an equitably efficient solution. Each equitably efficient solution can be achieved with the appropriate values of control parameters $\bar{y}$.

The complete and sufficient parameterization of the set of equitably efficient solutions $\hat{Y}_{0e}$ can be achieved by using the reference point method for the problem (8). This method uses aspiration levels (i.e., those values of the evaluation function that satisfy both parties) as control parameters. The aspiration levels are those values of the evaluation function that satisfy both parties.

The scalarizing function in the reference point method is as follows:

$$s(y, \bar{y}) = \min_{1 \leqslant i \leqslant m} \big(\bar{T}_i(y) - \bar{T}_i(\bar{y})\big) + \varepsilon \cdot \sum_{i=1}^{m} \big(\bar{T}_i(y) - \bar{T}_i(\bar{y})\big), \tag{10}$$

where:

y – an evaluation vector; $y = (y_1, y_2, \ldots, y_m)$,

$\overline{T}(y)$ – a cumulative ordered evaluation vector; $\overline{T}(y) = \big(\overline{T}_1(y), \overline{T}_2(y), \ldots, \overline{T}_m(y)\big)$,

$\bar{y})$ – a vector of aspiration levels; $\bar{y} = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m)$,

$T(\bar{y})$ – a cumulative ordered vector of aspiration levels;
$T(\bar{y}) = \big(T_1(\bar{y}), T_2(\bar{y}), \ldots, T_m(\bar{y})\big)$,

ε – an arbitrary small positive adjustment parameter.

This scalarizing function is called the achievement function. The aim is to find a solution that approaches the specific requirements (i.e., the aspiration levels) as closely as possible.

Maximizing this function with respect to x determines equitably efficient solution $\bar{y}$ and equitably efficient decision $\hat{x}$; it should be noted that this, in turn, depends on aspiration levels $\bar{y}$ (Kostreva *et al.*, 2005; Ogryczak *et al.*, 2008).

5. SET OF NEGOTIATIONS

The aim of complex negotiations is not only to achieve an agreement between the parties (even one beneficial for both parties) but also to find a solution that meets the expectations of both parties to the greatest extent possible and is superior to any solution attainable without negotiations.

Before starting the negotiations, the parties should consider the result that will be achieved if the negotiations are unsuccessful: the status quo point. This point is the result that can be achieved by each party without negotiating with the other. If the parties can achieve result $ys = (y_1s, y_2s)$ without negotiations (i.e., Party 1 can achieve result y_1s, and Party 2 – result y_2s), then neither party will agree to the less desirable result. During the negotiations, the parties want to improve the solution as related to this point. The status quo point determines the relative strengths of the parties in the negotiations and the impact of those strengths on the result.

The set of negotiations is a collection of equitably dominated evaluation values dominating the status quo point.

The set of negotiations is as follows:

$$B(\hat{Y}_{0e}, ys) = \{\hat{y} = (\hat{y}_1, \hat{y}_2) \in \hat{Y}_{0e} \wedge \hat{y}_1 \geqslant y_1s \wedge \hat{y}_2 \geqslant y_2s\}, \tag{11}$$

where $\hat{y} = (\hat{y}_1, \hat{y}_2) \in \hat{Y}_{0e}$ is an equitably non-dominated vector, and $ys = (y_1s, y_2s)$ is the status quo point (i.e., the result that can be achieved by both parties without an agreement).

A set of negotiations covers the points from the set of equitably non-dominated results, which give each party at least as much as it can achieve individually (i.e., without negotiation).

The parties wish to find a decision ($\hat{x} \in X_0$) in which the corresponding evaluation vector $\hat{y} = (\hat{y}_1, \hat{y}_2) = \big(f_1(\hat{x}), f_2(\hat{x})\big)$ belongs to the set of negotiations $B(\hat{Y}_{0e}, ys)$ (Luce, Raiffa, 1996; Straffin, 2004; Raiffa *et al.*, 2003).

6. METHOD FOR SOLUTION SELECTION

The solution to the multi-criteria optimization problem (8) is the set of equitably efficient decisions. In order to resolve the problem, one solution should be selected for evaluation by both parties. Since the solution is a whole set, the parties select the solution with the help of an interactive computer system, which allows for a controlled overview of the whole set. Each party participating in the negotiation determines its proposed solutions as aspiration levels. These are the values of the evaluation of the individual negotiation issuesthat each party would like to achieve and constitute the control parameters of the scalarizing function. For these values, the system indicates different equitably efficient solutions for an analysis corresponding to the currentof the control parameters. The aim is to find solutions that conform to the parties' specific requirements as closely as possible (i.e., aspiration levels).

The method of decision selection is as follows:

1) Initial arrangements.
2) Iterative algorithm (i.e., proposals for further decisions).
   a) Interaction with the system – the parties define their proposals for the individual subjects of the negotiations as aspiration levels $\bar{y}_1$ and $\bar{y}_2$.
   b) Calculations (i.e., providing another equitably efficient solution – $\hat{y} = (\hat{y}_1, \hat{y}_2)$).
   c) Evaluation of obtained solutions $\hat{y} = (\hat{y}_1, \hat{y}_2)$ (i.e., the parties may either accept or reject this solution). In the latter case, the parties submit new proposals representing the new values of their aspiration levels $\bar{y}_1$ and $\bar{y}_2$, and a new solution is determined (return to Step 2.2).
3) Determination of a decision that meets the requirements of both parties.

The choice of decision is not a single act of optimization but a dynamic process for searching for solutions. This means that the parties can learn and change their preferences during the process. Comparing the results of evaluations $\hat{y}_1$ and $\hat{y}_2$ to their corresponding aspiration points $\bar{y}_1$ and $\bar{y}_2$, we see that each party possesses information about what is and is not achievable as well as how far the parties' proposals $\bar{y}_1$ and $\bar{y}_2$ are from possible solutions $\hat{y}_1$ and $\hat{y}_2$. This enables the parties to make the appropriate modifications to their proposals and to supply new aspiration levels. These levels of aspiration are determined adaptively during the learning process, which ends when the parties find a decision that enables them to achieve the results that meet their aspirations (or are as close as possible to these aspirations).

The method of decision selection is shown in Figure 1.

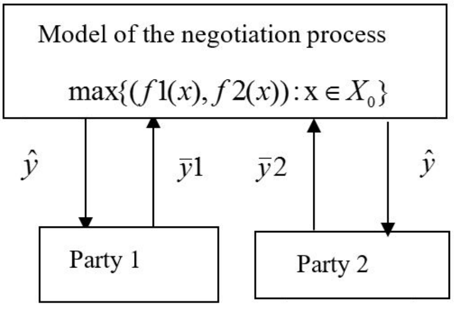

Fig. 1. Method of decision selection

This method of decision selection does not impose a rigid scenario on the parties and allows them to change their preferences while solving the problem. As we see, the parties learn about the problem during the negotiation. The computer does not replace the parties in the selection of the solution. It should be noted that the entire process of solution selection is controlled by both parties.

7. EXAMPLE

To illustrate the method of multi-criteria optimization for decision selection in a process of negotiations, the following example is presented (Ogryczak, 2002).

The problem related to negotiations is the following:

- Party 1 and Party 2 are the parties to the negotiations,
- $n = 2$ is the number of subjects for the negotiations,
- $x = (x_1, x_2) \in X_0$ is a solution; i.e., a decision to be agreed upon by the parties belonging to the feasible decision set, $X_0 \subset R^2$, x_1 is a decision concerning the first subject of the negotiations, x_2 is a decision concerning the second subject of the negotiations,
- $X_0 = \{(x_1, x_2) \in R^2 : x_1 + 3 \cdot x_2 \leqslant 63, 5 \cdot x_1 + 4 \cdot x_2 \leqslant 117, x_1 \geqslant 0, x_2 \geqslant 0\}$ is the feasible decision set,
- $f_1 : X_0 \to R^1 f_1(x) = x_1$ is the function of evaluation of the decision by Party 1,
- $f_2 : X_0 \to R^1 f_2(x) = x_2$ is the function of evaluation of the decision by Party 2,
- and $ys = (ys_1, ys_2) = (10, 10)$ is the status quo point.

The negotiation process is modeled as a multi-criteria optimization problem with the vector function of objective $f = (f_1, f_2)$:

$$\max_x \{ \big(f_1(x), f_2(x)\big) : \quad x \in X_0 \}, \tag{12}$$

where:

x – a vector of decision variables; $x = (x_1, x_2) \in X_0$,
y – the vector function that maps decision space X_0 into evaluation space $Y_0 \subseteq R^2$; $y = f(x_1, x_2)$,
X_0 – the feasible decisions set.

In the multi-criteria model (12), the individual solutions are evaluated using evaluation vector $y = (f_1, f_2)$, where f_1 is an evaluation function of decision x by Party 1 and f_2 is an evaluation function of decision x by Party 2.

As the first step of the multi-criteria analysis, a single-criterion optimization of the evaluation function for each party is performed. The result is a matrix of implementation goals that contains the values for all of the criteria of each party obtained by solving two single-criterion problems. This matrix allows for an estimation of the range of changes of each evaluation function in the feasible set as well as provides some information about the conflicting nature of the evaluation function.

Table 1. *Matrix of implementation of goals with utopia vector*

Optimization criterion		Solution $\hat{y}_1$	$\hat{y}_2$
Function	f_1	23.4	0
Function	f_1	0	21
Utopia vector	(y_1u, y_2u)	23.4	21

The matrix of the implementation of goals generates a utopia vector representing the best value for each separate criterion. An analysis of Table 1 shows that the negotiation forces of both parties are similar.

For each iteration, the price of fairness (POF) for each party is calculated (Bertsimas *et al.*, 2011). This is the quotient of the difference between the utopia value of a solution and the value from the solution of the multi-criteria problem as related to the utopia value:

$$POF = \frac{yiu - \hat{y}i}{yiu}, \; i = 1, 2, \tag{13}$$

where:

yiu – the utopia value of party i, $i = 1, 2$,
$\hat{y}i$ – the value from the solution of the multi-criteria problem of party i, $i = 1, 2$

The value of the POF is a number between 0 and 1. POF values closer to zero are preferred by the parties, as the solution is closer to a utopia solution. The closer the values of the POFs of both parties get to each other, the more the solution will be considered to be improved.

The parties control the decision-making regarding the solution, giving their propositions in the form of aspiration levels; these represent the desired values of their evaluation functions, and the system provides solutions that correspond to the current values of the parameters and can be analyzed by both parties. The multi-criteria analysis is presented in Table 2.

Table 2. *Interactive analysis of seeking solution*

Iteration		Party 1 $\hat{y}_1$	Party 2 $\hat{y}_2$
1.	Aspiration levels $\bar{y}$	23.4	21
	Solution $\hat{y}$	9	18
	POF	0.61	0.14
2.	Aspiration levels $\bar{y}$	18	16
	Solution $\hat{y}$	9	18
	POF	0.61	0.14
3.	Aspiration levels $\bar{y}$	16	14
	Solution $\hat{y}$	10.6	16
	POF	0.54	0.23
4.	Aspiration levels $\bar{y}$	15	13
	Solution $\hat{y}$	11.4	15
	POF	0.51	0.28
5.	Aspiration levels $\bar{y}$	14	12
	Solution $\hat{y}$	12.2	14
	POF	0.47	0.33
6.	Aspiration levels $\bar{y}$	13	12
	Solution $\hat{y}$	13	13
	POF	0.44	0.38

At the beginning of the analysis, the parties determine their preferences in the form of aspiration levels equal to the utopia vector. The solution is clearly more favorable for Party 2 yet unacceptable for Party 1, as it is worse than its point of reference. The cost of the POFs for Party 1 amounts to 0.61, while this number is 0.14 for Party 2. To improve the solution, both parties will reduce their requirements within the next iteration. The solution has not changed.

In the subsequent iterations, the parties keep on reducing their requirements. Each time, the solutions are more favorable for Party 1 and less favorable for Party 2. The values of the cost of the POFs for Party 1 become smaller, while they increase for Party 2 (and they get closer to each other). In Iteration 6, the cost of the POFs is 0.44 for Party 1 and 0.38 for Party 2.

Further attempts to change the solution do not result in an approximation of the POF values of the parties. The POF value of Party 1 cannot be improved any further. It is not possible to make the values of the POF more equal for both parties. This is the maximum that Party 1 can achieve. For Iteration 6, the solution of the negotiation process is the following decision: $\hat{x}^6 = (13, 13)$.

The final selection of a specific solution depends on the preferences of the parties. This example shows that the presented method enables the parties to get to know their decision-making possibilities within an interactive analysis and to search for a solution that would be satisfactory for both parties.

8. CONCLUSIONS

The paper presents a multi-criteria optimization approach to modeling a negotiation process to be used to support a decision selection. The model of the negotiation process as a multi-criteria optimization problem allows us to create variants of the decision and to track their consequences.

The method of an interactive analysis based on the reference point method is applied for a multi-criteria problem with a cumulative ordered evaluation vector.

This method is characterized by the following:

– the use of information about the parties' preferences in the form of aspiration points (i.e., the values of a goal function that are fully satisfactory to both) with the optimal option of the scalarizing achievement function in order to organize the interactions between the parties,

– the assumption that the preferences of the parties are not completely fixed and may change during the decision-making process.

This method enables us to identify solutions that are tailored to the parties' preferences. The numerical example shows that the appropriate computational problem can be solved efficiently by using standard optimization software.

This procedure does not determine the ultimate solution but supports and teaches the parties about the specific negotiation problem. The final decision must be made by the parties involved in the negotiations.

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