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DEFINITION AND INTERPOLATION OF DISCRETE METRIC FOR MESH GENERATION ON 3D SURFACES

The article concerns the problem of a definition of the control space from a set of discrete data (metric description gathered from different sources) and its influence on the efficiency of the generation process with respect to 2D and 3D surface meshes. Several methods of metric interpolation between these discrete points are inspected, including an automated selection of proper method. Some aspects of the procedures of creation and employment of the mesh control space based on the discrete set of points are presented. The results of using different variations of these methods are also included.

Keywords: *mesh generation, parametric surface, discrete metric, Delaunay triangulation*

DEFINICJA I INTERPOLACJA DYSKRETNEJ METRYKI DLA TWORZENIA SIATEK NA POWIERZCHNIACH TRÓJWYMIAROWYCH

Artykuł opisuje zagadnienie definicji przestrzeni kontrolnej (sterującej procesem generowania siatek) na podstawie dyskretnych danych (opisu metryki pozyskanego z różnych źródeł) oraz jej wpływu na wydajność procesu generacji siatek na płaszczyźnie oraz powierzchniach trójwymiarowych. Rozpatrywane są różne metody interpolacji metryki w obszarach pomiędzy dyskretnymi punktami ze zdefiniowaną metryką, włącznie z automatyczną metodą wyboru odpowiedniej metody interpolacji. Przedstawione są zagadnienia związane z procesem tworzenia i wykorzystywania przestrzeni kontrolnej opartej na informacji z dyskretnego zbioru punktów. Załączone są także przykładowe wyniki zastosowania różnych wariantów opisywanych metod.

Słowa kluczowe: *generacja siatek, powierzchnie parametryczne, dyskretna metryka, triangulacja Delaunaya*

1. Introduction

In many areas such as numerical computation or computer graphics there are used meshes, defined as an adequate partitioning of the modeled domain into elements with the required geometrical shape.

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An important issue during the construction of meshes is supervising the quality of the created elements with respect to their size and (optionally) stretching in the given direction. In the usual approach, the concept of metric [1, 2, 3, 4, 5] and control space [6, 7] is introduced. Similar method was used in the mesh generator developed by Authors [8], where meshes are created using the Delaunay property. The article concerns the problem of influence of the definition of the metric and of the control space on the efficiency of the generation process (with respect to 2D and 3D surface meshes).

Requirements regarding the shape and size of elements may come from various sources (curvature of surfaces or boundary curves, user specification, adaptation data from the simulation process, etc.). In order to use information about metric coming from multiple sources of different type, an uniform representation can be used in form of a set of discrete nodes. In case of metric specification defined in a continuous manner, an additional algorithm of selecting the proper set of discrete nodes is required, which represents the continuous space with expected precision.

The set of nodes with specified metric value is the basis of the construction of the control space. From the formal point of view, the control space is treated as a covering of the domain, divided into subdomains. Thus, the control space itself may be treated as a mesh structure. In the developed generator this structure can be stored in a various ways: regular grid (rectangular) or triangular mesh. In the second case, the points with the prescribed metric are used as nodes of this triangulation.

While creating or modifying the resultant mesh, the generator refers to the data stored in a particular elements of the control mesh. In the presented approach, the main information obtained from the control space is the value of metric defined in the nodes of the control space. In order to calculate the value of metric at any given point of the domain, it's necessary to adequately interpolate the metric between the control nodes. The appropriateness of the interpolation is mainly dependent on the selection of nodes.

In this article the following problems are described: metric definition and employment, procedure of creation of the mesh control space, and some methods of metric interpolation from a discrete set of points. The results of using different variations of these methods are also included.

2. Definition and Employment of Metric

The metric at any point of the domain is defined by means of the matrix M_s, which is used for transformation of the coordinates of points. The connection between the defined metric and the desired stretching (l_s, l_t) of element in the direction α is given as follows:

$$M_s = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} l_s & 0 \\ 0 & l_t \end{bmatrix} \tag{1}$$

where l_s is the required length of edges along the selected direction (given by α), and l_t is the required length of edges in the orthogonal direction.

For 3D surfaces given through a parametric mapping $u(s,t)$ an additional matrix M_p (calculated from the first fundamental form $\mathbf{I}(\mathbf{x},\mathbf{y}) \equiv \langle D\mathbf{p}(\mathbf{x}), D\mathbf{p}(\mathbf{y})\rangle$ of the parametric surface patch $\mathbf{p}(u,v)$, following the equation 2) has to be used to account for the potential distortion of the parameterization.

$$M_p M_p^T = \begin{bmatrix} \langle \mathbf{p}_u, \mathbf{p}_u \rangle & \langle \mathbf{p}_u, \mathbf{p}_v \rangle \\ \langle \mathbf{p}_u, \mathbf{p}_v \rangle & \langle \mathbf{p}_v, \mathbf{p}_v \rangle \end{bmatrix} \tag{2}$$

The resultant matrix M is then evaluated as a product of the two matrices M_s and M_p. Thus the defined matrix representation of the metric is used during the generation process for transformation of points coordinates.

2.1. Employment of Metric for Mesh Generation

If the parametric surfaces are used in the geometrical description of the modeled domain, each of them is discretized separately in its own parametric space. For each operation, which requires the evaluation of geometrical properties (length of an edge, area of an element, angles, etc.) the metric is calculated and used for transformation of coordinates of the mesh points.

During the construction of the mesh the metric transformation must be applied in the following phases:

1. *Generation of boundary nodes.* Nodes are placed iteratively, starting from one of the ends of the boundary segment. For each new node, a control space is used for establishing the metric for this point. The placement of the new node p_{i+1} is then established in a way, which should satisfy the formula:

   $$d(p_i, p_{i+1})_{M_a} = 1 \tag{3}$$

   where M_a is an average of metrics M_i and M_{i+1}.
   If any boundary is adjacent to several surface patches, the discretization process of such contour is performed in the parametric space and with the metric defined in one selected surface patch. After the discretization is complete, the placement of nodes is evaluated according to the metric defined in each of the adjacent surface patch and is refined if needed.
2. *Triangulation of boundary nodes.* The generation method used for creation of triangular mesh is based on the Delaunay property, which can be enforced in different ways. The first operation for each point inserted into the triangulation is searching for the containing triangle, which operation doesn't require metric specification. Next, one of the Delaunay retriangulation method is used (empty cavity or edge swapping). For both methods the proper metric (for the inserted point) is set at the beginning and treated as constant in the subsequent retriangulation steps.
3. *Insertion of inner nodes.* After the triangulation of all boundary nodes, the mesh is refined by insertion of inner nodes. During this operation, the elements with

the worst quality are selected for introduction of a new node. The metric is used for establishing the quality of triangles and calculating the coordinates of the new nodes to be inserted. Since the new node is being inserted in the circumcenter of the selected triangle, which can be quite distant from the corrected triangle, an additional check is made for conformity of the currently set metric and the metric at the inserted point. If these two metrics are too different, the insertion of point is abandoned.

4. *Smoothing.* The metric transformation is used for establishing the geometrical properties used by the smoothing procedures. Depending on the type of the method, the metric is retrieved from the control space for the coordinates of the inspected point (e.g. Laplace smoothing) or for the middle of the inspected elements.
5. *Conversion to quadrilaterals.* In the presented method of mesh generation, an indirect frontal method of conversion of the triangular mesh into the quadrilateral one is used. The value of the metric is set for the middle point of each front edge, used as a base of the new quadrilateral to be formed. The metric is being used for evaluating lengths of edges and inner angles, which directly influences the process of conversion. Additionally, the metric is used for determining the ordering of the front edges and during all operations of local smoothing in the neighborhood of the created quadrilaterals.

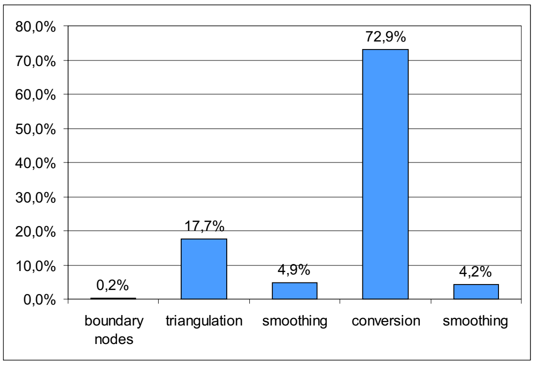

Fig. 1. Control space calls during different phases of mesh generation

Figure 1 presents typical percentage of calls to the control space for establishing the proper value of metric, at the subsequent stages of the mesh generation process.

3. Definition of Control Space

During the generation of the mesh, a separate control space is created and used for each surface patch. The control space provides the information about the desired size and stretching of elements throughout the discretized domain.

The control space is set up before the process of generation starts, and is used during all subsequent phases of mesh generation. The control space is defined basing on information gathered from different sources.

Depending on the user requirements, the following data can be used:

- curvature of the surface patches (calculated analytically or approximated),
- curvature of the boundary curves,
- user-defined size map (for discrete points, curves, subdomains or whole surface patch),
- adaptation data (usually given in discrete points — nodes of the mesh from the previous adaptation step),
- proximity of domain boundaries,
- small features.

3.1. Definition of Metric in the Nodes of Control Space

To increase the possibility of precise definition of size and stretching of elements throughout the domain, the metric can be defined in each discrete node in a few ways (similar approach was proposed in [9]):

- (P_i, M_i) — defines the precise metric M_i in the i^{th} control point, and is used together with the nearby control nodes for interpolation of the metric in some neighborhood (depending on the placement of control nodes and the selected method of interpolation);
- (P_i, M_i, r_i) — metric M_i is treated as constant in the neighborhood of the point P_i with the radius r_i. Outside this circle, the metric is interpolated together with the nearby control nodes;
- $(P_i, M_i, r_i, M_i', r_i')$ — in the point P_i and its neighborhood with radius r_i the constant metric M_i is assumed. Outside this radius the metric is gradually changed from M_i to M_i' in the range of the second radius r_i'. Outside the circle with radius $r_i + r_i'$, the metric M_i' is used for interpolation (see Fig. 2).

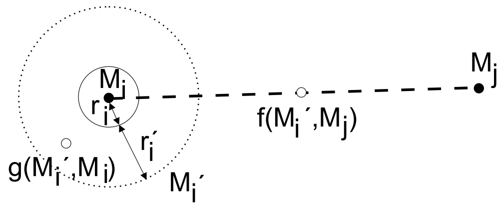

Fig. 2. Extended metric definition in discrete point

3.2. Construction of the Control Mesh

The process of constructing the control mesh can be divided into the following steps:

1. *Establishing the set of discrete points*, depending on the source of data.

2. *Triangulation of this set.* The triangulation method used here is identical with the one used for creation of the proper mesh, with metric set to identity. Additionally, for points placed too close to each other, the resultant metric is calculated as a minimum of both metrics, and is introduced in one point only.
3. *Refinement of the control mesh.* Since in some cases the given set of control nodes can be distributed in the domain in an irregular manner, an additional refinement procedure can be used. If required, additional nodes can be inserted into the control mesh, with the ascribed metric value calculated from the current control mesh. The goal of this operation is to increase the overall geometrical quality of the control mesh, which usually helps to locate the containing triangle of the inspected point in a possibly short time. Since searching of the containing triangle is the initial step of retrieving metric information from the control step, the overall efficiency of the generation process can be increased as well.
4. *Classification of the interpolation type.* Optionally, should different interpolation methods be used during the generation, all control triangles have to be classified according to the required interpolation type (Sec. 4.2).

4. Utilization of the Control Space

During the mesh generation, each time the local metric has to be established at the given coordinates, the proper value is retrieved from the control space. This operation can be summarized in the following manner:

1. *Proximity check.* If the coordinates of the inspected point are close enough to the point, where the metric was calculated most recently, the current metric is considered valid, and no further operation is needed. The maximum distance between points, which can be called "close enough" is calculated in the metric space (i.e. it's dependent on the required length of edges according to the current metric with a scaling factor). This proceeding is based on the assumption that the proper metric shouldn't significantly change over the single element defined by the metric itself.
2. *Search for the containing triangle*, which is done by traversing the triangles in the mesh in the direction of the given point. The additional quadtree structure (built during the triangulation step) is used for selection of good starting triangle.
3. *Check for the extended metric definition.* After the triangle is located, its vertices are examined. If the metric definition in one of the vertices of the triangle has a nonzero radius r or r' (see Sec. 3.1) and the inspected point is located within the 1 st or 2 nd neighborhood, the proper metric is immediately returned. If the point is located in such neighborhood of several vertices, the minimum metric is calculated.
4. *Interpolation.* Depending on the selected interpolation method (or the classification type of the triangle — for automatic mode), the proper set of vertices is gathered and the metric is calculated as described in Sec. 4.1.

4.1. Interpolation of Metric in the Control Mesh

If the metric should be interpolated from the control space data, the average matrix is calculated from the one of the following set of control nodes (Fig. 3):

V_s — single arbitrary vertex of the triangle,
V_t — all vertices of the triangle,
V_o — all vertices of all the triangles which have this point theirs circumcircles.

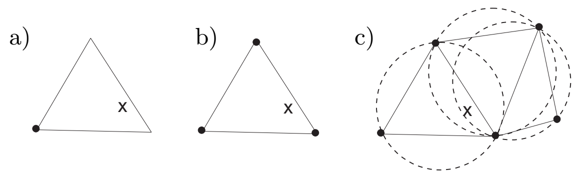

Fig. 3. Selection of vertices for metric interpolation: a) single vertex of the containing triangle V_s; b) all vertices of the containing triangle V_t; c) all vertices of all triangles containing inspected point (marked with $\times$) in theirs circumcircles V_o

After the proper set V_* of vertices is selected, the resultant metric in the point P is calculated following the formula:

$$M^P = \frac{1}{\sum \omega_i} \sum M_i \omega_i \tag{4}$$

where summation is carried out for all $i \in V_*^P$ and

$$\omega_i = d(P, P_i)^{-2} \tag{5}$$

4.2. Automatic classification of control triangles

The method of selecting the vertices used for the calculation of the average metric can be chosen arbitrarily for all triangles of the control space, or it can be automatically ascribed for the single control elements. The automatic selection allows to use the simplest interpolation method, which is still able to give proper metric for points within the control triangle.

If the interpolation method is to be chosen automatically, a preliminary classification of the control triangles is required after the triangulation of all control nodes. The classification is carried out by calculation of the metric in the representative points (middle point of the triangle, middle points of the triangle edges) within the triangle, using two different interpolation methods (M_t and M_o, calculated from sets V_t and V_o respectively). Next, the conformity coefficient of these metrics is assessed according to the Formula 6 [10].

$$\delta = \max D_{ij}, \quad \text{where} \quad D = M_o M_t^{-1} \tag{6}$$

If the established coefficient is smaller then a given accuracy ϵ for all tested points, the method M_t is selected as less costly. In such case the metrics in the vertices of the triangle are additionally compared, and if they all are identical (with the same conformity coefficient and accuracy as before), the simplest method M_s is selected for this triangle.

5. Results

Three different examples were elected for illustrating the described aspects of using discrete control space. In first two examples the 2D mesh was generated for a rectangular domain. However, the number and placement of control nodes with the defined metric was different in these cases. The third example shows mesh generated for 3D analytical surface, where various sets of discrete control nodes were generated based on the curvature of the surface.

The meshes were generated for various methods of selection of control nodes for metric interpolation, including the automatic classification of control mesh elements. The effects of control mesh refinements by insertion of additional nodes were also compared. Meshing times given in the statistics were obtained at 2.7 GHz Intel Pentium IV.

5.1. Example

Figures 4 and 5 present meshes generated for different methods of metric interpolation within the control mesh. The metric (identical for both meshes) is prescribed in four discrete points, marked on Figure 6a, where the isotropic metric M_1 defines edge lengths to be 10 times smaller then in the isotropic metric M_2. The created control mesh is presented on the Figure 6b.

a) b)

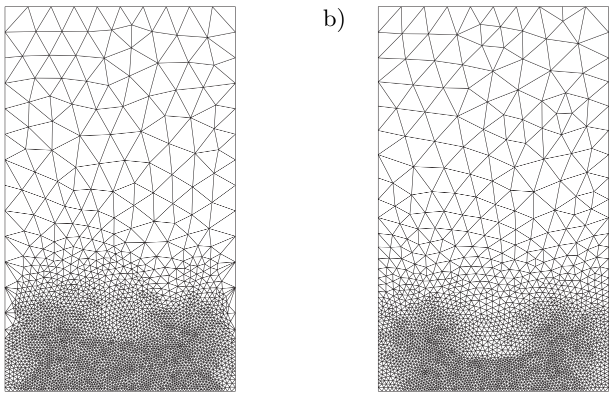

Fig. 4. Triangular meshes for metric interpolation from a) control vertices of V_t; b) control vertices of V_o

Table 1 gives the quality indicators of conformity of these meshes (both triangular and quadrilateral) to the defined control space, expressed by the edge lengths calculated in the local metric (average value μ and standard deviation σ).

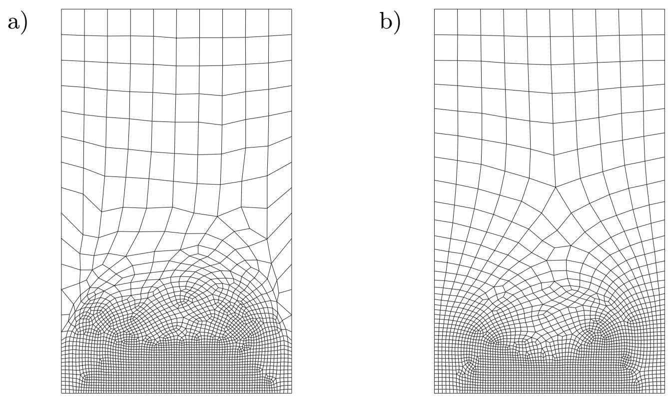

Fig. 5. Quadrilateral meshes for metric interpolation from a) control vertices of V_t; b) control vertices of V_o

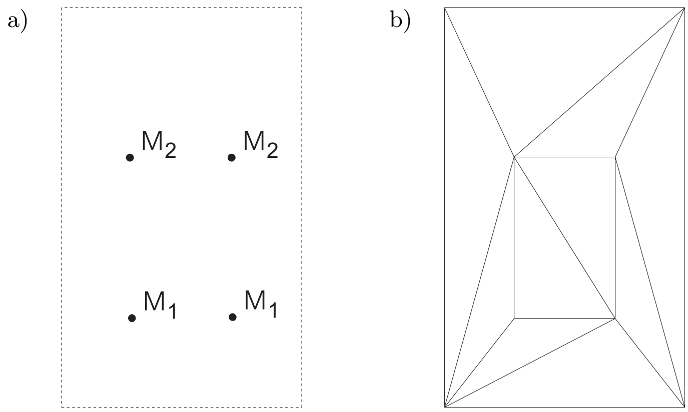

Fig. 6. Control space for meshes of Figure 4: a) set of control nodes; b) triangulation of control nodes

Two metric interpolation methods were used (based on sets V_t and V_o, denoted with proper subscripts) for both mesh generation and evaluation. NP and NE is the number of mesh points and elements in the resultant mesh.

Table 1
Statistics for meshes of Figures 4 and 5

	NP	NE	μ_t	σ_t	μ_o	σ_o
T_t	3063	5970	**1.046**	**0.17**	0.975	0.21
T_o	2730	5284	1.098	0.23	**1.044**	**0.15**
Q_t	2680	2602	**1.047**	**0.23**	0.979	0.25
Q_o	2641	2553	1.048	0.21	**0.986**	**0.19**

5.2. Example

Figures 7a and 8a present triangular meshes generated for relatively larger number of control points (prescribed by user with the extended metric definition (Sec. 3.1), placed in an irregular manner (in both cases the initial set of control points is identical). However, in case of mesh of Figure 8a the control mesh is additionally improved by insertion of inner nodes. The appropriate control meshes are presented on Figures 7b and 8b.

a) b)

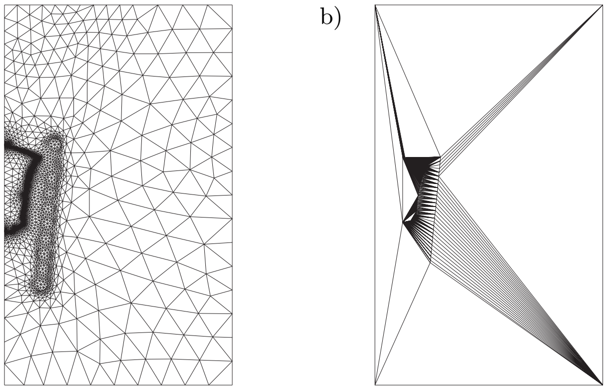

Fig. 7. Mesh generation for irregular distribution of control nodes: a) created mesh; b) control mesh

a) b)

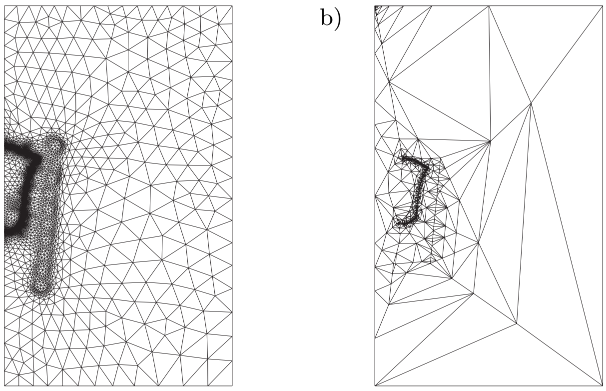

Fig. 8. Mesh generation for irregular distribution of control nodes with insertion of additional inner nodes: a) created mesh and b) control mesh

As can be seen on Figure 7b, nonuniform placement of control nodes may lead to creation of badly shaped triangles, which can decrease the efficiency of using this control space structure. Elongated triangles typically render the process of finding the containing triangle (which is done by traversing mesh triangles in the direction of the given point) more time consuming. Additionally, if the third method of interpolation is selected, the average number of vertices in the set V_o is increased.

Table 2
Statistics for meshes of Figures 7a and 8a created using various metric interpolation methods

	NT 10^3	NT/s 10^3	μ	σ	μ_o	σ_o
V_t	25.0	6.1	1.003	0.19	1.008	0.22
V_o	25.5	5.7	1.003	0.18	1.003	0.18
V_x	25.5	5.8	1.003	0.19	1.005	0.19
V_t+I	28.0	11.5	1.006	0.18	1.016	0.18
V_o+I	28.7	10.1	1.012	0.18	1.012	0.18
V_x+I	28.7	10.2	1.012	0.17	1.012	0.17

Table 2 presents the comparison of the statistics and generation time for meshes created with and without insertion of additional control points, and using different method of metric interpolation. V_x denotes selection of methods V_s, V_t and V_o based on the classification of single triangles of the control mesh. Letter I denotes using insertion of inner nodes into the control mesh. NT is the number of triangles in the resultant mesh, NT/s is the speed (number of triangles per sec.) of the mesh generation process. μ and σ are the mean edge length and standard deviation measured with the metric interpolated with method, which was used for mesh generation. Values of μ_o and σ_o are calculated using V_o metric interpolation method in all cases.

Table 3
Statistics for control meshes of Figures 7b and 8b

	m_s [%]	m_t [%]	m_o [%]	N_s [%]	N_t [%]	N_o [%]
V_x	0.6	75.0	24.4	4.3	80.1	15.6
V_x+I	1.9	73.4	24.7	11.3	55.0	37.7

Table 3 presents the results of classification of control mesh triangles, according to the required metric interpolation type (Sec. 4.2). Values m_s, m_t and m_o show the percentage of calls to the control space for establishing the metric value, where the appropriate metric interpolation types (V_s, V_t and V_o) were used, basing on the classification of the control triangles. N_s, N_t and N_o are the percentages of control triangles classified for various groups. In the second mesh, where additional inner control nodes were inserted, the total number of elements in the control mesh increased from 622 to 1466 triangles.

The number of triangles qualified for the third category (N_o) was larger in the control mesh with additional control nodes. However, the number of calls for the control space resulting in using this kind of interpolation (m_o) wasn't influenced much, which seems to be caused by the specific character of this example. On the other hand, the evident increase of the efficiency of the process of mesh generation for control space meshes refined with additional nodes can be observed in a majority of tests, especially for preliminary irregular placement of control nodes.

5.3. Example

Figures 9 and 10 present triangular meshes generated for 3D analytical surface defined by formula (7).

$$f(x,y) = 1.5\sin(2x)\cos\left(\frac{y}{5}\right)\exp\left(-\frac{x^2+y^2}{1000}\right), \quad x,y \in [-6,6] \tag{7}$$

The description of element sizing and stretching was automatically recognized from the surface curvature in a regular set of points, which was then used by the mesh generator as a set of control nodes with metric description.

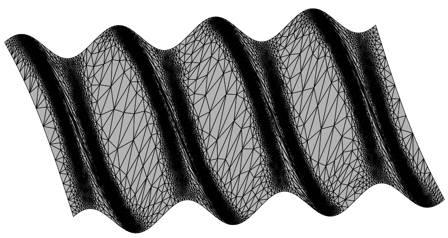

Fig. 9. Triangular mesh of an analytical surface

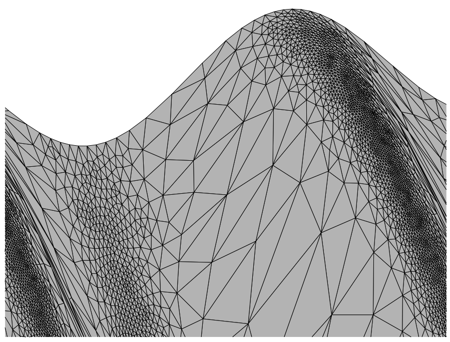

Fig. 10. Closeup of the mesh of Figure 9

Table 4 presents the results of mesh generation basing on various number (40 000, 10 000, 2500) of control nodes with metric description. Like before, V_x denotes method using the automatic classification of control triangles. The letter I denotes insertion of inner nodes into the control mesh, while R denotes version of the algorithm, where calls to the control space for obtaining the value of the proper metric may be skipped, if the considered point is close enough to the last point, where the metric was established. NP_c is the number of control nodes, NT number of triangles in the resultant mesh, NT/s and t_g are the speed and the overall time of the mesh generation process. Since the number of control points (and therefore the size of the control mesh) is relatively large, as compared with the size of the resultant mesh, the amount of time t_c required for the creation of control spaces is given.

Table 4

Statistics for the surface mesh of Figure 9

	NP_c 10^3	NT 10^3	NT/s 10^3	t_g $[s]$	t_c $[s]$	μ_o	σ_o
V_x+IR	40.0	42.7	4.48	9.5	2.63	0.985	0.18
V_x+I	40.0	42.8	3.24	13.2	2.70	0.984	0.18
V_o	40.0	42.7	3.05	14.0	1.33	0.983	0.18
V_x+IR	10.0	37.7	5.48	6.9	0.5	0.976	0.19
V_x+I	10.0	37.9	4.42	8.6	0.52	0.975	0.16
V_o	10.0	37.9	4.36	8.7	0.28	0.973	0.19
V_x+IR	2.5	25.8	6.62	3.9	0.11	0.943	0.22
V_x+I	2.5	26.1	5.37	4.9	0.11	0.938	0.22
V_o	2.5	26.0	5.27	4.9	0.06	0.938	0.22

Table 5

Statistics for control mesh used for the generation of mesh of Figure 9

	m_s [%]	m_t [%]	m_o [%]	m_r [%]	N_s [%]	N_t [%]	N_o [%]
V_x+IR	0.6	24.1	48.7	26.6	9.1	30.3	60.6
V_x+I	0.9	32.8	66.3	–	9.1	30.3	60.6
V_o	–	–	100.0	–	–	–	–
V_x+IR	0.2	1.6	71.6	26.6	5.2	4.2	90.6
V_x+I	0.2	2.1	97.7	–	5.2	4.2	90.6
V_o	–	–	100.0	–	–	–	–
V_x+IR	0.3	0.4	72.3	27.0	0.8	1.8	97.4
V_x+I	0.4	0.5	99.1	–	0.8	1.8	97.4
V_o	–	–	100.0	–	–	–	–

Table 5 presents the statistics for control meshes used for generation of meshes, described in Table 4. The additional m_r value shows the amount of calls, which were recognized obsolete because of the proximity of the subsequently inspected points. The values N_s, N_t and N_o show the percentage of control triangles classified for the appropriate metric interpolation method.

6. Summary

A visible influence of the selection of the control space on the efficiency of the generation process can be seen for the used method of the construction of the meshes. In the example 5.2 the automated introduction of additional control nodes allowed to nearly double the speed of overall triangulation process, while noting relatively low cost of the control space construction. For the first two examples (5.1,5.2) this time of control space creation didn't exceed 0.05s even in the worst case (for about 1500 control nodes).

For a large initial number of discrete control nodes the procedures of introduction of additional control nodes and classification of these triangles with respect to the used metric interpolation method can prove to be rather costly. But even in this case, the time of control mesh creation is much shorter then the time of the generation of the resultant mesh. In the worst case time of control space creation was 4 times lower then the generation time (for 40 000 control nodes, with classification of control triangles). In this case however, the density of the control space was evidently too large.

More attention should be focused on developing the method of the proper preliminary selection of the control nodes for cases, where the amount of control nodes with given metric is to numerous. However, such method shouldn't cause the loss of information, resulting in the decreasing of the quality of the resultant mesh. On the other hand it should be fast enough to justify its employment.

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