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ENERGY REDISTRIBUTION IN AUTONOMOUS HYBRIDIZATION OF AGENT-BASED COMPUTING

Abstract *Evolutionary multi-agent systems (EMAS) are very good at dealing with difficult, multi-dimensional problems. Research is currently underway to improve this algorithm, giving agents even more freedom not only to solve the problem, but also to make decisions about the behavior of the algorithm. One way is to hybridize this algorithm with other existing algorithms to create the Hybrid Evolutionary Multi Agent-System (HEMAS). Unfortunately, such connections generate problems in the form of unbalanced agent energy levels. One solution is to use an agent energy redistribution operator. The article presents three different proposals for such redistribution operators, compared them with each other and selected the best based on the results of numerous experiments.*

Keywords agent-based computing, hybrid metaheuristics, nature-inspired algorithms

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1. Introduction

Despite the increase in computer performance, not all problems can be resolved in a timely manner. Some problems solved by deterministic algorithms take too long or are too complicated (adding dozens of cities to the TSP problem is such an example). However, some problems do not have deterministic solutions. In these and other cases, novel stochastic methods can help. If other solutions do not meet the assumed goals, you can use metaheuristics. Their big advantage is that they do not require information about the characteristics of the search space. One of the advantages is the ability to tune the algorithm through the selection of parameters (cf. iRace [11]) or the ability to combine with other algorithms to create hybrid algorithms (cf. Talbi [17]). We are still looking for new metaheuristics because it is impossible to find a single method that will solve all problems with the same accuracy (cf. Wolpert and MacReady [18]).

An example of such an algorithm is EMAS, which has been with us since 1996 [5]. It is a kind of combination of the evolutionary method with the agent paradigm, resulting in a program in which agents are part of the computational process that searches the area of search and consists of, among others, decentralized selections, giving offspring or death. There is no central control, it can all be easily paralleled, which reduces the computation time. After careful analysis [4, 15], it has become a solid base for attempts to combine with other ideas, giving some interesting hybrid algorithms ([8] and [12]). However, when connecting, one may encounter the problem of redistributing the energy of agents making their own decisions to use algorithms that do not use energy ([9]). This article looks at several solutions to this problem and carefully examines the best of them.

The article, after reminding how the EMAS algorithm works, describes in detail its hybrid HEMAS along with the problem of energy redistribution. The main contribution of this paper is proposal of three solutions in the form of Proportional redistribution operator, Ranking redistribution operator and Tournament redistribution operator. After experimenting with the parameters for Tournament redistribution operator, is presented the results of the comparative experiments of all operators. The next part includes the results of the experiment examining the influence of the selected operator on the algorithm. At the end is a summary and plan for further research.

2. Hybrid versions of EMAS

EMAS (*evolutionary multi-agent system*), may perceived as "proactive" alternative to classical evolutionary computation techniques [10], hoped by the authors to relieve the evolutionary metaheuristics from several inconsistencies with the real-life evolution, such as e.g. lack of global control, and asynchronous reproduction. In this system, solutions (genotypes) are entrusted to agents, handling and improving their solution during realization of several types of actions available to them. In this way agents can reproduce, die or migrate among the islands. The selection mechanism is implemented using resources (agents compete for the resources, only a rich agent can reproduce, the poor agent will die) [5]. During meetings, agent exchange the resources (the worse one gives a part of its resource to the better one). For the schematic view on EMAS one can refer to the Figure 1. It is to note, that correctness of the EMAS as global universal optimizer has been formally proven using Markov Chain based models, inspired by the theoretical works of Michael Vose [3,4]. EMAS has also many extensions, e.g. immunological one [1,2] and was applied to solve different single and multi-criteria problems.

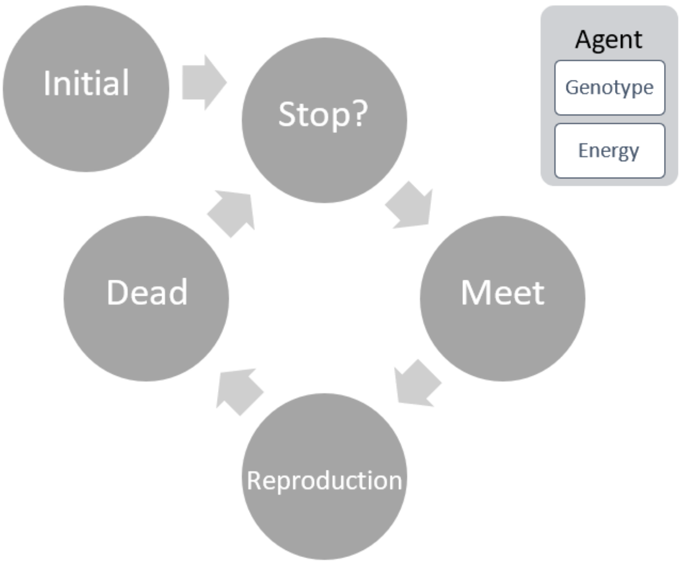


Figure 1. Diagram of Evolutionary Multi Agent Systems

Hybrid Evolutionary Multi Agent-System (HEMAS) is an algorithm built on the EMAS algorithm with an additional hybridization step. Hybridization step, as shown in the Figure 2, was placed after dead step, but before the step in which we check whether the algorithm should be terminated. It is put here because this step can significantly speed up the completion of the entire algorithm. This step has 3 parts as described below.

Optimization condition. In this part, we check if the agents are willing to participate in the hybridization step. This can happen for a variety of reasons. For example, an agent may not improve his solution for a long time, lose many matches, or for other reasons. Reasons for participating (as well as refusing to participate) can vary, and there are plans to investigate this. In this particular solution, it is given the opportunity to participate in this step every 500 cycles of the HEMAS algorithm. Note that each of the algorithms included in this step may have specific requirements. Failure to meet these requirements results in failure to run the given algorithm. If neither of the algorithms are satisfied, this entire step is skipped until the next possibility. In this implementation, each of the algorithms used required at least 20 agents willing to participate to run.

Run optimization algorithms. At this stage, algorithms are run to improve the agents' performance. All algorithms are run here. There is no limit to the number of algorithms or the length of their operation, however, it is worth choosing the algorithm values so that they support EMAS and not replace it (let's limit the number of calls to the evaluation function, because it depends on how long our HEAMS algorithm will run). There is no obligation for the algorithms to use agents or energy. Each algorithm can operate in a different way and store values along with solutions (e.g. speed and direction). We do not require the algorithm to use agents, but if it does not use agents, one change is required. If an agent is to die and a new agent is to be created, do the following instead. Change the solution value of a given agent and reset other parameters (except energy). Proceed with this agent as with a newly created one. With this solution we do not lose energy and algorithms do not need to know how to create new agent. This solution uses two algorithms: particle swarm optimization (PSO) and differential evolution (DE). This combination gives very good results [9], therefore we continue the chosen direction.

Energy redistribution. At this stage, the main task is to be performed by the energy redistribution operator. All agents who participated in the previous stage take part in it. Everyone, regardless of the number or type of algorithms, goes to this one redistribution operator together. In the following part are described different ways of distribution and the effect of these operators on the final results.

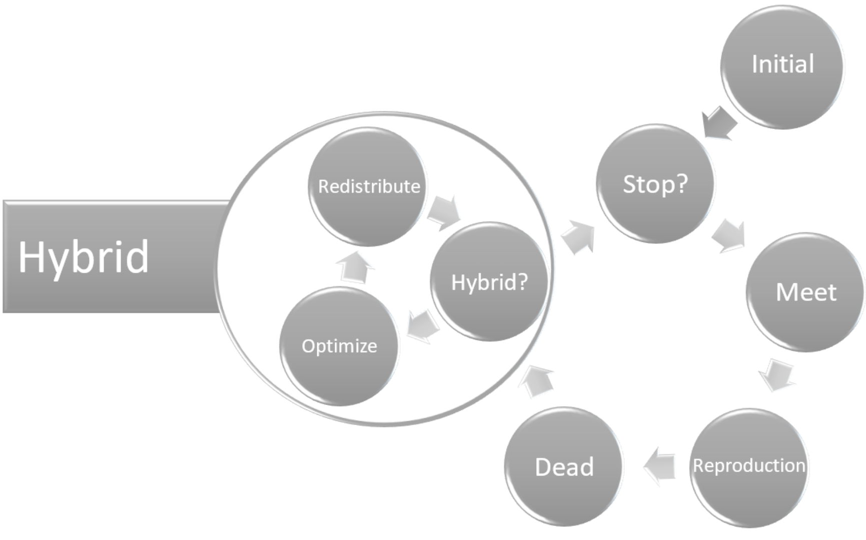


Figure 2. Diagram of Hybrid Evolutionary Multi Agent Systems

3. Energy redistribution in HEMAS

An important aspect in the EMAS algorithm is the energy of the agents. Its quantity determines whether the agent will be able to reproduce (its solution will be passed on) or die (its solution will be removed from the pool). The HEMAS hybrid step, when run for a group of agents, can significantly change agent solutions. It can save promising solutions or strengthen already good results. All these changes, however, can be lost if we do not change the amount of energy in the agents. Without this change, there may be a situation where an agent with a new, better solution, but with an old energy level, dies before passing on his solution. Unfortunately, the algorithms used in this step often do not consume energy or have different energy utilization mechanisms.Therefore, just like w [14], it was necessary to introduce a special way of combining these algorithms with each other. To prevent interference with the algorithms, we decided not to change them and to solve this problem by using the energy redistribution operator. However, this goal can be achieved by various methods, hence this research and article. As mentioned in article [9], agents completing a hybrid step in HEMAS have energy prior to that step. Therefore, their current amount of energy is disproportionate to the quality of the solutions they have. This should be fixed before proceeding with the algorithmic steps. In another case, you may just lose your changes. To solve this, an energy redistribution mechanism was introduced. It involves reallocating energy to the agents according to their present state. There are several different ways to do this. In this article we will try to analyze several different approaches, compare them with each other and finally choose the best one.

In order to precisely define the proposed redistribution mechanisms, let us assume, that one of agents undergoing the hybridization will be described as: $Ag \ni ag_i = (e_i, g_i)$ where $Ag = \mathbb{R}^+ \cup \{0\} \times R^d$ is a set of all possible agents, d is the number of dimensions of the real-value space (problem domain), e_i and g_i are energy and genotype of the i-th agent respectively. Now, the set of agents which will undergo the hybridization will be described as: $2^{Ag} \supset \gamma_b = \{ag_1, \ldots, ag_k\}$, where $k \in \mathbb{N}$, k is the number of the agents in the set which will undergo the hybridization. Now a hybridization function will transfer γ_b into γ_a: $hyb : 2^{Ag} \to 2^{Ag}$, and a proper redistribution operator will work on the energy of γ_a transferring it into energy of γ_a.

3.1. Proportional redistribution operator

The first way to distribute energy, proposed in the article [9], is to allocate energy to agents proportionally to their fitnesses. As a result of the operation of this operator, agents will always have exactly enough energy so that the energy ratio between the agents corresponds to the ratio of their solutions.

This is resolved as follows:

1. The energy of all agents is summed up. In this way, we obtain an energy bank that we will use in the next steps. At this point, all agents have 0 energy for the moment. They do not die, because the death of agents is possible only in dead steps of the HEMAS algorithm. $\mathbb{R} \ni s = \sum_{i=1}^{k} e_i$; $ag_i \in \gamma_a$.

2. Next, we sum up all fitness values of the agents. $\mathbb{R} \ni fitnesses = \sum_{i=1}^{k} f(g_i); ag_i \in \gamma_a$, where $f : \mathbb{R}^d \to \mathbb{R}$ is the fitness function.
3. Then, we sum proportions. $\mathbb{R} \ni proportions = \sum_{i=1}^{k} \frac{fitnesses}{f(g_i)}; ag_i \in \gamma_a$.
4. Finally, we allocate a portion of the energy from s to each agent so that so as to maintain the ratio of fitness and energy. $\forall i, e_i = \frac{s \cdot fitnesses}{f(g_i) \cdot proportions}$, where $ag_i \in \gamma_a$.

3.2. Ranking redistribution operator

Another solution is the ranking operator. It builds a ranking of agents based on their solutions. Then it allocates energy depending on the ranking position. The agent who wins the ranking, will take the most energy. The agent in the last place gets her the least. Energy comes from all agents that are involved in this step of the algorithm. An important difference that distinguishes this method from the proportional operator is that with this method, the agents get energy directly proportional to their rank. Of course, the place in the ranking depends on their solution, but the difference in energy obtained between agents with very similar solutions may be different than what they would get in the proportional operator. The differences in the amount of energy received from place to place are constant. This solution, therefore, does not so strong promote very good solutions and it does not so strong punish weak solutions.

Let us assume the ranking function $r : 2^{Ag} \to 2^{Ag} \times \mathbb{N}$. This function will assign the rank to each of agents from γ_a, based on agents' fitnesses. It will produce the ranking set: $rs = \{(ag_1, r_1), \ldots, (ag_k, r_k)\}$. Now the energy will be set according to the following equation: $\forall i, e_i = \frac{2 \cdot s \cdot (1+k-r_i)}{k \cdot (k+1)}$, where $ag_i \in \gamma_a$.

3.3. Tournament redistribution operator

The most complex operator in action is the last operator discussed here: the tournament operator. Its operation is based on a series of meetings of two or more agents, hereinafter referred to as tournaments. Agents do not report to the tournament themselves, but are randomly selected from the entire pool of agents taking part in this step. Only one agent can win each tournament. The winner takes the prize which is energy. An agent can win a lot of tournaments and get a lot of energy that way. After a series of such tournaments, the energy should more reflect the quality of agents' solutions.

We need the following parameters to implement this operator:

- number of tournaments,
- number of agents in each tournament,
- entry fee to the tournament,
- amount of the prize for winning the tournament.

The first two parameters should be selected experimentally. We'll do this in 4.3. Unfortunately, we cannot have a fixed entry fee to the tournament, as agents with more and less energy can appear here. It is possible that there will be agents who do not have any energy at the time of joining this step of the algorithm. Therefore, as an entry fee to all competitions, we take all energy from the agents. We determine the size of the prize by dividing the amount of energy from the entry fees by the number of tournaments. Thus, each tournament will have the same prize.

Thus, assuming in the simplest case (two-agent tournament) a prize being a part of the sum of the energy $p \propto s$, two agents are randomly selected from the γ_a: ag_t and ag_u $(u, t \in \mathbb{N})$. Now, if $f(g_t) > f(g_u)$, $e_t \leftarrow e_t + p$. Such meetings are performed until all the energy from s is redistributed.

4. Experimental results

This section presents and discusses the experimental results obtained for HEMAS and the various energy redistribution operators.

All experiments were done using Prometheus which has 2403 TFlops, runs on Linux CentOS 7 and which is part of PL-Grid[1].

A platform based on jMetal[2] was used to run the tests. This platform has some improvmens done by dr Leszek Siwik. It was used to prepare, inter alia, calculations [13] and [16]. All implementations of algorithms, problems and other components come from this platform and can be found here: https://bitbucket.org/lesiwik/modelowaniesymulacja2018. JMetal version used was 5.6 and Java 13.0.2.

4.1. Benchmarks

In order to compare the energy redistribution operators, the following problems were used: Rastrigin, Ackley, Sphere, Schwefel, Griewank. Each of the problems was used in the following sizes: 100, 300, 500, 1000 and 2000. Additionally, in order to confirm a wide range of solutions, the algorithm was tested on 24 problems from CEC 2005, each in three sizes: 10, 30 and 50. All problems are implemented in jmetal [6]. During the implementation we have leveraged the knowledge gathered while devloping many computing frameworks, e.g. [7].

4.2. Configuration

Main parameters of the HEMAS algorithm are as follows:

- population size: 50,
- initial agent energy: 10,
- reproduction predicate: energy above 20,
- death predicate: energy equal to 0,
- crossover operator: SBX Crossover, (Distribution Index 5, Crossover Probability 1),

[1] http://www.plgrid.pl/en

[2] jMetal [6] is an object-oriented Java-based framework aimed at the development, experimentation, and study of metaheuristics for solving optimization problems. http://jmetal.github.io/jMetal/

- mutation operator: Polynomial Mutation (Distribution Index 10, Mutation Probability 0.002),
- reproduction energy transfer: 10,
- meet energy transfer: 1,
- hybrid predicate: algorithm calls frequency – every 500 HEMAS cycles.

PSO algorithm parameters:

- max iterations: 3,
- optimization predicate: energy below 3,
- minimal population size: 20.

DE algorithm parameters:

- max iterations: 3,
- optimization predicate: energy above 17,
- minimal population size: 20.

Tournament Redistribute Operator parameters selected was:

- the number of tournaments: 20,
- the size of the tournament groups: 3.

Number of starts of each tested case: 30. End condition: $1000 \ldots d$ (size of the problem) calls of the evaluation function.

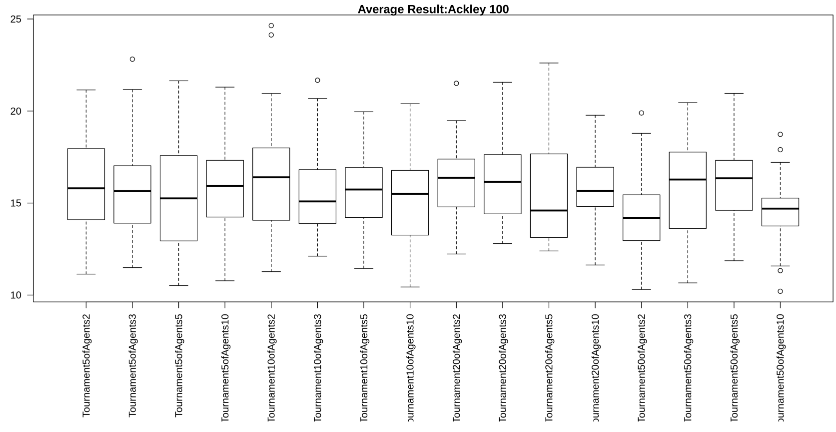
4.3. Selection of parameters for Tournament Energy Redistribution Operator

Tournament Redistribute Operator parameters tested was:

- the number of tournaments: 5, 10, 20, 50,
- the size of the tournament groups: 2, 3, 5, 10.

The operator was tested on a subset of problems because of the many combinations tested. For this set, 3 problems (Ackley, Rastrigin and Sphere) were selected in 3 dimensions (100, 300, 500), which gives us 9 test cases. The differences in the results of individual combinations on Figure 3, Figure 4 and Figure 5 did not differ significantly from each other. Therefore, further search was abandoned and the following parameters were selected: number of tournaments 20 and size of the tournament groups 3.

a)



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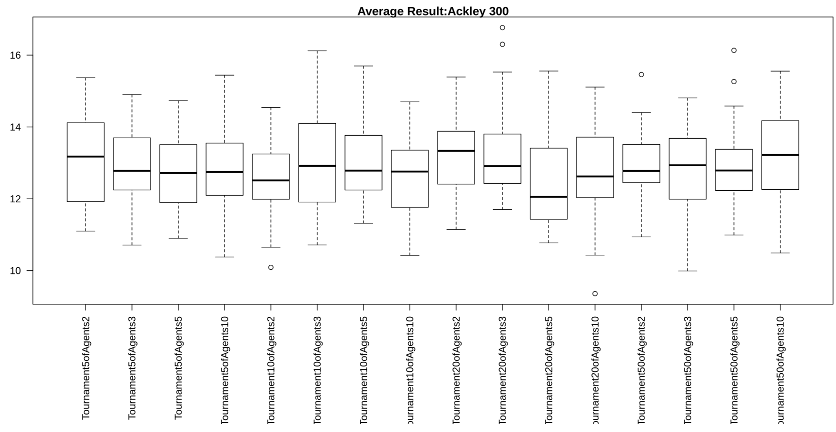

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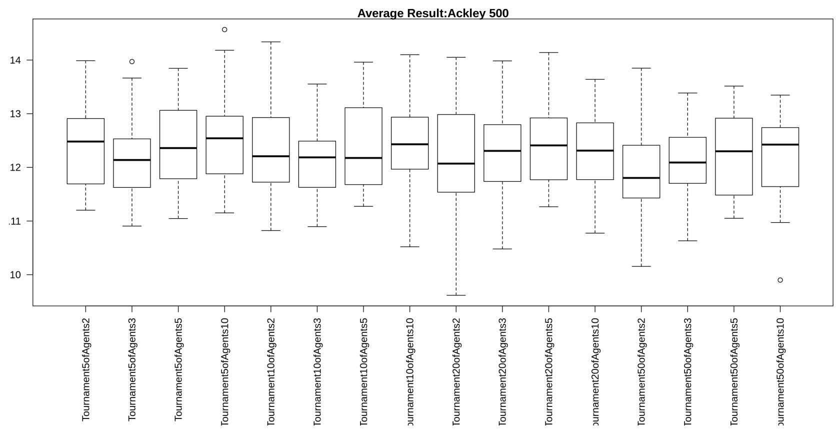
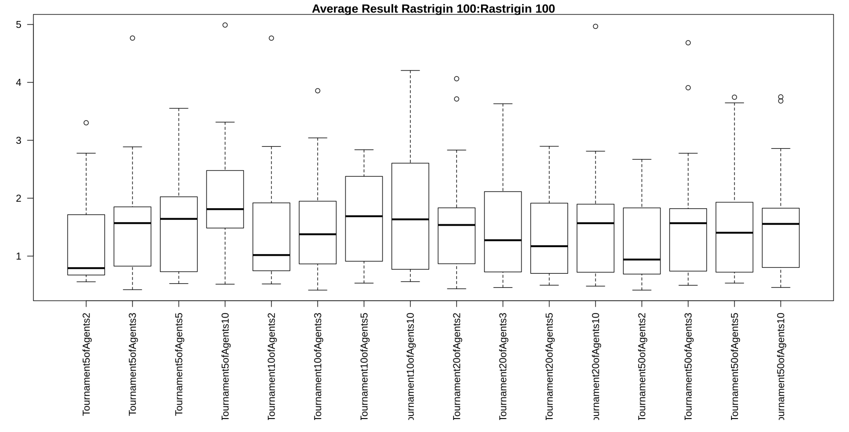


Figure 3. Best fitness values for the selected benchmark functions obtained by HEMAS with tournament redistribution operator with different parameters: a) 100-dimensional Ackley function; b) 300-dimensional Ackley function; c) 500-dimensional Ackley function

a)



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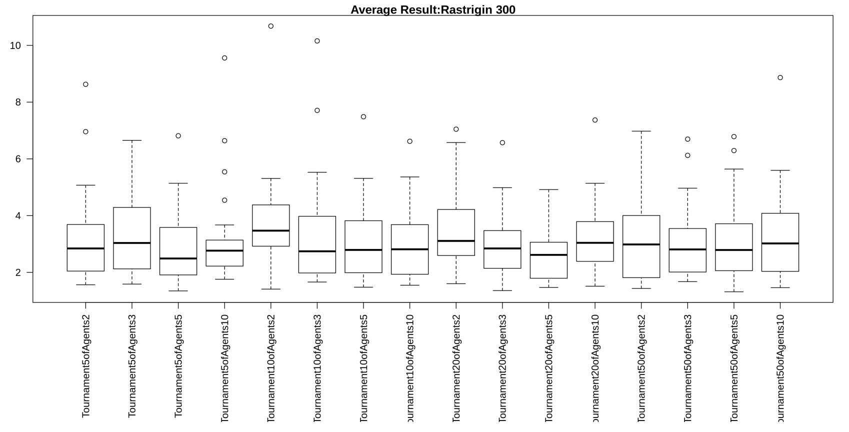

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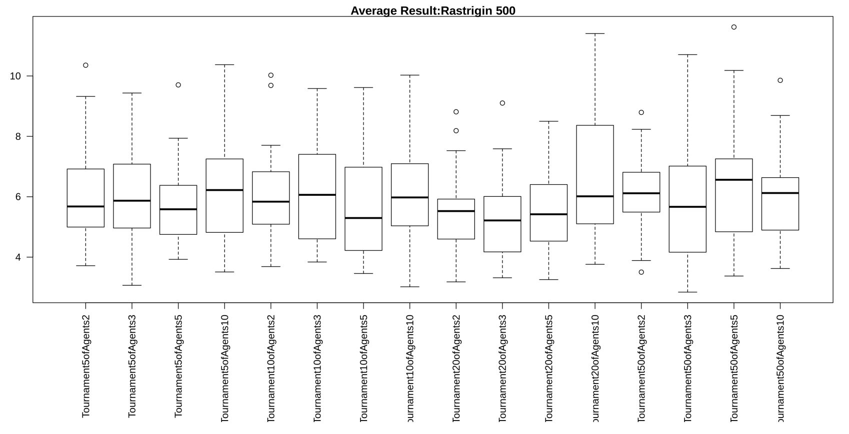

Figure 4. Best fitness values for the selected benchmark functions obtained by HEMAS with tournament redistribution operator with different parameters: a) 100-dimensional Rastrigin function; b) 300-dimensional Rastrigin function; c) 500-dimensional Rastrigin function

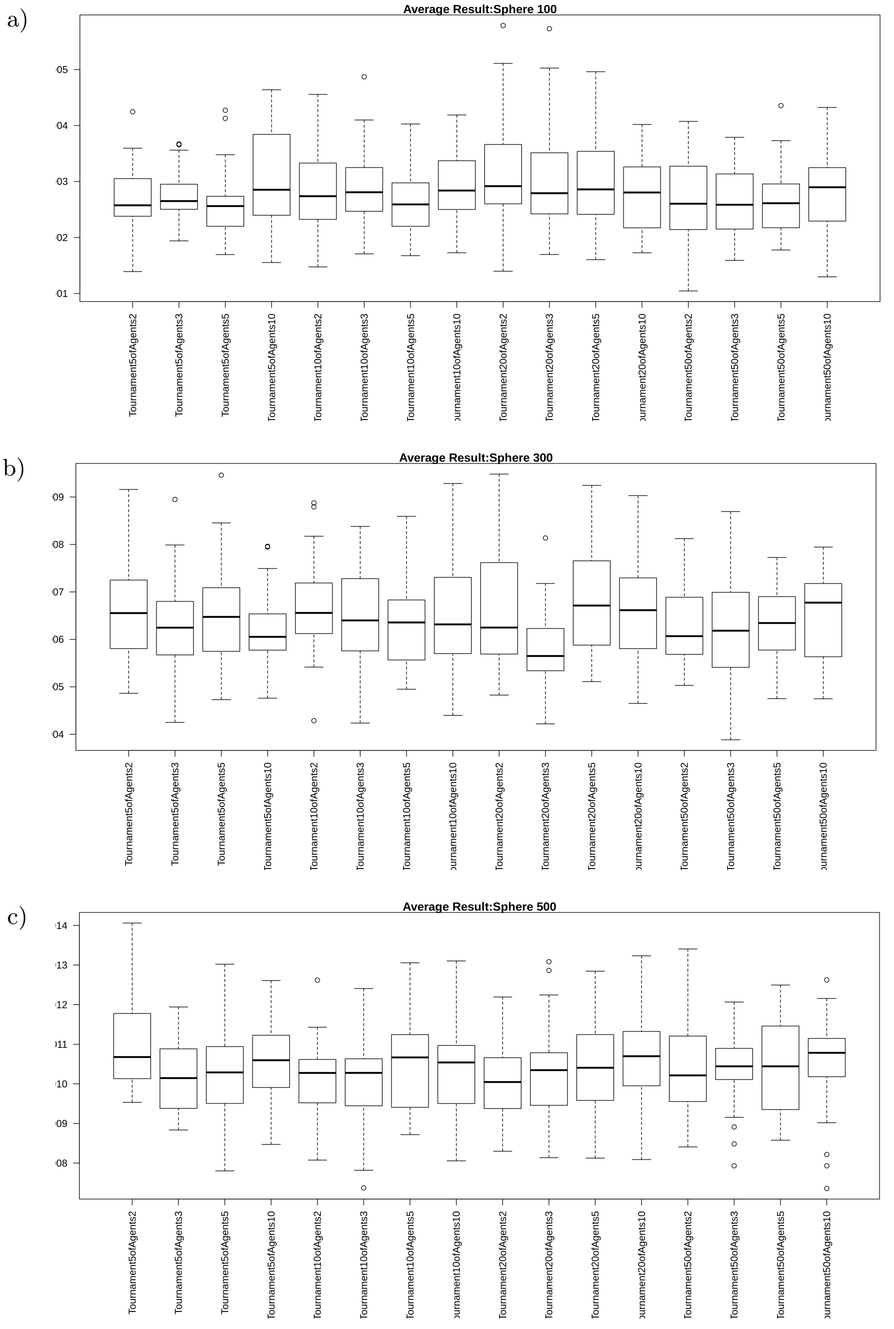

Figure 5. Best fitness values for the selected benchmark functions obtained by HEMAS with tournament redistribution operator with different parameters: a) 100-dimensional Sphere function; b) 300-dimensional Sphere function; c) 500-dimensional Sphere function

4.4. Compare the redistribution operators

Having selected parameters for the tournament operator, we can proceed to compare all redistribution operators with each other. We will use the same 9 test cases for testing as in section 4.3. The box-plot with all final results can be compared in Figure 6.

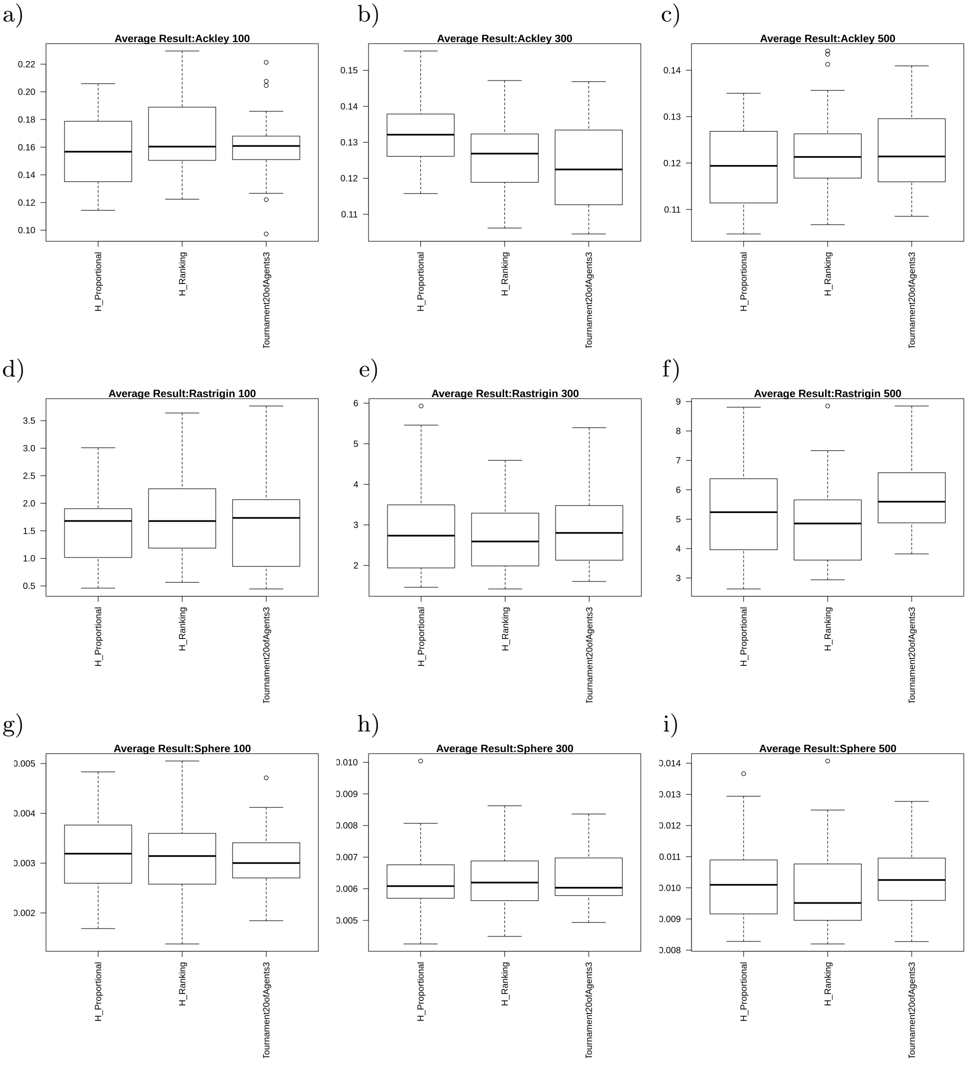

Figure 6. Best fitness values for the selected benchmark functions obtained by HEMAS with proportional, ranking and tournament redistribution operators: a) 100D Ackley function; b) 300D Ackley function; c) 500D Ackley function; d) 100D Rastrigin function; e) 300D Rastrigin function; f) 500D Rastrigin function; g) 100D Sphere function; h) 300D Sphere function; i) 500D Sphere function

In each sub-graph, the results of the algorithm with the proportional operator are on the left, in the middle is the algorithm with the ranking operator, and on the right are the results of the algorithm with the tournament operator. At first glance, choosing the best operator is difficult, therefore, the Kruskal-Wallis tests were used. After analyzing the results, the proportional operator is the most statistically different from the basic algorithm. Therefore, in further tests, it is the algorithm with this operator that will be used.

4.5. Comparing the impact of the redistribution operator on HEMAS algorithm

After an in-depth analysis of the results of the various redistribution operators and their various parameters, the most promising was selected. Now the time has come for a detailed comparison of the algorithm version with and without this operator. Thanks to this, it will be possible to determine whether and what influence the selected operator has on the final result. First, the HEMAS algorithm was compared with and without the selected operator on the Ackley, Griewank, Rastrigin, Schwefel and Sphere problem set, each in five sizes 100 dimensions, 300 dimensions, 500 dimensions, 1000 dimensions and 2000 dimensions.

In Table 1, 2, 3 and 4 can be found median, mean, standard deviation, maximum and minimum of all 30 runs for all problems and sizes. In each table, on top are HEMAS results and on bottom are results from HEMAS with the Proportional operator. In Table 1 it can be read that the HEMAS results for Ackley problems at all magnitudes are weaker than those obtained by HEMAS with the Proportional operator. It is similar with the solutions to the Griewank problem (still Table 1), here also the results of the algorithm with the operator are much better than the results of the algorithm without the operator. The same table show the results from the Rastrigin problem. In the case of the 100 dimensions Rastrigin problem, it can be seen that the better part of the algorithm's results without the operator coincides with the weaker part of the algorithm's results with the operator. For size 300 dimensions and 500 dimensions, HEMAS with operator results better than HEMAS without operator. For the size of 1000 dimensions, the results start to be similar and for the size of 2000 dimensions the operator can see that he negatively influenced the final results. For all five versions of the Schwefel problem, no significant differences can be seen between the solutions of both versions of the algorithm. The greatest profit from the use of the operator can be seen in the results of the Sphere problems. In each of the five cases, the operator helped significantly.

Due to the above, it was decided to study the algorithms on a wider set of problems. For this purpose, issue set from IEEE Congress on Evolutionary Computation 2005 (CEC2005) was used. Each of the 24 problems was solved in 3 sizes: 10 dimensions, 30 dimensions and 50 dimensions. In each of the tables, HEMAS algorithm is on the top and HEMAS algorithm with the redistribution operator on the bottom. Table 2 shows the results from the first eight problems. You will notice that the results for first problem are better for the operator version of HEMAS. The remaining seven problems were resolved in a similar way. From Tables 3 and 4, where there are problems 9 to 24, the results for problems 15, 16, 17, 21 and 22 in the 30 dimensions and 50 dimensions sizes stand out, where the algorithm with the operator gave weaker results. The above problems have thoroughly analyzed the version of the algorithm on very difficult cases, thanks to which it was found that the algorithm does not always work perfectly for problem sizes between 30 and 50. Of course, one cannot expect any algorithm to solve all problems equally well.

Table 1

Results of HEAMS with operator and without operator for tested problems

	HEMAS without redistribute operator				
	Mean	Median	SD	Minimum	Maximum
Ackley 100	$2.62 \cdot 10^{-1}$	$2.61 \cdot 10^{-1}$	$4.21 \cdot 10^{-2}$	$1.79 \cdot 10^{-1}$	$3.65 \cdot 10^{-1}$
Ackley 300	$2.91 \cdot 10^{-1}$	$2.80 \cdot 10^{-1}$	$4.86 \cdot 10^{-2}$	$2.36 \cdot 10^{-1}$	$4.75 \cdot 10^{-1}$
Ackley 500	$3.47 \cdot 10^{-1}$	$3.17 \cdot 10^{-1}$	$9.64 \cdot 10^{-2}$	$2.30 \cdot 10^{-1}$	$6.23 \cdot 10^{-1}$
Ackley 1000	$5.01 \cdot 10^{-1}$	$4.55 \cdot 10^{-1}$	$1.96 \cdot 10^{-1}$	$2.90 \cdot 10^{-1}$	1.20
Ackley 2000	$5.40 \cdot 10^{-1}$	$5.12 \cdot 10^{-1}$	$1.55 \cdot 10^{-1}$	$2.78 \cdot 10^{-1}$	$9.14 \cdot 10^{-1}$
Griewank 100	$8.67 \cdot 10^{-1}$	$8.78 \cdot 10^{-1}$	$9.79 \cdot 10^{-2}$	$6.86 \cdot 10^{-1}$	1.03
Griewank 300	1.02	1.01	$6.66 \cdot 10^{-2}$	$8.64 \cdot 10^{-1}$	1.17
Griewank 500	1.07	1.09	$1.13 \cdot 10^{-1}$	$8.82 \cdot 10^{-1}$	1.54
Griewank 1000	1.28	1.25	$1.07 \cdot 10^{-1}$	1.11	1.53
Griewank 2000	1.96	1.54	1.49	1.37	9.79
Rastrigin 100	5.45	5.23	1.96	1.74	9.40
Rastrigin 300	$1.49 \cdot 10^{1}$	$1.49 \cdot 10^{1}$	3.66	7.27	$2.33 \cdot 10^{1}$
Rastrigin 500	$2.59 \cdot 10^{1}$	$2.49 \cdot 10^{1}$	6.74	$1.71 \cdot 10^{1}$	$4.57 \cdot 10^{1}$
Rastrigin 1000	$6.05 \cdot 10^{1}$	$5.30 \cdot 10^{1}$	$2.14 \cdot 10^{1}$	$3.55 \cdot 10^{1}$	$1.1 \cdot 10^{2}$
Rastrigin 2000	$2.01 \cdot 10^{2}$	$1.83 \cdot 10^{2}$	$9.79 \cdot 10^{1}$	$9.42 \cdot 10^{1}$	$5.55 \cdot 10^{2}$
Schwefel 100	$5.28 \cdot 10^{3}$	$5.40 \cdot 10^{3}$	$6.29 \cdot 10^{2}$	$3.92 \cdot 10^{3}$	$6.28 \cdot 10^{3}$
Schwefel 300	$1.62 \cdot 10^{4}$	$1.64 \cdot 10^{4}$	$9.04 \cdot 10^{2}$	$1.39 \cdot 10^{4}$	$1.81 \cdot 10^{4}$
Schwefel 500	$2.82 \cdot 10^{4}$	$2.83 \cdot 10^{4}$	$1.18 \cdot 10^{3}$	$2.53 \cdot 10^{4}$	$3.04 \cdot 10^{4}$
Schwefel 1000	$5.59 \cdot 10^{4}$	$5.60 \cdot 10^{4}$	$1.16 \cdot 10^{3}$	$5.32 \cdot 10^{4}$	$5.84 \cdot 10^{4}$
Schwefel 2000	$1.15 \cdot 10^{5}$	$1.15 \cdot 10^{5}$	$3.17 \cdot 10^{3}$	$1.06 \cdot 10^{5}$	$1.21 \cdot 10^{5}$
Sphere 100	$6.71 \cdot 10^{-3}$	$6.55 \cdot 10^{-3}$	$1.73 \cdot 10^{-3}$	$3.67 \cdot 10^{-3}$	$1.13 \cdot 10^{-2}$
Sphere 300	$2.15 \cdot 10^{-2}$	$2.14 \cdot 10^{-2}$	$2.60 \cdot 10^{-3}$	$1.61 \cdot 10^{-2}$	$2.61 \cdot 10^{-2}$
Sphere 500	$3.45 \cdot 10^{-2}$	$3.46 \cdot 10^{-2}$	$3.23 \cdot 10^{-3}$	$2.60 \cdot 10^{-2}$	$4.18 \cdot 10^{-2}$
Sphere 1000	$7.16 \cdot 10^{-2}$	$7.08 \cdot 10^{-2}$	$5.77 \cdot 10^{-3}$	$6.24 \cdot 10^{-2}$	$8.79 \cdot 10^{-2}$
Sphere 2000	$1.43 \cdot 10^{-1}$	$1.39 \cdot 10^{-1}$	$1.46 \cdot 10^{-2}$	$1.27 \cdot 10^{-1}$	$2.12 \cdot 10^{-1}$
	HEMAS with Proportional Operator				
	Mean	Median	SD	Minimum	Maximum
Ackley 100	$1.66 \cdot 10^{-1}$	$1.63 \cdot 10^{-1}$	$2.68 \cdot 10^{-2}$	$1.14 \cdot 10^{-1}$	$2.22 \cdot 10^{-1}$
Ackley 300	$1.30 \cdot 10^{-1}$	$1.28 \cdot 10^{-1}$	$8.79 \cdot 10^{-3}$	$1.09 \cdot 10^{-1}$	$1.51 \cdot 10^{-1}$
Ackley 500	$1.21 \cdot 10^{-1}$	$1.24 \cdot 10^{-1}$	$8.17 \cdot 10^{-3}$	$1.01 \cdot 10^{-1}$	$1.32 \cdot 10^{-1}$
Ackley 1000	$1.22 \cdot 10^{-1}$	$1.22 \cdot 10^{-1}$	$4.61 \cdot 10^{-3}$	$1.12 \cdot 10^{-1}$	$1.30 \cdot 10^{-1}$
Ackley 2000	$1.03 \cdot 10^{-1}$	$1.03 \cdot 10^{-1}$	$8.04 \cdot 10^{-3}$	$8.00 \cdot 10^{-2}$	$1.19 \cdot 10^{-1}$

Table 1 (cont.)

Griewank 100	$4.66 \cdot 10^{-1}$	$4.60 \cdot 10^{-1}$	$8.35 \cdot 10^{-2}$	$2.99 \cdot 10^{-1}$	$6.12 \cdot 10^{-1}$
Griewank 300	$4.54 \cdot 10^{-1}$	$4.55 \cdot 10^{-1}$	$8.53 \cdot 10^{-2}$	$3.21 \cdot 10^{-1}$	$6.55 \cdot 10^{-1}$
Griewank 500	$4.81 \cdot 10^{-1}$	$4.76 \cdot 10^{-1}$	$6.95 \cdot 10^{-2}$	$3.26 \cdot 10^{-1}$	$6.74 \cdot 10^{-1}$
Griewank 1000	$5.84 \cdot 10^{-1}$	$5.62 \cdot 10^{-1}$	$5.97 \cdot 10^{-2}$	$4.89 \cdot 10^{-1}$	$7.05 \cdot 10^{-1}$
Griewank 2000	$5.22 \cdot 10^{-1}$	$5.47 \cdot 10^{-1}$	$1.09 \cdot 10^{-1}$	$1.81 \cdot 10^{-1}$	$7.22 \cdot 10^{-1}$
Rastrigin 100	1.61	1.60	$9.91 \cdot 10^{-1}$	$4.57 \cdot 10^{-1}$	4.87
Rastrigin 300	2.91	2.69	1.16	1.57	6.44
Rastrigin 500	6.07	6.12	1.54	3.20	9.53
Rastrigin 1000	$4.50 \cdot 10^{1}$	$4.61 \cdot 10^{1}$	9.65	$2.63 \cdot 10^{1}$	$6.35 \cdot 10^{1}$
Rastrigin 2000	$5.19 \cdot 10^{2}$	$5.28 \cdot 10^{2}$	$5.47 \cdot 10^{1}$	$4.14 \cdot 10^{2}$	$6.16 \cdot 10^{2}$
Schwefel 100	$5.05 \cdot 10^{3}$	$5.21 \cdot 10^{3}$	$6.12 \cdot 10^{2}$	$3.79 \cdot 10^{3}$	$6.04 \cdot 10^{3}$
Schwefel 300	$1.59 \cdot 10^{4}$	$1.60 \cdot 10^{4}$	$7.74 \cdot 10^{2}$	$1.42 \cdot 10^{4}$	$1.73 \cdot 10^{4}$
Schwefel 500	$2.76 \cdot 10^{4}$	$2.78 \cdot 10^{4}$	$1.34 \cdot 10^{3}$	$2.43 \cdot 10^{4}$	$3.06 \cdot 10^{4}$
Schwefel 1000	$5.67 \cdot 10^{4}$	$5.65 \cdot 10^{4}$	$1.95 \cdot 10^{3}$	$5.25 \cdot 10^{4}$	$6.29 \cdot 10^{4}$
Schwefel 2000	$1.15 \cdot 10^{5}$	$1.15 \cdot 10^{5}$	$2.54 \cdot 10^{3}$	$1.09 \cdot 10^{5}$	$1.20 \cdot 10^{5}$
Sphere 100	$2.91 \cdot 10^{-3}$	$2.76 \cdot 10^{-3}$	$7.18 \cdot 10^{-4}$	$1.85 \cdot 10^{-3}$	$5.24 \cdot 10^{-3}$
Sphere 300	$6.30 \cdot 10^{-3}$	$6.33 \cdot 10^{-3}$	$6.93 \cdot 10^{-4}$	$4.73 \cdot 10^{-3}$	$7.56 \cdot 10^{-3}$
Sphere 500	$1.04 \cdot 10^{-2}$	$1.02 \cdot 10^{-2}$	$1.25 \cdot 10^{-3}$	$8.17 \cdot 10^{-3}$	$1.30 \cdot 10^{-2}$
Sphere 1000	$2.29 \cdot 10^{-2}$	$2.29 \cdot 10^{-2}$	$1.43 \cdot 10^{-3}$	$2.02 \cdot 10^{-2}$	$2.55 \cdot 10^{-2}$
Sphere 2000	$3.38 \cdot 10^{-2}$	$3.56 \cdot 10^{-2}$	$7.70 \cdot 10^{-3}$	$1.82 \cdot 10^{-2}$	$4.72 \cdot 10^{-2}$

Table 2

Results of HEAMS with operator and without operator for CEC problems (1–8)

	HEMAS without redistribute operator				
	Mean	Median	SD	Minimum	Maximum
CEC_1 10	$-450 \cdot 10^{2}$	$-450 \cdot 10^{2}$	$1.46 \cdot 10^{-1}$	$-450 \cdot 10^{2}$	$-449 \cdot 10^{2}$
CEC_1 30	$-449 \cdot 10^{2}$	$-449 \cdot 10^{2}$	$3.34 \cdot 10^{-1}$	$-450 \cdot 10^{2}$	$-448 \cdot 10^{2}$
CEC_1 50	$-449 \cdot 10^{2}$	$-449 \cdot 10^{2}$	$4.50 \cdot 10^{-1}$	$-450 \cdot 10^{2}$	$-448 \cdot 10^{2}$
CEC_2 10	$-394 \cdot 10^{2}$	$-406 \cdot 10^{2}$	$4.00 \cdot 10^{1}$	$-446 \cdot 10^{2}$	$-245 \cdot 10^{2}$
CEC_2 30	$4.78 \cdot 10^{3}$	$4.65 \cdot 10^{3}$	$1.84 \cdot 10^{3}$	$1.06 \cdot 10^{3}$	$9.48 \cdot 10^{3}$
CEC_2 50	$2.09 \cdot 10^{4}$	$2.05 \cdot 10^{4}$	$4.62 \cdot 10^{3}$	$1.35 \cdot 10^{4}$	$3.05 \cdot 10^{4}$
CEC_3 10	$1.32 \cdot 10^{6}$	$1.08 \cdot 10^{6}$	$8.63 \cdot 10^{5}$	$1.99 \cdot 10^{5}$	$3.57 \cdot 10^{6}$
CEC_3 30	$1.62 \cdot 10^{7}$	$1.51 \cdot 10^{7}$	$6.63 \cdot 10^{6}$	$5.82 \cdot 10^{6}$	$3.39 \cdot 10^{7}$
CEC_3 50	$3.54 \cdot 10^{7}$	$3.72 \cdot 10^{7}$	$1.06 \cdot 10^{7}$	$1.78 \cdot 10^{7}$	$5.69 \cdot 10^{7}$
CEC_4 10	$-347 \cdot 10^{2}$	$-356 \cdot 10^{2}$	$7.04 \cdot 10^{1}$	$-438 \cdot 10^{2}$	$-156 \cdot 10^{2}$
CEC_4 30	$1.42 \cdot 10^{4}$	$1.40 \cdot 10^{4}$	$4.96 \cdot 10^{3}$	$5.41 \cdot 10^{3}$	$2.54 \cdot 10^{4}$
CEC_4 50	$5.68 \cdot 10^{4}$	$5.58 \cdot 10^{4}$	$1.36 \cdot 10^{4}$	$2.53 \cdot 10^{4}$	$8.15 \cdot 10^{4}$
CEC_5 10	$-252 \cdot 10^{2}$	$-271 \cdot 10^{2}$	$5.90 \cdot 10^{1}$	$-306 \cdot 10^{2}$	$-496 \cdot 10^{1}$
CEC_5 30	$7.41 \cdot 10^{3}$	$7.94 \cdot 10^{3}$	$1.87 \cdot 10^{3}$	$3.16 \cdot 10^{3}$	$1.06 \cdot 10^{4}$
CEC_5 50	$1.89 \cdot 10^{4}$	$1.89 \cdot 10^{4}$	$2.73 \cdot 10^{3}$	$1.29 \cdot 10^{4}$	$2.39 \cdot 10^{4}$
CEC_6 10	$8.92 \cdot 10^{2}$	$5.38 \cdot 10^{2}$	$1.16 \cdot 10^{3}$	$4.12 \cdot 10^{2}$	$6.50 \cdot 10^{3}$
CEC_6 30	$1.50 \cdot 10^{3}$	$9.15 \cdot 10^{2}$	$2.16 \cdot 10^{3}$	$6.50 \cdot 10^{2}$	$1.26 \cdot 10^{4}$
CEC_6 50	$1.19 \cdot 10^{3}$	$1.15 \cdot 10^{3}$	$2.96 \cdot 10^{2}$	$8.02 \cdot 10^{2}$	$2.44 \cdot 10^{3}$

Table 2 (cont.)

	HEMAS without redistribute operator				
	Mean	Median	SD	Minimum	Maximum
CEC_7 10	$3.20 \cdot 10^2$	$3.12 \cdot 10^2$	$5.89 \cdot 10^1$	$1.80 \cdot 10^2$	$4.08 \cdot 10^2$
CEC_7 30	$1.92 \cdot 10^3$	$1.92 \cdot 10^3$	$1.02 \cdot 10^2$	$1.70 \cdot 10^3$	$2.11 \cdot 10^3$
CEC_7 50	$3.06 \cdot 10^3$	$3.08 \cdot 10^3$	$7.82 \cdot 10^1$	$2.80 \cdot 10^3$	$3.14 \cdot 10^3$
CEC_8 10	$-119 \cdot 10^2$	$-119 \cdot 10^2$	$9.70 \cdot 10^{-2}$	$-120 \cdot 10^2$	$-119 \cdot 10^2$
CEC_8 30	$-119 \cdot 10^2$	$-119 \cdot 10^2$	$5.46 \cdot 10^{-2}$	$-119 \cdot 10^2$	$-119 \cdot 10^2$
CEC_8 50	$-119 \cdot 10^2$	$-119 \cdot 10^2$	$3.83 \cdot 10^{-2}$	$-119 \cdot 10^2$	$-119 \cdot 10^2$
	HEMAS with Proportional Operator				
	Mean	Median	SD	Minimum	Maximum
CEC_1 10	$-450 \cdot 10^2$	$-450 \cdot 10^2$	$1.30 \cdot 10^{-1}$	$-450 \cdot 10^2$	$-449 \cdot 10^2$
CEC_1 30	$-450 \cdot 10^2$	$-450 \cdot 10^2$	$5.52 \cdot 10^{-1}$	$-450 \cdot 10^2$	$-447 \cdot 10^2$
CEC_1 50	$-449 \cdot 10^2$	$-450 \cdot 10^2$	4.44	$-450 \cdot 10^2$	$-425 \cdot 10^2$
CEC_2 10	$-406 \cdot 10^2$	$-409 \cdot 10^2$	$2.60 \cdot 10^1$	$-444 \cdot 10^2$	$-335 \cdot 10^2$
CEC_2 30	$4.13 \cdot 10^3$	$4.11 \cdot 10^3$	$1.35 \cdot 10^3$	$1.91 \cdot 10^3$	$7.60 \cdot 10^3$
CEC_2 50	$1.87 \cdot 10^4$	$1.83 \cdot 10^4$	$4.57 \cdot 10^3$	$9.42 \cdot 10^3$	$3.19 \cdot 10^4$
CEC_3 10	$1.28 \cdot 10^6$	$1.09 \cdot 10^6$	$8.52 \cdot 10^5$	$2.21 \cdot 10^5$	$3.93 \cdot 10^6$
CEC_3 30	$1.48 \cdot 10^7$	$1.46 \cdot 10^7$	$4.07 \cdot 10^6$	$6.73 \cdot 10^6$	$2.39 \cdot 10^7$
CEC_3 50	$3.73 \cdot 10^7$	$3.72 \cdot 10^7$	$1.23 \cdot 10^7$	$1.69 \cdot 10^7$	$6.51 \cdot 10^7$
CEC_4 10	$-360 \cdot 10^2$	$-387 \cdot 10^2$	$7.24 \cdot 10^1$	$-436 \cdot 10^2$	$-191 \cdot 10^2$
CEC_4 30	$1.90 \cdot 10^4$	$1.84 \cdot 10^4$	$6.75 \cdot 10^3$	$7.69 \cdot 10^3$	$3.14 \cdot 10^4$
CEC_4 50	$5.91 \cdot 10^4$	$5.84 \cdot 10^4$	$1.09 \cdot 10^4$	$3.90 \cdot 10^4$	$8.08 \cdot 10^4$
CEC_5 10	$-237 \cdot 10^2$	$-265 \cdot 10^2$	$9.07 \cdot 10^1$	$-306 \cdot 10^2$	$1.50 \cdot 10^2$
CEC_5 30	$7.40 \cdot 10^3$	$7.81 \cdot 10^3$	$1.86 \cdot 10^3$	$3.15 \cdot 10^3$	$1.06 \cdot 10^4$
CEC_5 50	$1.89 \cdot 10^4$	$1.89 \cdot 10^4$	$2.78 \cdot 10^3$	$1.29 \cdot 10^4$	$2.39 \cdot 10^4$
CEC_6 10	$1.21 \cdot 10^3$	$5.57 \cdot 10^2$	$1.88 \cdot 10^3$	$4.30 \cdot 10^2$	$8.31 \cdot 10^3$
CEC_6 30	$2.25 \cdot 10^3$	$8.39 \cdot 10^2$	$4.16 \cdot 10^3$	$5.89 \cdot 10^2$	$1.62 \cdot 10^4$
CEC_6 50	$1.30 \cdot 10^3$	$1.05 \cdot 10^3$	$8.38 \cdot 10^2$	$7.35 \cdot 10^2$	$4.36 \cdot 10^3$
CEC_7 10	$3.14 \cdot 10^2$	$3.01 \cdot 10^2$	$6.55 \cdot 10^1$	$1.80 \cdot 10^2$	$4.19 \cdot 10^2$
CEC_7 30	$1.90 \cdot 10^3$	$1.92 \cdot 10^3$	$9.02 \cdot 10^1$	$1.74 \cdot 10^3$	$2.05 \cdot 10^3$
CEC_7 50	$3.05 \cdot 10^3$	$3.08 \cdot 10^3$	$7.37 \cdot 10^1$	$2.85 \cdot 10^3$	$3.16 \cdot 10^3$
CEC_8 10	$-119 \cdot 10^2$	$-119 \cdot 10^2$	$1.12 \cdot 10^{-1}$	$-120 \cdot 10^2$	$-119 \cdot 10^2$
CEC_8 30	$-119 \cdot 10^2$	$-119 \cdot 10^2$	$6.05 \cdot 10^{-2}$	$-119 \cdot 10^2$	$-119 \cdot 10^2$
CEC_8 50	$-119 \cdot 10^2$	$-119 \cdot 10^2$	$4.15 \cdot 10^{-2}$	$-119 \cdot 10^2$	$-119 \cdot 10^2$

Table 3

Results of HEAMS with operator and without operator for CEC problems (9–16)

	HEMAS without redistribute operator				
	Mean	Median	SD	Minimum	Maximum
CEC_9 10	$-330 \cdot 10^2$	$-330 \cdot 10^2$	$4.48 \cdot 10^{-1}$	$-330 \cdot 10^2$	$-328 \cdot 10^2$
CEC_9 30	$-329 \cdot 10^2$	$-329 \cdot 10^2$	$8.59 \cdot 10^{-1}$	$-330 \cdot 10^2$	$-326 \cdot 10^2$
CEC_9 50	$-328 \cdot 10^2$	$-328 \cdot 10^2$	1.25	$-330 \cdot 10^2$	$-324 \cdot 10^2$
CEC_10 10	$-311 \cdot 10^2$	$-313 \cdot 10^2$	8.32	$-327 \cdot 10^2$	$-281 \cdot 10^2$
CEC_10 30	$-241 \cdot 10^2$	$-243 \cdot 10^2$	$1.46 \cdot 10^1$	$-263 \cdot 10^2$	$-209 \cdot 10^2$
CEC_10 50	$-149 \cdot 10^2$	$-142 \cdot 10^2$	$4.52 \cdot 10^1$	$-249 \cdot 10^2$	$-498 \cdot 10^1$

Table 3 (cont.)

CEC_11 10	$9.57 \cdot 10^{1}$	$9.57 \cdot 10^{1}$	1.40	$9.32 \cdot 10^{1}$	$9.88 \cdot 10^{1}$
CEC_11 30	$1.18 \cdot 10^{2}$	$1.17 \cdot 10^{2}$	2.49	$1.13 \cdot 10^{2}$	$1.23 \cdot 10^{2}$
CEC_11 50	$1.43 \cdot 10^{2}$	$1.43 \cdot 10^{2}$	4.24	$1.34 \cdot 10^{2}$	$1.49 \cdot 10^{2}$
CEC_12 10	$1.01 \cdot 10^{3}$	$4.54 \cdot 10^{2}$	$1.42 \cdot 10^{3}$	$-420 \cdot 10^{2}$	$5.65 \cdot 10^{3}$
CEC_12 30	$4.59 \cdot 10^{4}$	$3.92 \cdot 10^{4}$	$2.50 \cdot 10^{4}$	$7.31 \cdot 10^{3}$	$1.00 \cdot 10^{5}$
CEC_12 50	$1.98 \cdot 10^{5}$	$1.74 \cdot 10^{5}$	$1.27 \cdot 10^{5}$	$5.04 \cdot 10^{4}$	$5.67 \cdot 10^{5}$
CEC_13 10	$4.62 \cdot 10^{3}$	$4.62 \cdot 10^{3}$	$6.31 \cdot 10^{-1}$	$4.62 \cdot 10^{3}$	$4.62 \cdot 10^{3}$
CEC_13 30	$1.15 \cdot 10^{4}$	$1.15 \cdot 10^{4}$	$1.06 \cdot 10^{2}$	$1.12 \cdot 10^{4}$	$1.16 \cdot 10^{4}$
CEC_13 50	$3.01 \cdot 10^{4}$	$3.01 \cdot 10^{4}$	$2.40 \cdot 10^{2}$	$2.92 \cdot 10^{4}$	$3.03 \cdot 10^{4}$
CEC_14 10	$-296 \cdot 10^{2}$	$-296 \cdot 10^{2}$	$2.63 \cdot 10^{-1}$	$-297 \cdot 10^{2}$	$-296 \cdot 10^{2}$
CEC_14 30	$-287 \cdot 10^{2}$	$-287 \cdot 10^{2}$	$3.44 \cdot 10^{-1}$	$-288 \cdot 10^{2}$	$-286 \cdot 10^{2}$
CEC_14 50	$-278 \cdot 10^{2}$	$-278 \cdot 10^{2}$	$5.99 \cdot 10^{-1}$	$-279 \cdot 10^{2}$	$-277 \cdot 10^{2}$
CEC_15 10	$3.70 \cdot 10^{2}$	$5.24 \cdot 10^{2}$	$2.08 \cdot 10^{2}$	$1.20 \cdot 10^{2}$	$7.03 \cdot 10^{2}$
CEC_15 30	$1.29 \cdot 10^{2}$	$1.20 \cdot 10^{2}$	$2.57 \cdot 10^{1}$	$1.20 \cdot 10^{2}$	$2.63 \cdot 10^{2}$
CEC_15 50	$1.31 \cdot 10^{2}$	$1.22 \cdot 10^{2}$	$2.47 \cdot 10^{1}$	$1.20 \cdot 10^{2}$	$2.39 \cdot 10^{2}$
CEC_16 10	$2.61 \cdot 10^{2}$	$2.49 \cdot 10^{2}$	$2.78 \cdot 10^{1}$	$2.20 \cdot 10^{2}$	$3.17 \cdot 10^{2}$
CEC_16 30	$1.86 \cdot 10^{2}$	$1.21 \cdot 10^{2}$	$1.55 \cdot 10^{2}$	$1.20 \cdot 10^{2}$	$6.55 \cdot 10^{2}$
CEC_16 50	$1.22 \cdot 10^{2}$	$1.20 \cdot 10^{2}$	4.39	$1.20 \cdot 10^{2}$	$1.38 \cdot 10^{2}$
	HEMAS with Proportional Operator				
	Mean	Median	SD	Minimum	Maximum
CEC_9 10	$-330 \cdot 10^{2}$	$-330 \cdot 10^{2}$	$9.53 \cdot 10^{-2}$	$-330 \cdot 10^{2}$	$-329 \cdot 10^{2}$
CEC_9 30	$-329 \cdot 10^{2}$	$-330 \cdot 10^{2}$	2.42	$-330 \cdot 10^{2}$	$-321 \cdot 10^{2}$
CEC_9 50	$-323 \cdot 10^{2}$	$-329 \cdot 10^{2}$	$1.33 \cdot 10^{1}$	$-330 \cdot 10^{2}$	$-275 \cdot 10^{2}$
CEC_10 10	$-309 \cdot 10^{2}$	$-311 \cdot 10^{2}$	8.74	$-324 \cdot 10^{2}$	$-280 \cdot 10^{2}$
CEC_10 30	$-245 \cdot 10^{2}$	$-246 \cdot 10^{2}$	$1.37 \cdot 10^{1}$	$-272 \cdot 10^{2}$	$-21 \ \cdot 10^{2}$
CEC_10 50	$-124 \cdot 10^{2}$	$-126 \cdot 10^{2}$	$6.43 \cdot 10^{1}$	$-242 \cdot 10^{2}$	$3.85 \cdot 10^{1}$
CEC_11 10	$9.56 \cdot 10^{1}$	$9.55 \cdot 10^{1}$	1.55	$9.31 \cdot 10^{1}$	$9.97 \cdot 10^{1}$
CEC_11 30	$1.19 \cdot 10^{2}$	$1.19 \cdot 10^{2}$	3.17	$1.13 \cdot 10^{2}$	$1.24 \cdot 10^{2}$
CEC_11 50	$1.45 \cdot 10^{2}$	$1.44 \cdot 10^{2}$	5.51	$1.37 \cdot 10^{2}$	$1.55 \cdot 10^{2}$
CEC_12 10	$1.74 \cdot 10^{3}$	$1.17 \cdot 10^{3}$	$2.88 \cdot 10^{3}$	$-395 \cdot 10^{2}$	$1.33 \cdot 10^{4}$
CEC_12 30	$4.63 \cdot 10^{4}$	$4.24 \cdot 10^{4}$	$2.02 \cdot 10^{4}$	$1.70 \cdot 10^{4}$	$8.58 \cdot 10^{4}$
CEC_12 50	$1.78 \cdot 10^{5}$	$1.52 \cdot 10^{5}$	$9.01 \cdot 10^{4}$	$6.27 \cdot 10^{4}$	$4.44 \cdot 10^{5}$
CEC_13 10	$4.62 \cdot 10^{3}$	$4.62 \cdot 10^{3}$	$3.50 \cdot 10^{-1}$	$4.62 \cdot 10^{3}$	$4.62 \cdot 10^{3}$
CEC_13 30	$1.15 \cdot 10^{4}$	$1.15 \cdot 10^{4}$	$1.15 \cdot 10^{2}$	$1.12 \cdot 10^{4}$	$1.16 \cdot 10^{4}$
CEC_13 50	$3.01 \cdot 10^{4}$	$3.02 \cdot 10^{4}$	$2.45 \cdot 10^{2}$	$2.93 \cdot 10^{4}$	$3.03 \cdot 10^{4}$
CEC_14 10	$-296 \cdot 10^{2}$	$-296 \cdot 10^{2}$	$3.84 \cdot 10^{-1}$	$-297 \cdot 10^{2}$	$-295 \cdot 10^{2}$
CEC_14 30	$-287 \cdot 10^{2}$	$-287 \cdot 10^{2}$	$3.36 \cdot 10^{-1}$	$-288 \cdot 10^{2}$	$-286 \cdot 10^{2}$
CEC_14 50	$-278 \cdot 10^{2}$	$-278 \cdot 10^{2}$	$4.82 \cdot 10^{-1}$	$-279 \cdot 10^{2}$	$-277 \cdot 10^{2}$
CEC_15 10	$3.30 \cdot 10^{2}$	$2.76 \cdot 10^{2}$	$1.93 \cdot 10^{2}$	$1.20 \cdot 10^{2}$	$5.54 \cdot 10^{2}$
CEC_15 30	$4.30 \cdot 10^{2}$	$4.39 \cdot 10^{2}$	$1.09 \cdot 10^{2}$	$1.73 \cdot 10^{2}$	$6.86 \cdot 10^{2}$
CEC_15 50	$4.52 \cdot 10^{2}$	$4.49 \cdot 10^{2}$	$9.50 \cdot 10^{1}$	$2.17 \cdot 10^{2}$	$5.72 \cdot 10^{2}$
CEC_16 10	$2.60 \cdot 10^{2}$	$2.53 \cdot 10^{2}$	$2.20 \cdot 10^{1}$	$2.19 \cdot 10^{2}$	$3.02 \cdot 10^{2}$
CEC_16 30	$3.51 \cdot 10^{2}$	$3.27 \cdot 10^{2}$	$1.11 \cdot 10^{2}$	$1.46 \cdot 10^{2}$	$5.49 \cdot 10^{2}$
CEC_16 50	$3.68 \cdot 10^{2}$	$3.66 \cdot 10^{2}$	$6.05 \cdot 10^{1}$	$1.43 \cdot 10^{2}$	$4.90 \cdot 10^{2}$

Table 4

Results of HEAMS with operator and without operator for CEC problems (17–24)

	HEMAS without redistribute operator				
	Mean	Median	SD	Minimum	Maximum
CEC_17 10	$2.72 \cdot 10^2$	$2.71 \cdot 10^2$	$1.80 \cdot 10^1$	$2.38 \cdot 10^2$	$3.06 \cdot 10^2$
CEC_17 30	$1.81 \cdot 10^2$	$1.22 \cdot 10^2$	$1.03 \cdot 10^2$	$1.21 \cdot 10^2$	$5.54 \cdot 10^2$
CEC_17 50	$1.34 \cdot 10^2$	$1.22 \cdot 10^2$	$5.16 \cdot 10^1$	$1.21 \cdot 10^2$	$3.96 \cdot 10^2$
CEC_18 10	$9.04 \cdot 10^2$	$9.81 \cdot 10^2$	$1.96 \cdot 10^2$	$4.37 \cdot 10^2$	$1.09 \cdot 10^3$
CEC_18 30	$7.25 \cdot 10^2$	$7.23 \cdot 10^2$	$7.55 \cdot 10^1$	$4.29 \cdot 10^2$	$8.80 \cdot 10^2$
CEC_18 50	$7.03 \cdot 10^2$	$7.16 \cdot 10^2$	$1.06 \cdot 10^2$	$1.88 \cdot 10^2$	$8.09 \cdot 10^2$
CEC_19 10	$9.49 \cdot 10^2$	$9.91 \cdot 10^2$	$1.14 \cdot 10^2$	$5.57 \cdot 10^2$	$1.08 \cdot 10^3$
CEC_19 30	$7.03 \cdot 10^2$	$7.13 \cdot 10^2$	$1.06 \cdot 10^2$	$2.87 \cdot 10^2$	$8.92 \cdot 10^2$
CEC_19 50	$7.60 \cdot 10^2$	$7.50 \cdot 10^2$	$5.39 \cdot 10^1$	$6.92 \cdot 10^2$	$9.52 \cdot 10^2$
CEC_20 10	$8.81 \cdot 10^2$	$9.73 \cdot 10^2$	$2.16 \cdot 10^2$	$3.20 \cdot 10^2$	$1.06 \cdot 10^3$
CEC_20 30	$7.48 \cdot 10^2$	$7.39 \cdot 10^2$	$4.52 \cdot 10^1$	$6.78 \cdot 10^2$	$9.04 \cdot 10^2$
CEC_20 50	$7.05 \cdot 10^2$	$7.19 \cdot 10^2$	$7.38 \cdot 10^1$	$4.45 \cdot 10^2$	$8.46 \cdot 10^2$
CEC_21 10	$1.28 \cdot 10^3$	$1.27 \cdot 10^3$	$2.88 \cdot 10^2$	$6.64 \cdot 10^2$	$1.63 \cdot 10^3$
CEC_21 30	$6.41 \cdot 10^2$	$5.23 \cdot 10^2$	$2.46 \cdot 10^2$	$4.87 \cdot 10^2$	$1.28 \cdot 10^3$
CEC_21 50	$8.32 \cdot 10^2$	$7.89 \cdot 10^2$	$2.85 \cdot 10^2$	$5.14 \cdot 10^2$	$1.35 \cdot 10^3$
CEC_22 10	$1.19 \cdot 10^3$	$1.16 \cdot 10^3$	$6.64 \cdot 10^1$	$1.13 \cdot 10^3$	$1.32 \cdot 10^3$
CEC_22 30	$1.18 \cdot 10^3$	$1.19 \cdot 10^3$	$9.72 \cdot 10^1$	$9.86 \cdot 10^2$	$1.42 \cdot 10^3$
CEC_22 50	$1.16 \cdot 10^3$	$1.14 \cdot 10^3$	$1.17 \cdot 10^2$	$9.93 \cdot 10^2$	$1.42 \cdot 10^3$
CEC_23 10	$1.13 \cdot 10^3$	$1.17 \cdot 10^3$	$1.91 \cdot 10^2$	$5.28 \cdot 10^2$	$1.41 \cdot 10^3$
CEC_23 30	$1.22 \cdot 10^3$	$1.21 \cdot 10^3$	$1.08 \cdot 10^2$	$9.77 \cdot 10^2$	$1.46 \cdot 10^3$
CEC_23 50	$1.24 \cdot 10^3$	$1.25 \cdot 10^3$	$1.40 \cdot 10^2$	$9.41 \cdot 10^2$	$1.50 \cdot 10^3$
CEC_24 10	$5.01 \cdot 10^2$	$4.60 \cdot 10^2$	$1.04 \cdot 10^2$	$4.60 \cdot 10^2$	$7.72 \cdot 10^2$
CEC_24 30	$5.06 \cdot 10^2$	$4.79 \cdot 10^2$	$1.03 \cdot 10^2$	$4.17 \cdot 10^2$	$8.23 \cdot 10^2$
CEC_24 50	$1.17 \cdot 10^3$	$1.25 \cdot 10^3$	$2.31 \cdot 10^2$	$6.54 \cdot 10^2$	$1.42 \cdot 10^3$
	HEMAS with Proportional Operator				
	Mean	Median	SD	Minimum	Maximum
CEC_17 10	$2.70 \cdot 10^2$	$2.69 \cdot 10^2$	$1.94 \cdot 10^1$	$2.38 \cdot 10^2$	$3.18 \cdot 10^2$
CEC_17 30	$4.62 \cdot 10^2$	$4.33 \cdot 10^2$	$1.04 \cdot 10^2$	$1.80 \cdot 10^2$	$7.07 \cdot 10^2$
CEC_17 50	$5.1 \cdot 10^2$	$5.14 \cdot 10^2$	$6.03 \cdot 10^1$	$3.1 \cdot 10^2$	$6.27 \cdot 10^2$
CEC_18 10	$8.45 \cdot 10^2$	$9.63 \cdot 10^2$	$2.47 \cdot 10^2$	$3.14 \cdot 10^2$	$1.08 \cdot 10^3$
CEC_18 30	$7.42 \cdot 10^2$	$7.78 \cdot 10^2$	$1.23 \cdot 10^2$	$2.28 \cdot 10^2$	$8.60 \cdot 10^2$
CEC_18 50	$9.26 \cdot 10^2$	$9.05 \cdot 10^2$	$8.12 \cdot 10^1$	$7.45 \cdot 10^2$	$1.13 \cdot 10^3$
CEC_19 10	$9.19 \cdot 10^2$	$9.73 \cdot 10^2$	$1.73 \cdot 10^2$	$3.67 \cdot 10^2$	$1.08 \cdot 10^3$
CEC_19 30	$7.54 \cdot 10^2$	$7.59 \cdot 10^2$	$7.98 \cdot 10^1$	$3.97 \cdot 10^2$	$8.70 \cdot 10^2$
CEC_19 50	$8.48 \cdot 10^2$	$8.67 \cdot 10^2$	$1.16 \cdot 10^2$	$4.18 \cdot 10^2$	$1.03 \cdot 10^3$
CEC_20 10	$9.26 \cdot 10^2$	$9.83 \cdot 10^2$	$1.44 \cdot 10^2$	$4.97 \cdot 10^2$	$1.07 \cdot 10^3$
CEC_20 30	$7.50 \cdot 10^2$	$7.52 \cdot 10^2$	$9.01 \cdot 10^1$	$3.38 \cdot 10^2$	$9.17 \cdot 10^2$
CEC_20 50	$8.83 \cdot 10^2$	$8.92 \cdot 10^2$	$8.23 \cdot 10^1$	$6.26 \cdot 10^2$	$1.08 \cdot 10^3$
CEC_21 10	$1.22 \cdot 10^3$	$1.24 \cdot 10^3$	$2.98 \cdot 10^2$	$6.61 \cdot 10^2$	$1.60 \cdot 10^3$
CEC_21 30	$9.15 \cdot 10^2$	$8.75 \cdot 10^2$	$1.22 \cdot 10^2$	$6.31 \cdot 10^2$	$1.22 \cdot 10^3$
CEC_21 50	$1.07 \cdot 10^3$	$1.07 \cdot 10^3$	$1.55 \cdot 10^2$	$8.83 \cdot 10^2$	$1.30 \cdot 10^3$
CEC_22 10	$1.18 \cdot 10^3$	$1.16 \cdot 10^3$	$5.19 \cdot 10^1$	$1.12 \cdot 10^3$	$1.32 \cdot 10^3$

Table 4 (cont.)

CEC_22 30	$1.35 \cdot 10^3$	$1.35 \cdot 10^3$	$6.98 \cdot 10^1$	$1.19 \cdot 10^3$	$1.54 \cdot 10^3$
CEC_22 50	$1.34 \cdot 10^3$	$1.35 \cdot 10^3$	$6.15 \cdot 10^1$	$1.20 \cdot 10^3$	$1.45 \cdot 10^3$
CEC_23 10	$1.10 \cdot 10^3$	$1.1 \cdot 10^3$	$1.34 \cdot 10^2$	$7.76 \cdot 10^2$	$1.36 \cdot 10^3$
CEC_23 30	$1.24 \cdot 10^3$	$1.25 \cdot 10^3$	$1.12 \cdot 10^2$	$1.03 \cdot 10^3$	$1.40 \cdot 10^3$
CEC_23 50	$1.27 \cdot 10^3$	$1.29 \cdot 10^3$	$1.22 \cdot 10^2$	$1.01 \cdot 10^3$	$1.49 \cdot 10^3$
CEC_24 10	$5.47 \cdot 10^2$	$4.60 \cdot 10^2$	$1.93 \cdot 10^2$	$4.60 \cdot 10^2$	$1.16 \cdot 10^3$
CEC_24 30	$5.94 \cdot 10^2$	$4.91 \cdot 10^2$	$2.36 \cdot 10^2$	$4.71 \cdot 10^2$	$1.18 \cdot 10^3$
CEC_24 50	$1.17 \cdot 10^3$	$1.24 \cdot 10^3$	$2.46 \cdot 10^2$	$6.22 \cdot 10^2$	$1.48 \cdot 10^3$

4.6. Discussion of the results

For most problems, you can see a large or significant positive effect of the redistribution operator. For some problems you cannot see the difference and only for individual cases the operator does not have a positive effect on the final result. Since the operation of the operator is not long, it does not use the evaluation function, so in our case it does not affect the duration of the entire algorithm. Therefore, one should assume the correct way is to use the algorithm along with this operator. The analysis of various operators did not reveal new areas of interest to be investigated, so this part of the research can be considered finished.

5. Conclusion

At the beginning of the article, the tournament energy redistribution operator was introduced. The results of the experiments showed that neither the number of tournaments nor the number of agents per tournament significantly influenced the final results. So we selected 20 tournaments with 3 agents each as the best ones for further comparison.

We then compared the tournament energy redistribution operator (with the parameters just selected), the ranking energy redistribution operator, and the proportional energy redistribution operator. The results obtained from the experiments showed that the proportional energy redistribution operator is the best.

That is why it was thoroughly tested in the last part of the research. The series of tests shows that the algorithm with the proportional redistribution operator performs much better compared to the algorithm without the operator. Therefore, in future research related to adding new algorithms of its hybrid or the analysis of algorithm selection methods by agents, the version of the algorithm with this operator will be used.

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References

[1] Byrski A., Kisiel-Dorohinicki M.: Immune-Based Optimization of Predicting Neural Networks. In: V.S. Sunderam, G.D. van Albada, P.M.A. Sloot, J.J. Dongarra (eds.), *Computational Science – ICCS 2005. ICCS 2005*, *Lecture Notes in Computer Science*, vol. 3516, pp. 703–710, Berlin–Heidelberg, 2005. doi: 10.1007/11428862_96.

[2] Byrski A., Kisiel-Dorohinicki M.: Agent-Based Evolutionary and Immunological Optimization. In: Y. Shi, G.D. van Albada, J.J. Dongarra, P.M.A. Sloot (eds.), *Computational Science – ICCS 2007. ICCS 2007*, *Lecture Notes in Computer Science*, vol. 4488, pp. 928+, Berlin–Heidelberg, 2007. doi: 10.1007/978-3-540-72586-2_129.

[3] Byrski A., Schaefer R.: Formal model for agent-based asynchronous evolutionary computation. In: *2009 IEEE Congress on Evolutionary Computation*, pp. 78–85, 2009. doi: 10.1109/CEC.2009.4982933.

[4] Byrski A., Schaefer R., Smolka M., Cotta C.: Asymptotic guarantee of success for multi-agent memetic systems, *Bulletin of the Polish Academy of Sciences: Technical Sciences*, vol. 61(1), pp. 257–278, 2013. doi: 10.2478/bpasts-2013-0025.

[5] Cetnarowicz K., Kisiel-Dorohinicki M., Nawarecki E.: The Application of Evolution Process in Multi-Agent World (MAW) to the Prediction System. In: M. Tokoro (ed.), *Proceedings of the Second International Conference on Multiagent Systems (ICMAS'96)*, pp. 26–32, AAAI Press, 1996.

[6] Durillo J.J., Nebro A.J.: jMetal: A Java framework for multi-objective optimization, *Advances in Engineering Software*, vol. 42(10), pp. 760–771, 2011. doi: 10.1016/j.advengsoft.2011.05.014.

[7] Faber L., Piętak K., Byrski A., Kisiel-Dorohinicki M.: Agent-Based Simulation in AgE Framework. In: A. Byrski, Z. Oplatkova, M. Carvalho, M. Kisiel- Dorohinicki (eds.), *Advances in Intelligent Modelling and Simulation: Simulation Tools and Applications*, *Studies in Computational Intelligence*, vol. 416, pp. 55–83, Springer, Berlin, Heidelberg, 2012.

[8] Godzik M., Grochal B., Piekarz J., Sieniawski M., Byrski A., Kisiel-Dorohinicki M.: Differential Evolution in Agent-Based Computing. In: *Intelligent Information and Database Systems. ACIIDS 2019*, *Lecture Notes in Computer Science*, vol. 11432, pp. 228–241, Springer, Cham, 2019. doi: 10.1007/978-3-030-14802-7_20.

[9] Godzik M., Idzik M., Piętak K., Byrski A., Kisiel-Dorohinicki M.: Autonomous Hybridization of Agent-Based Computing. In: *Computational Collective Intelligence. 12th International Conference, ICCCI 2020, Da Nang, Vietnam, Nov. 30–Dec. 3, 2020, Proceedings*, vol. 12496, pp. 139–151, Springer International Publishing, 2020.

[10] Kisiel-Dorohinicki M., Dobrowolski G., Nawarecki E.: Agent Populations as Computational Intelligence. In: *Neural Networks and Soft Computing*, *Advances in Soft Computing*, vol. 19, pp. 608–613, Physica-Verlag HD, Heidelberg, 2003. doi: 10.1007/978-3-7908-1902-1_93.

[11] López-Ibáñez M., Dubois-Lacoste J., Cáceres L.P., Birattari M., Stützle T.: The irace package: Iterated racing for automatic algorithm configuration, *Operations Research Perspectives*, vol. 3, pp. 43–58, 2016. doi: 10.1016/j.orp.2016.09.002.

[12] Placzkiewicz L., Sendera M., Szlachta A., Paciorek M., Byrski A., Kisiel-Dorohinicki M., Godzik M.: Hybrid Swarm and Agent-Based Evolutionary Optimization. In: Y. Shi, H. Fu, Y. Tian, V.V. Krzhizhanovskaya, M.H. Lees, J.J. Dongarra, P.M.A. Sloot (eds.), *Computational Science – ICCS 2018. ICCS 2018*, *Lecture Notes in Computer Science*, vol. 10861, pp. 89–102, Springer, 2018. doi: 10.1007/978-3-319-93701-4_7.

[13] Podsiadło K., Łoś M., Siwik L., Woźniak M.: An Algorithm for Tensor Product Approximation of Three-Dimensional Material Data for Implicit Dynamics Simulations. In: Y. Shi, H. Fu, Y. Tian, V.V. Krzhizhanovskaya, M.H. Lees, J. Dongarra, P.M.A. Sloot (eds.), *Computational Science – ICCS 2018*, pp. 156–168, Springer International Publishing, Cham, 2018.

[14] Polnik W., Stobiecki J., Byrski A., Kisiel-Dorohinicki M.: Ant colony optimization–evolutionary hybrid optimization with translation of problem representation, *Computational Intelligence*, vol. 37(2), pp. 891–923, 2021. doi: 10.1111/coin.12439.

[15] Schaefer R., Byrski A., Smolka M.: The island model as a Markov dynamic system, *International Journal of Applied Mathematics and Computer Science*, vol. 22(4), pp. 971–984, 2012. doi: 10.2478/v10006-012-0072-z.

[16] Siwik L., Łoś M., Kisiel-Dorohinicki M., Byrski A.: Hybridization of Isogeometric Finite Element Method and Evolutionary Multi-Agent System as a Tool-Set for Multiobjective Optimization of Liquid Fossil Fuel Reserves Exploitation with Minimizing Groundwater Contamination, *Procedia Computer Science*, vol. 80, pp. 792–803, 2016. doi: 10.1016/j.procs.2016.05.369.

[17] Talbi E.G.: A Taxonomy of Hybrid Metaheuristics, *Journal of Heuristics*, vol. 8, pp. 541–564, 2002.

[18] Wolpert D.H., Macready W.G.: No Free Lunch Theorems for Optimization, *IEEE Transactions on Evolutionary Computation*, vol. 67(1), pp. 67–82, 1997.

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