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COMPARISON OF TWO KINDS OF FUZZY ARITHMETIC, LR AND OFN, APPLIED TO FUZZY OBSERVATION OF THE COFFERDAM WATER LEVEL

Abstract *This paper presents certain important aspects of the fuzzy logic extension, one of which is OFN. It includes basic definitions of that discipline. It also compares fuzzy logic arithmetic with the arithmetic of ordered fuzzy numbers in L-R notation. Computational experiments were based on fuzzy observation of the impounding basin. The results of the study show that there is a connection between the order of OFN number and trend of changes in the environment. The experiment was carried out using computer software developed specially for that purpose. When comparing the arithmetic of fuzzy numbers in L-R notation with the arithmetic of ordered fuzzy numbers on the grounds of the experiment, it has been concluded that with fuzzy numbers it is possible to expand the scope of solutions in comparison to fuzzy numbers in classic form. The symbol of OFN flexibility is the possibility to determine the X number that always satisfies the equation $A+X = C$, regardless of the value of arguments. Operations performed on OFN are less complicated, as they are performed in the same way regardless the sign of the input data and their results are more accurate in the majority of cases. The promising feature of ordered fuzzy numbers is their lack of rapidly growing fuzziness. Authors expect to see implication of that fact in practice in the near future.*

Keywords fuzzy logic, fuzzy number, Ordered Fuzzy Numbers

1. Introduction

The history of artificial intelligence shows that new ideas were often inspired by natural phenomena. Many tourists come back with passion to beaches at the Oceanside and many sailors sail on tide water. The phenomenon of high and low tides, although well known, has been stimulating imagination and provoking reflexion on the perfection of the Creation for many ages. The casual observer is unable to precisely specify water level decline, but he or she can easily describe it using fuzzy concepts such as "less and less", "little" and "a bit". The same applies to the increase of water level in the observed basin. The observer can describe it using such linguistic terms like "more", "lots of" or "very much". Such linguistic description of reality is characteristic to powerful and dynamically developing discipline of artificial intelligence like fuzzy logic. The author of Fuzzy logic is an American professor of the Columbia University in New York City and of Berkeley University in California – Lotfi A. Zadeh, who published the paper entitled "Fuzzy sets" in the journal "Information and Control" in 1965 [1]. He defined the term of a fuzzy set there, thanks to which imprecise data could be described using values from the interval (0,1). The number assigned to them represents their degree of membership in this set. It is worth mentioning that in his theory L. Zadeh used the article on 3-valued logic published 45 years before by a Pole – Jan Łukasiewicz [2]. That is why many scientists [24] in the world regard this Pole as the "father" of fuzzy logic. Next decades saw rapid development of fuzzy logic. As next milestones of the history of that discipline one should necessarily mention L-R representation of fuzzy numbers proposed by D. Dubois and H. Prade [3, 4], which enjoys great successes today.

Coming back to the original analogy, an observer can see a trend, i.e. general increase during rising tide or decrease during low tide, regardless of momentary fluctuations of the water surface level. This resembles a number of macro and microeconomic mechanisms where trends and time series can be observed. The most obvious example of that seems to be the bull and bear market on stock exchanges, which indicates to the general trend, while shares of individual companies may temporarily fall or rise. The aim is to capture the environmental context of changes in the economy or another limited part of reality. Changes in an object described using fuzzy logic seem to be thoroughly studied in many papers. But it is not necessarily the case as regards linking those changes with a trend. Perhaps this might be the opportunity to apply generalization of fuzzy logic which are, in the opinion of authors of that concept, W. Kosiński [8, 22, 23] and his team [25, 26], Ordered Fuzzy Numbers. As the basis for experiments, let us assume the example of the dam and the impounding basin presented in the figure (Fig. 1). Letter A indicates the water level measured during last evening measurement. Then there was a rapid surge of water in the night. While the measurement taken in the morning was marked using letter C. Measurements were imprecise to some extent due to rapid changes of weather conditions. It is also known that during the last measurement, the safety valve z2 was open and then the valve z1 activated. The management of the dam faces the problem of reporting rapid surges of water to the disaster recovery canter.

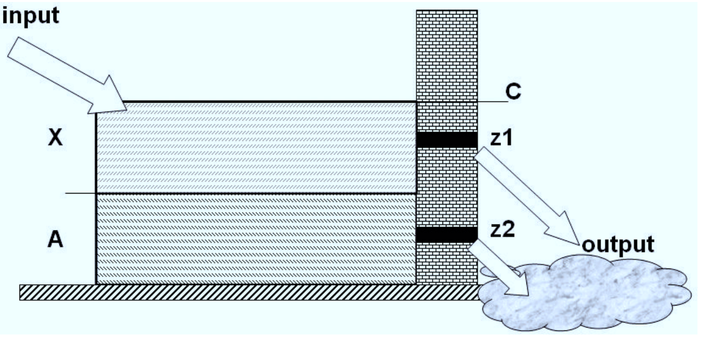

Figure 1. The diagram of water flow in the impounding basin.

2. Theoretical background description of OFN

2.1. Some definitions of OFN

Each operation on fuzzy numbers, regardless if it is addition, subtraction, division or multiplication, can increase the carrier value, i.e. the areas of non-accuracy. Several operations performed on given *L-R* numbers can result in numbers that are too broad and, as a result, they can become less useful. Solving equations using conventional operations on fuzzy numbers [10] is usually impossible either. An $A + X = C$ equation can always be solved using conventional operations on fuzzy numbers only when A is a real number. First attempts to redefine new operations on fuzzy numbers were undertaken at the beginning of the 1990-ties by Witold Kosiński and his PhD student – P. Słysz [5]. Further studies of W. Kosiński published in cooperation with P. Prokopowicz and D. Ślęzak [6, 7, 9] led to introduction of the ordered fuzzy numbers model – *OFN*.

Definition 1. *An ordered fuzzy number A was identified with an ordered pair of continuous real functions defined on the interval* $[0, 1]$*, i.e.,* $A = (f, g)$ *with* $f, g\colon [0, 1] \to R$ *as continuous functions.*

We call f and g the up and down-parts of the fuzzy number A, respectively. In order to comply with the classical denotation of fuzzy sets (numbers), the independent variable of both functions f and g is denoted by y, and their values by x. [8]

Continuity of those two parts shows that their images are limited by specific intervals. They are named respectively: UP and DOWN. The limits (real numbers) of those intervals were marked using the following symbols: $UP = (l_A, 1_A^-)$ and $DOWN = (1_A^+, p_A)$.

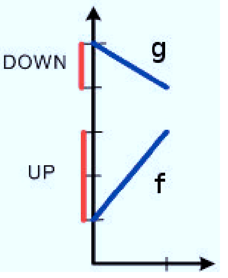

Figure 2. Ordered fuzzy number.

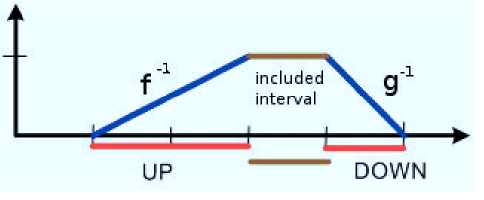

Figure 3. OFN presented in a way referring to fuzzy numbers.

If both functions that are parts of the fuzzy number are strictly monotonic, then there are their inverse functions x_{up}^{-1} and x_{down}^{-1} defined in respective intervals UP and DOWN and the following assignment is valid:

$$l_A := x_{up}(0),\ 1_A^{-1} := x_{up}(1),\ 1_A^{+1} := x_{down}(1),\ p_A := x_{down}(0). \tag{1}$$

If a constant function equal to 1 is added within the interval $[1_A^-, 1_A^+]$ we get UP and DOWN with one interval, which can be treated as a carrier. Then the membership function $\mu_A(x)$ of the fuzzy set defined on the R set is defined by the following formulas:

$$\begin{array}{ll} \mu_A(x) = 0 & \text{for } x \notin [l_A, p_A] \\ \mu_A(x) = x_{up}^{-1}(x) & \text{for } x \in UP \\ \mu_A(x) = x_{down}^{-1}(x) & \text{for } x \in DOWN. \end{array} \tag{2}$$

The fuzzy set defined in that way gets an additional property which is called order. Whereas the following interval is the carrier:

$$UP \cup [1_A^+, 1_A^-] \cup DOWN \tag{3}$$

The limit values for up and down parts are:

$$\begin{array}{l} \mu_A(l_A) = 0 \\ \mu_A(1_A^-) = 1 \\ \mu_A(1_A^+) = 1 \\ \mu_A(p_A) = 0 \end{array} \tag{4}$$

Generally, it can be assumed that ordered fuzzy numbers are of trapezoid form. Each of them can be defined using four real numbers:

$$A = (l_A, 1_A^-, 1_A^+, p_A). \tag{5}$$

The figures below (Fig. 4) show sample ordered fuzzy numbers including their characteristic points.

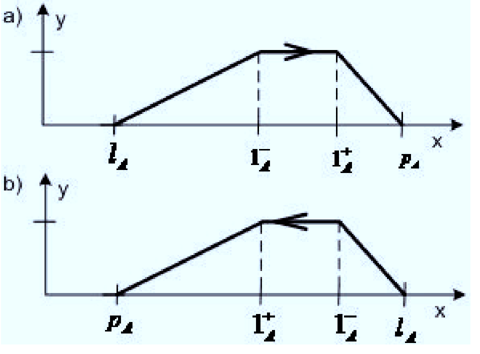

Figure 4. Fuzzy number that is ordered a) positively b) negatively.

Functions f_A, g_A correspond to parts up_A , $down_A \subseteq R^2$ respectively, so that:

$$up_A = (f_A(y), y)\colon y \in [0,1] \tag{6}$$

$$down_A = (g_A(y), y)\colon y \in [0,1] \tag{7}$$

The orientation corresponds to the order of graphs f_A and g_A. The figure below (Fig. 5) shows the graphic interpretation of two opposite fuzzy numbers and the real number χ_0.

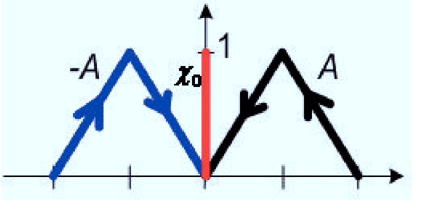

Figure 5. Opposite numbers and the real number.

Opposite numbers are reversely ordered [11].

Definition 2. *A* ***membership function of an ordered fuzzy number*** *A is the function* $\mu_A\colon R \to [0,1]$ *defined for* $x \in R$ *as follows [6, 8]:*

$$\mu(x) = \begin{cases} f^{-1}(x) \ \ \textit{if}\ x \in [f(0), f(1)] = [l_A, 1_A^-] \\ g^{-1}(x) \ \ \textit{if}\ x \in [g(1), g(0)] = [l_A^+, p1_A] \\ 1 \ \ \textit{if}\ x \in [l_A^-, 1_A^+] \end{cases}$$

The above membership function can be used in the control rules similarly to the way membership of classic fuzzy numbers is used. All quantities that can be found in the fuzzy control describe selected part of the reality. Process of determining this value is called **fuzzy observation**.

Definition 3. ***Reversal of the orientation*** *of the ordered fuzzy number A consists in the replacement of the part up (function f_A) with the part down (function g_A). That operation is described as follows [5]:*

$$B = A|^{-} \Leftrightarrow g_A = f_A \wedge f_B = g_A$$

where:

- *A is an ordered fuzzy number defined by the pair of functions,*
- *B is the result of the operation consisting in reversal of OFN orientation,*
- *The sign $|^{-}$ is a symbol of OFN orientation reversal.*

The number obtained in that way is called a reversed OFN number or a reversed orientation number.

2.2. Arithmetic operations in OFN

The operation of adding two pairs of such functions is defined as the pair-wise addition of their elements, i.e., if $(f1, g1)$ and $(f2, g2)$ are two ordered fuzzy numbers, then $(f1+f2, g1+g2)$ will be just their sum. It is interesting to notice that as long as we are dealing with an ordered fuzzy number represented by pairs of affine functions of the variable $y \in [0, 1]$, its so-called classical counterpart, i.e., a membership function of the variable x is just of trapezoidal type. For any pair of affine functions (f, g) of $y \in [0, 1]$ we form a quaternion of real numbers according to the rule $[f(0), f(1), g(1), g(0)]$ which correspond to the four numbers $[l_A, 1^-_A, 1^+_A, p_A]$ as was mentioned in previous paragraph. If $(f, g) = A$ is a base pair of affine functions and $(e, h) = B$ is another pair of affine functions, then the set of typical operation will be uniquely represented by the following formulas respectively:

- addition $A + B = (f + e, g + h) = C$,

$$C \rightarrow [f(0) + e(0), f(1) + e(1), g(1) + h(1), g(0) + h(0)] \tag{8}$$

- scalar multiplication $C = \lambda A = (\lambda f, \lambda g)$,

$$C \rightarrow [\lambda f(0), \lambda f(1), \lambda g(1), \lambda g(0)] \tag{9}$$

- subtraction $A - B = (f - e, g - h) = C$

$$C \rightarrow [f(0) - e(0), f(1) - e(1), g(1) - h(1), g(0) - h(0)] \tag{10}$$

- multiplication $A * B = (f * e, g * h) = C$

$$C \rightarrow [f(0) * e(0), f(1) * e(1), g(1) * h(1), g(0) * h(0)] \tag{11}$$

2.3. Association of OFN order with the environmental trend

In order to explain calculations presented in this sections, authors made the following assumptions concerning the context of changes taking place in the studied object (the impounding basin).

- **close context** – understood us the trend visible locally in the basin. It defines the trend of the object, i.e. if it is gradually filled or if the water level gradually falls. It is defined locally by the management of the dam,
- **further (environmental) context** – understood as the global trend for the observed area. It specifies trend of specific section of the river, intensity of precipitation as well as the set of other regional data which give better image of the environment. It is defined in the disaster recovery centre.

Two context types described above will be associated with the order of OFN numbers as follows. As the current water level state is reported to the disaster recovery centre, numbers that represent that state will always be oriented according to the environmental trend (reported to the centre). Whereas order of fuzzy numbers that represent changes of the water level in the impounding basin will be consistent with the local trend defined by the management of the dam. As a result, the order will be positive when the basin will be gradually filled, regardless the rate of that process. Whereas negative order can be observed when the outflow of water from the basin starts. Trend order changes themselves, as well as boundary conditions of the moment when they should occur, will make a separate process defined be the disaster recovery centre and the dam management respectively. Nothing stands in the way to ultimately use known segmentation methods for those processes and to determine the trend. Such issues are currently present in numerous publications and their detailed description is beyond the scope of this paper. Authors of that study assumed that the order equivalent to the trend changes is provided by a trusted third party and represents the expert's opinion concerning short-term weather forecast in the region important for the water level in the impounding basin as well as for the basin itself.

3. An experimental comparison of fuzzy numbers arithmetic

3.1. Elementary arithmetic operations

To automate computational experiments, authors of this study developed dedicated programme Ordered FN, which is also an efficient aid for graphic interpretation of operations being performed on ordered fuzzy numbers. Additionally, it is equipped with the Calc L-R module. Owing to that, it is possible to compare some operations on L-R fuzzy numbers. Ordered FN is equipped with an additional module which is started using the Calc L R button. It includes procedures to calculate the sum, the difference and the product of L R numbers. This allows to compare some results obtained from operation on L-R and OFN numbers. The introduced fuzzy number has a form of (m, α, β) where α and β are left and right-side dispersions. Shown here is an example of the software application. It includes the summary of simple arithmetic operation results in OFN and L-R notation. As shown in the attached figures (Fig. 6), subtraction of 3-3 is different for LR than for OFN numbers.

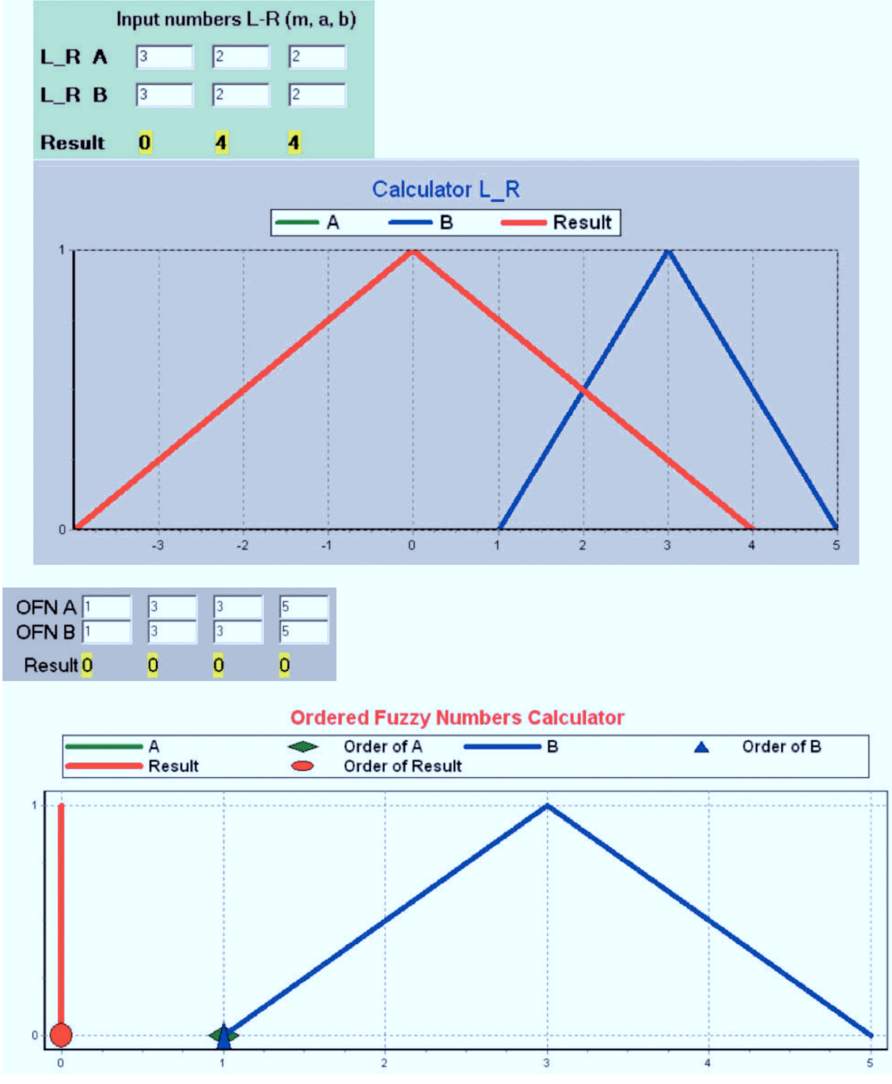

Figure 6. Comparison of the difference between the same numbers in L-R and OFN form.

In the first case (operation on OFN) the result was real zero. Whereas the second result (operation on L-R) amounts to fuzzy zero. Comparison of the results for subtraction of two numbers in L-R and OFN notation presented in the figure on. 8 leads to the conclusion that OFN subtraction reduced the fuzziness. One may easily notice that the carrier of the resulting number is smaller in case of fuzzy numbers. Thus a more precise application of calculation results, e.g. for control, is possible.

3.2. Comparison of calculations on L-R and OFN numbers

Majority of operations performed on L-R numbers, regardless if they are additions or subtractions, increase the carrier value. Hence performance of several operations on those numbers can cause such big fuzziness that the resulting quantity will be useless. It is impossible to solve an $A + X = C$ equation using L-R representation through operations because for that interpretation $X + A + (-A) \neq X$ and $X * A * A^{-1} \neq X$. Whereas every inverse operation on L-R numbers will increase the carrier. It is often impossible to solve the equations using analytical (computations) method either. However, it is possible to break the stalemate using certain empirical methods. The situation is totally different in case of operation on ordered fuzzy numbers. It is possible to solve the above mentioned equation using an analytical method.

Example 1. *Problem: There was a rapid surge of water in the impounding basin at night. The management of the dam has to send reports to the disaster recovery centre including the value of the water level change comparing to the previous state. Unfavourable weather conditions do not allow for precise measurement.*
Data:

- $A(1, 1, 2)$ – *previous measurement* $[mln\ m^3]$,
- $C(5, 2, 3)$ – *current measurement* $[mln\ m^3]$.

Mathematic interpretation: Hence the problem comes down to determining X *number that satisfies the equation* $A + X = C$, $A(1, 1, 2) + X = C(5, 2, 3)$

Solution versions:
Version I:
Solution using computational method of OFN arithmetic.

$$A[0, 1, 1, 3] + X = C[3, 5, 5, 8],$$
$$X = C - A,$$
$$X = [3, 4, 4, 5]$$

Verification:

$$A + X = C$$
$$A[0, 1, 1, 3] + X[3, 4, 4, 5] = C[3, 5, 5, 8]$$

The verification proved coherence of OFN calculations. Positive order of both numbers was assumed for cumulative character of both local and global phenomena. Then those numbers were presented in OFN form to perform calculations and visualisation using Ordered FN calculator (Fig. 7).

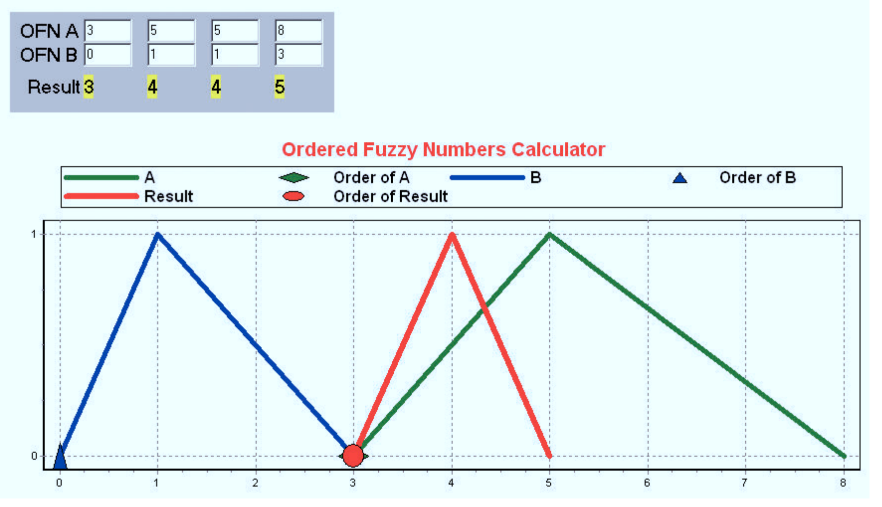

Figure 7. Solution of the equation $A + X = C$ using OFN.

Version II:
Solution using computational method of L-R arithmetic.

$$A(1,1,2) + X = C(5,2,3)$$
$$X = C - A$$
$$X = (4,4,4)$$

Verification:

$$A + X = C$$
$$A(1,1,2) + X(4,4,4) \neq C(5,2,3)$$

As a result of operations on L-R numbers we obtain the outcome $(4,4,4)$*. However, the verification through addition* $(A + X)$ *gives the result which is different from C. The correct result* $(4,1,1)$ *can only be achieved using the empirical method. It is problematic and not always feasible. The figure below* (Fig. 8) *shows the solution of the same operation for L-R numbers, obtained using Calc L-R module.*

Version III:
Summary of all solutions obtained by the computational method of OFN arithmetic using all versions of the order.

4. Conclusions

Operations performed on ordered fuzzy numbers are often more accurate than operations performed on classic fuzzy numbers. Results of operations performed on them

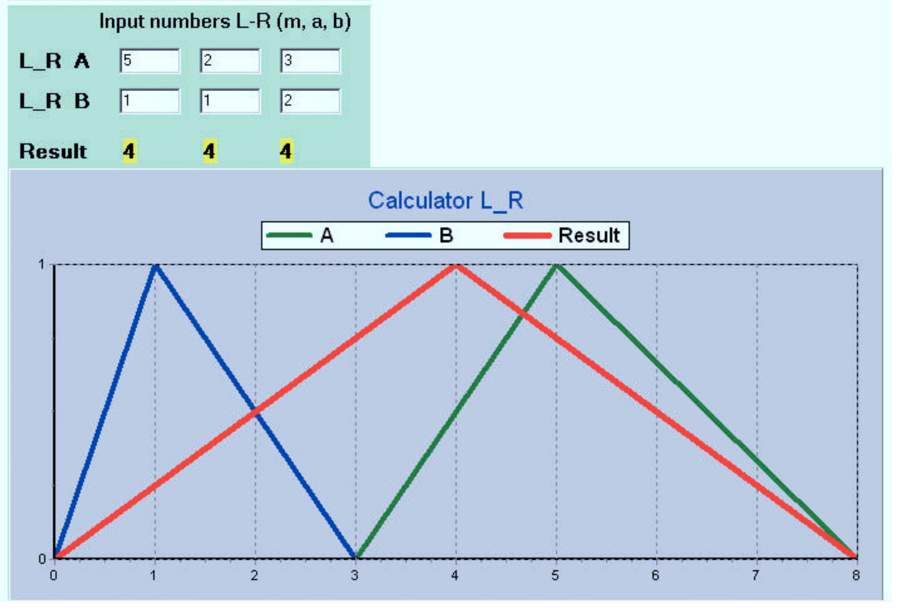

Figure 8. Solution of the equation $A + X = C$ using L-R numbers.

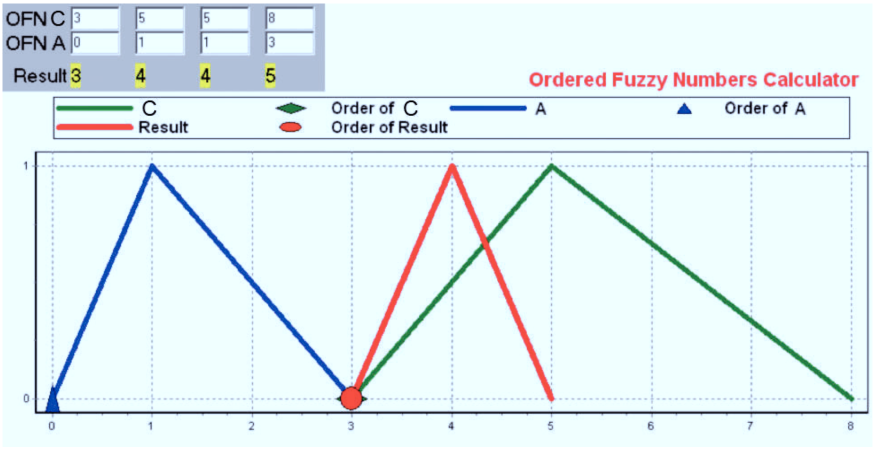

Figure 9. Results of $C[3, 5, 5, 8]$(positive order) $-A[0, 1, 1, 3]$ (positive order).

are the same as those obtained from operations on real numbers. Performing multiple operations not necessarily causes large increase of the carrier. The situation is different for L-R fuzzy numbers, where several operations often lead to numbers characterized by high fuzziness. An infinitesimal carrier is interpreted as a real number

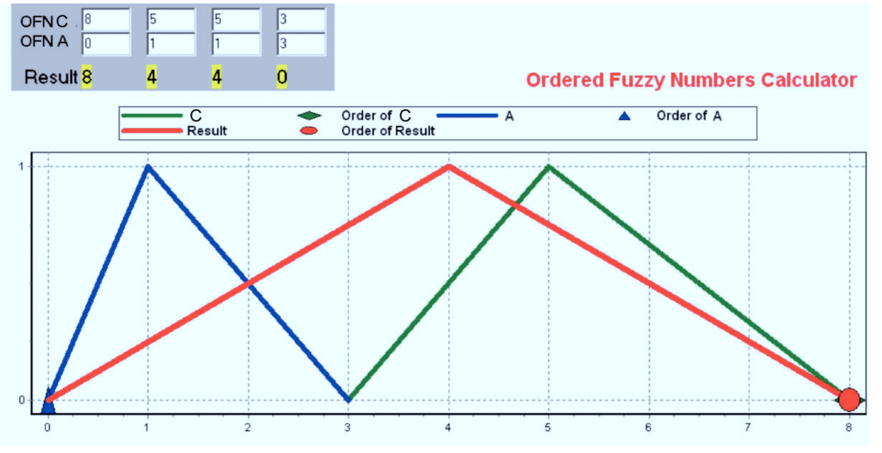

Figure 10. Result of $C[8,5,5,3]$ (negative order) $-A[0,1,1,3]$ (positive order).

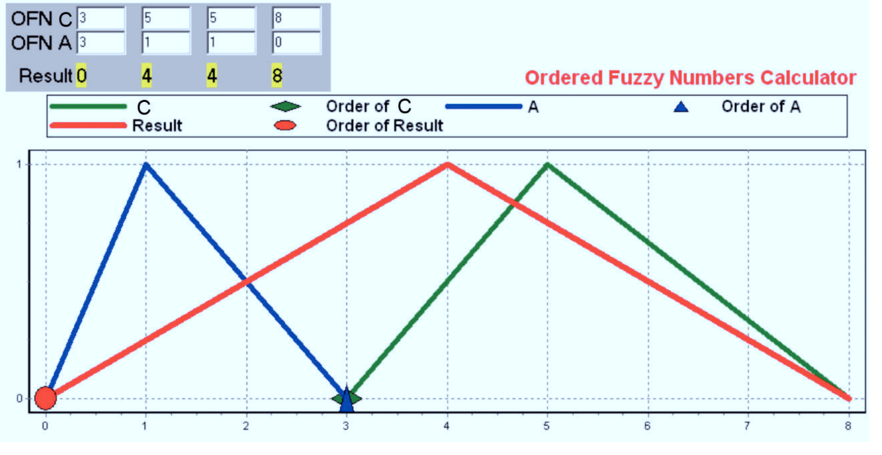

Figure 11. Result of $C[3,5,5,8]$ (negative order) $-A[3,1,1,0]$ (positive order).

and thus, for OFN numbers, one can apply commutative and associative property of multiplication over addition.

The possibility to perform back inference on them allows to reproduce input data by solving an appropriate equation. This very property is an added value that makes this fuzzy logic extension worth to promulgate. Calculations performed on ordered fuzzy numbers are easy and accurate. It is worth to mention the multiplication here, where the same procedure is used for all ordered fuzzy numbers regardless their sign. Whereas multiplication of L-R numbers is different for two positive numbers from that for two negative ones. Another completely different procedure is used for multiplication of numbers of indefinite signs and for fuzzy zeros.

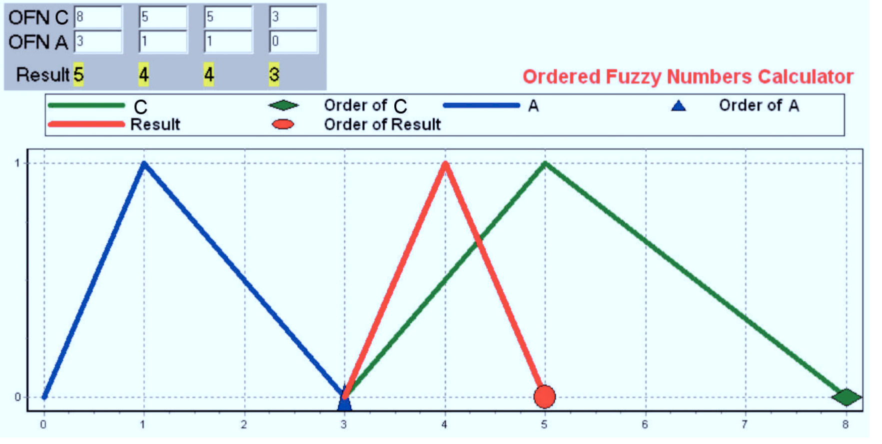

Figure 12. Result of $C[8, 5, 5, 3]$ (negative order) $-A[3, 1, 1, 0]$ (positive order).

It also seems to be very interesting to associate OFN numbers with the trend of changes taking place for studied part of the reality. We are convinced that new applications of this property of OFN, shown here in the example of the fuzzy observation of the impounding basin in unfavourable weather conditions, will be introduced with the passing of time. Hence it seems that introduction of OFN provides new possibilities to designers of highly dynamic systems. With this approach it is possible to define trend of changes, which gives new possibilities for the development of fuzzy control and it charts new ways of research in the fuzzy logic discipline. Broadening it by the theory of ordered fuzzy numbers seems to enable more efficient use of imprecise operations. Simple algorithmization of ordered fuzzy numbers allows to use them in a new control model. It also inspires researchers to search for new solutions. Authors did not use defuzzyfication operators in this paper, which in themselves are interesting subject of many researches. They will also contribute to development of the comparative calculator created here.

Although authors of this study do not fully share the enthusiasm of the creators of OFN as regards excellent prospects of this new fuzzy logic idea, but they are impressed by capabilities offered by the arithmetic operations performed using this notation. Even sceptics, who treat OFN with reserve as it is a generalisation of fuzzy logic, can benefit from this arithmetic. After all, OFN can be treated as an internal representation of fuzzy numbers (heedless of it's authors' intention). With this new kind of notation for fuzzy numbers and fuzzy control, it is possible to achieve clear and easily interpreted calculation, which can be arithmetically verified regardless of the input data type. Perhaps the OFN idea will become another paradigm of fuzzy logic, just like the object oriented programming paradigm has become dominant in software engineering after the structured programming paradigm. Whichever scenario wins, at least some aspects of OFN arithmetic seem to be hard to ignore a priori.

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